Woods Hole Oceanographic Institution
Woods Hole, Massachusetts

LONG RANGE SOUND TRANSMISSION
Interim Report No. 3

SOFAR POSITION TRIANGULATION

by

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SOFAR POSITION TRIANGULATION

A. THE INCOMING SIGNAL

Description

The first sounds ~~to~~ arrive(ing) at the receiver have ~~commence at~~ a low intensity and ~~with~~ long intervals between them. Gradually, as the intervals shorten, the sounds become louder until they blend into one loud sound which ceases abruptly[1]. This peak cut-off is so marked, that when recorded on suitable equipment synchronized with an accurate time piece, the instant of its occurrence may be determined within an accuracy of .01 seconds. In this report, all computations will be based on this interval as a limit of accuracy attainable.

True under any conditions? i.e. off the axis?

instant of cut off = .01 sec? No.

Duration of Signal

The theoretical duration of a signal from first arrival to peak cut off in the Atlantic Ocean is 1.20 seconds per 100 nautical miles of distance travelled. The recorded duration, however, is often less than this. This discrepancy may be due to the fact that, either the signal does not originate, or the receiver is not located there at the axial ~~axis~~ depth. Another reason for an obviously too short duration is that the first sounds to arrive are masked ~~to inaudibility~~ by ~~the inherent~~ noise level. It may be safely assumed that the observed duration time in seconds when multiplied by

not as sect. I data

"axial depth of 'sound channel'" - s.ch. first needs substantiation

[1] Long Range Sound Transmission. Interim Report No. 1 March 1, 1944 — January 20, 1945 by Maurice Ewing and J. Lamar Worzel Woods Hole Oceanographic Report No. 9. August 25, 1945.

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include an exhibit of G.P.W.'s curve

83.33 will give the minimum travel distance in nautical miles. Dr. G. P. Woollard has drawn a curve, based on data taken from actual receptions, from which travel distance may be closely estimated by the use of the time interval from the ~~beginning~~ point at which (1) ~~of~~ a marked increase in intensity ~~and frequency of the arriving~~ begins and (2) the intervals of silence ~~sounds~~ diminish to the peak cut-off.

It is the purpose of this report to discuss position triangulation from the standpoint of the difference in arrival times of the peak cut-off at three or more monitoring stations. The duration of the signal, at each station, is important as a check on the results thus obtained; or in the event of the signal being heard at only two stations, it may determine the point at which the signal originated. See later discussion on proposed Sofar Position Plotting Sheets, page 62.

Peak Cut-Off

The peak cut-off occurs at the instant the axial rays, i.e., those travelling the shortest path, + with minimum velocity reach the hydrophone[1]. The difference in time between the origin of the signal and the arrival of the peak cut-off at a monitoring station is known as the travel time. The axial sound velocity is the distance travelled divided by the travel time.

explain whether velocity is based on hyd. data or ? empirical data

B. MONITORING STATIONS

Description

not a description

~~A report on~~ the electronic gear, recording equipment, timing circuit and hydrophone construction, have been developed by the U. S. Navy

[1] Long Range Sound Transmission Interim Report No. 1 March 1, 1944 – January 20, 1945 by Maurice Ewing and J. Lamar Worzel Woods Hole Oceanographic Institution Report No. 9. August 25, 1945

Underwater Sound Laboratory at New London, Connecticut and a ~~and a~~ report on this work ~~report on this work~~ has been published[2].

Hydrophone Location

To gain the maximum ~~all possible~~ advantages of the use of the signal duration as discussed above, the hydrophone must be placed at axis depth. The reword

It must also be placed in such a position that it will cover as wide an aspect as possible without being shadowed by reefs, islands or sea mounts.

Its ~~It's~~ geographic position should be determined to an accuracy of ~~not more than~~ fifty feet. This figure is based on an average axial sound velocity of 4850 ft/sec and a measurable time factor of .01 seconds.

For a suggested method of determining the geographic position of a hydrophone see ~~please refer to~~ Appendix I.

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C. AXIAL SOUND VELOCITY

General

For purposes of position triangulation the difference in arrival times of the peak cut-off at three or more monitoring stations must be converted to difference of distance of the origin of the signal from each of two pairs of stations. This can only be determined if the axial sound velocity is accurately known.

Broadcast monitoring case!! p. 3 of G.P.W.'s manual

[2] Receiving, Recording and Timing Equipment for Ultra Long Distance Sound Ranging by W. B. Watkins and W. F. Saars., USN, USL. Completion Report No. 35., January 17, 1946.

Hydrographic Station Data

For areas in which hydrographic operations have been made, the axial sound velocity may be computed from existing data. This axial sound velocity is correct only in the vicinity of that hydrographic station and in large areas is subject to changes which must be averaged to determine a mean velocity for triangulation purposes.

Expand by discussing hydro-method of computing velocity

Question necessity for quoting Joe so fully?

Atlantic Ocean

For analyzing data obtained in the triangulation experiment, ~~to be~~ described later, a mean axial sound velocity of 4888 ft/sec was chosen. This value was obtained by M. Ewing and J. L. Worzel from thermal data covering that region of the Atlantic Ocean in which the experiment took place[1]. This value was further confirmed by hydrographic data obtained by the ~~USRV~~ R.V. Atlantis while enroute to and while occupying her monitoring station. The following account of ~~lowerings~~ the STATIONS made is quoted from notes written by J. L. Worzel, Scientist in charge.

specific reference

"The only ship equipped to make hydrographic observations was the Atlantis. Six Nansen reversing water bottles were used to determine the temperature depth relations, and the salinity determinations from the water samples were made at Woods Hole.

"A bathythermograph was rigged to make observations of the temperature depth relation to 700 fathoms. A standard 75 fathom BT was used with the modification of replacing the piston-bellows

[1] Long Range Sound Transmission. Interim Report No. 1. March 1, 1944 – January 20, 1945 by Maurice Ewing and J. ~~Lamar~~ Worzel, Woods Hole Oceanographic Institution. Report No. 9. August 25, 1945

pressure unit with a 4 1/2 turn bourdon pressure element. A slide holder was attached to the arm of the bourdon tube ~~at~~ about 6" from the center. The slide holder was skewed to about 45° in order to partially compensate for the pressure effect on the thermal element. The pressure scale of the instrument was calibrated in the pressure vessels at W.H.O.I. Woods Hole. There was insufficient time to calibrate the thermal scale adequately, so a partial calibration was made by comparing the water bottle readings with the Bathythermograph readings.

"On August 21, 1945, the 700 fathom BT was lowered twice from the Atlantis while underway at 3 knots. The traces were poor and the depths achieved were only 430 fathoms. Great difficulty was encountered due to the attempt to use the hydrographic winch as a BT winch. On August 24 the BT was lowered twice more, braking the drum slightly to prevent overrunning. The first lowering only reached a depth of 250 fathoms, but the second reached 550 fathoms before the wire became fouled. On August 26 the deep BT was lowered at two different times. The ship was slowed to 2 knots, and with about 1000 fathoms of wire out, depths of 600 and 650 fathoms were obtained. A reproduction of this deepest lowering while underway is shown in Plate 1. The near surface trace is poor due to the vibrations in the instrument while recovering it. The deep part of the trace shows a separation whose cause is unknown at present. The mean of the two traces appears to give data agreeable with that from water bottles.

"On August 27, 1945, the Atlantis reached the position where she was to anchor and a water bottle station was made in conjunction with the 700 fathom bathythermographs. Since only 6 water bottles were available, the data had to be obtained in three casts. Figure 5A is a plot of the water bottle data and the deep bathythermograph data for comparison purposes. The bottom point of each cast was used to assist in calibrating the 700 fathom bathythermograph, but no other points were considered.

"The agreement between the bathythermograph curves is within the possible fluctuations due to internal waves. Unfortunately the water bottle data did not overlap sufficiently to obtain any data on this possibility. The calibration of the instrument was not too well controlled, and the instrument was damaged before an adequate calibration could be obtained.

"Figures 5B and 5C give the temperature depth curves obtained at the other lowerings of the bathythermograph. These curves were converted to velocity depth curves by standard methods and these are plotted in Figure 5D. The extreme right hand curve ~~are~~ is the velocity data obtained from the water bottle observations. Note how characteristic the shape of the velocity depth curve is. None of the bathythermograph lowerings were sufficiently deep to determine the axis depth. They are good enough to show, however, that this instrument is feasible and that bathythermographs may be used to make observations to sufficient depths while the ship is underway. A more suitable winch and bathythermograph are being developed at present for future use.

"The depth of the sound channel axis at the Atlantis position was 640 fathoms. This is slightly less than that indicated (700 fathoms) by mean velocity depth curves for this portion of the ocean. The axial velocity was 4888 ft/sec in agreement with that obtained from the mean curves."

Pacific Ocean. O. W. Schreiber[3] in a report on axial sound velocity in the Pacific Ocean, points out that it varies from an average of 4845.3 ft/sec off California to an average of 4859.6 ft/sec off Hawaii. If we ~~quote~~ ignore the fact that he presents profiles showing that the rate of change in axial sound velocity is not uniform from Hawaii to California, thereby further complicating the problem, we may assume that the mean velocity value for this area is 4852.5 ft/sec. It is approximately 2000 miles between these (C + H) two points and, using this mean velocity, the travel time of a signal ~~travelling~~ traversing the entire distance would be 2506.40 seconds. If a signal originated 300 miles from California and 1700 miles from Hawaii the travel time from this point would be 2132.22 seconds to Hawaii and 373.82 seconds to California. Thus the signal's source must have been 1758.40 seconds travel time nearer California than Hawaii. This, when applied to 4852.5 ft/sec, would equal 1403.332 miles, an error of 3.332 miles.

revise

3 Long Range Sound Transmission. Interim Report No. 4. by O.W. Schreiber. Woods Hole Oceanographic Institution, July 29, 1946.

Conclusion

The use of average axial sound velocities to cover a wide area is accurate enough for air-sea rescue work but can not full~~y~~ take advantage of the possibilities of precise survey work offered by the equipment to be installed in the monitoring stations. It is therefore recommended that, as soon as there are three or more shore-connected hydrophones whose positions are known, attempts be made to gather empirical data on the axial sound velocity by the firing ~~of~~ bombs from ~~by~~ survey vessels ~~from~~ at known positions.

D. ATTEMPTS TO DETERMINE ~~DETERMINATION OF~~ AXIAL SOUND VELOCITY BY EMPIRICAL METHODS

The first SOFAR triangulation experiment was conducted in August and September 1945. The following account of the empirical methods used to determine the axial sound velocity for the analysis of data is quoted from excerpts of the notes made by J. L. Worzel, Scientist in Charge of the ~~USRV~~ "Atlantis."

Eleuthera

"The Eleuthera station was manned and operating at the time the plan of operations was made. All the personnel were alerted and instructed in their parts in the triangulation operations by radio and by mail. While the floating ~~other~~ stations were made ready and put to sea, Eleuthera repaired its ~~their~~ level recorders, which had been giving continuous trouble throughout all previous ~~their~~ operations and continued to do so until the station was abandoned, and made observations for the more accurate determinations of the location of the hydrophones. The personnel that operated the station were N. C. Steenland, M. Schalk, N. West, R. Dix, C.R.M., Dr. D. E. Kirkpatrick with Dr. J. A. Peoples in charge."

"Atlantis

"On the way out to the listening station, the USRV "Atlantis" fired charges (See Figure 1) three times a day in order to establish a figure for the axial velocity in the area in which the experiments were to take place. ......Bombs were dropped as shown in Figure 1, while the "Atlantis" was on drift station, to help give velocity data. All the navigation for the whole trip was the routine ship's navigation........

"Valor

"The USCG "Valor" fired four deep shots at four times to help obtain velocity data in the area. Since her position was accurately known by means of Loran, the distance to Eleuthera could be accurately determined and the velocity obtained. The positions at which bombs were dropped are circled in Figure 3......The personnel who operated the Valor station were Mr. William Saars with R. J. McCurdy in charge.

"Galaxy

"The U. S. S. "Galaxy" was provided with a hydrophone, amplifier, level recorder and chronometer to record the firing times of all shots, and with Mark 34 bomb cases, Mark 4 demolition blocks, and

Woods Hole detonators to make bombs.

"The Galaxy started to sea to commence her firing schedule, but ran into a storm which did quite a lot of damage to her and short-circuited her whole electrical system, so she had to return to port.

"Mason

"The U. S. S. Mason, DE 529, was obtained to replace the Galaxy as a firing ship. All the gear was moved from the Galaxy to the Mason, which immediately put to sea. This change only caused about two days delay in the schedule, since the Mason was ~~quite a bit~~ faster than the Galaxy.........

"Recordings were made of the time of release of the bombs and the direct sound and as many echoes as possible with the level recorder. This allowed accurate timing of the explosion instant, and a determination of the firing depth........

"The personnel on the USS Mason were C. R. Noll, C.R.M., with Mr. H. H. Robinson in charge of the bomb work.

"Velocity Determinations

"General

There are three types of data obtained in this work which may be used in determining velocity. The shots fired by the Atlantis allow determinations which are limited by the accuracy of the firing positions. These positions were obtained by ordinary good ships' navigation and dead reckoning. The shots fired by the Valor allow determinations again limited in accuracy by the navigation. The Valor, however, had Loran navigation. The triple reception shots with instant of explosion known, allow determinations limited only by the accuracy of the time measurements, by the necessary assumption of the same velocity on the paths to each receiver and by the position determination of all three stations.

"Atlantis velocity determinations

" From the Atlantis positions for each shot the distance to Eleuthera was calculated. (FOR FORMULAE USED SEE APPENDIX III) ~~using the standard solution for a spherical triangle. This is repeated below for convenience~~

Table I.

Experimental Velocity Values ~~from Atlantis Firing~~
at Shots Fired from RV ATLANTIS
Arrival recorded Eleuthera B.W.I. Lat. 24° 56.2', Long. 76° 06.3'

Shot	~~Lat.~~ ATLANTIS ~~Shot~~ Position (std. navigation) Lat.	~~Long.~~ Long	Elliptical distance (Naut. Miles)	Travel time (Seconds)	Velocity (ft/sec)	
322	35°-53.7'	65°-28.2'	900.1	1121.2	4881.1	
323	35-59.6	64.04.2	907.8	1121.7	4920.5	4892
324	35-34.5	63-46.2	901.4	1123.6	4877.6	4855
325	34-47.8	64-07.7	857.5	1066.5	4888.5	4879
326	33-10.9	64-41.5	775.4	961.0	4906.0	
328	32-18.3	64-22.0	752.6	943.3	4851.2	
329	32-04.6	63-38.7	784.1	970.9	4910.6	
330	31-39.0	62-11.0	838.1	1040.6	4897.0	
331	31-35.8	61-59.1	846.0	1049.9	4899.2	
332	31-23.4	61-27.3	865.9	1075.0	4897.5	
333	31-06.2	60-44.1	894.1	1114.3	4878.7	
335	30-36.4	59-16.8	955.5	1188.2	4889.7	4864
336	30-08.4	57-51.2	1019.5	1266.4	4894.4	
337	29-43.1	56-40.6	1074.6	1335.3	4893.3	
338	29-32.3	56-09.6	1099.3	1371.0	4875.2	
339	29-13.8	55-25.8	1134.8	1411.8	4887.3	
423	30-23.9	54-36.9	1187.6	1479.5	4880.7	4884
428	30-38.9	54-48.7	1180.5	1470.8	4880.2	

~~* An easier form for making these calculations has been brought to our attention by Mr. C. A. Speerl and is given in Appendix III.~~

pple rec

Shot	Lat.	Long.	Elliptical distance Naut. Miles	Travel time secs.	Velocity ft/sec
429	30-43.6	54-56.2	1175.1	1461.9	4887.6
446	30-48.6	55-05.3	1168.2	1456.2	4877.6
				Average	4888.7

"Valor velocity determinations

"The Valor positions should be much more reliable than the Atlantis positions since the Valor had Loran navigation.

"The reductions to velocity were made in the same way as those for the Atlantis given above. Table ~~3~~ II gives the positions and results of these computations.

Table ~~2~~ II

Experimental Velocity Values ~~from Valor Firing~~
Shots fired from USS VALOR
Arrivals recorded at Eleuthera, BWI Lat. 24° 56.2', Long. 76° 06.3'

Shot	~~Lat.~~ VALOR Position (LORAN) Lat.	~~Long.~~ Long	Elliptical distance Naut. Miles	Travel time secs.	Velocity ft/sec
345	38-27.3	64-28.7	1002.0	1244.3	4896.4
407A	39-17.3	63-06.2	1081.8	1345.2	4889.6
407	38-59.0	63-12.9	1064.6	1324.0	4889.2
420	38-08.7	64-24.6	990.0	1231.6	4887.6
				Average	4890.7

"Three position velocity determinations

"The best position for the three position fix as described in a later section was used as the best probable shot location. From this point (see figures 122, 123 a 124) the elliptical distances to the three stations were calculated as above. The measured distance was then subtracted from the calculated elliptical distance and the difference stepped off in the proper direction along the bearing line. The three isodistance lines then formed a triangle. These isodistance lines have then been drawn on the basis of a mean velocity which has been assumed to be the same to all three stations. By trial and error another velocity was chosen until the triangle became as small as desired. This velocity was then assumed to be the appropriate velocity to give the best position fixes. Table ~~5~~ III tives a resumé of the velocity and shot position for these shots.

TABLE III
Velocity Values from Three Station Fixes

Shot	Final Velocity	Final Shot Position Lat.	Final Shot Position Long.
371	indeterminate		
409	4874.6	29°04.9'	58°49.2'
410	4873.3	29°05.4'	58°49.8'
412	4857.9	28°29.3'	57°52.4'
	4868.6 mean		

"For shot 371 the triangle would not close within any reasonable velocity. The only conclusion which may be drawn is that some of the data is inaccurate for this shot.

Resumé of velocity determinations

"The mean velocities determined by the Atlantis firing and from the Valor firing largely confirms the figure of 4888 feet per second obtained from thermal data as the mean value for the axial velocity in this region of the Atlantic Ocean[1]. The values determined by the three position fixes were subject to errors of location of each receiving point. There are too few values determined by this method for the average to ~~satisfactorily~~ take care of the errors. Since the figures above confirm the mean velocity obtained from the temperature depth curves, the velocity of 4888 feet per second or .8039 mi/sec will be used for the velocity in this report."

[1] Long Range Sound Transmission Interim Report No 1. March 1, 1944 — January 20, 1945 by Maurice Ewing and J. Lamar Worzel. Woods Hole Oceanographic Institution Report No. 9. August 25, 1945

Atlantic Ridge Project 11-45

In a further attempt to confirm the axial sound velocity to be used as an average in the North Atlantic and to explore the effect of diffraction around sea mounts and islands the U. S. S. Muir with Wm. Von Arx in charge of Scientific data set sail ~~for~~ from Philadelphia in November 1945. Plate 2 shows her course and the positions of events 505 to 586 inclusive.

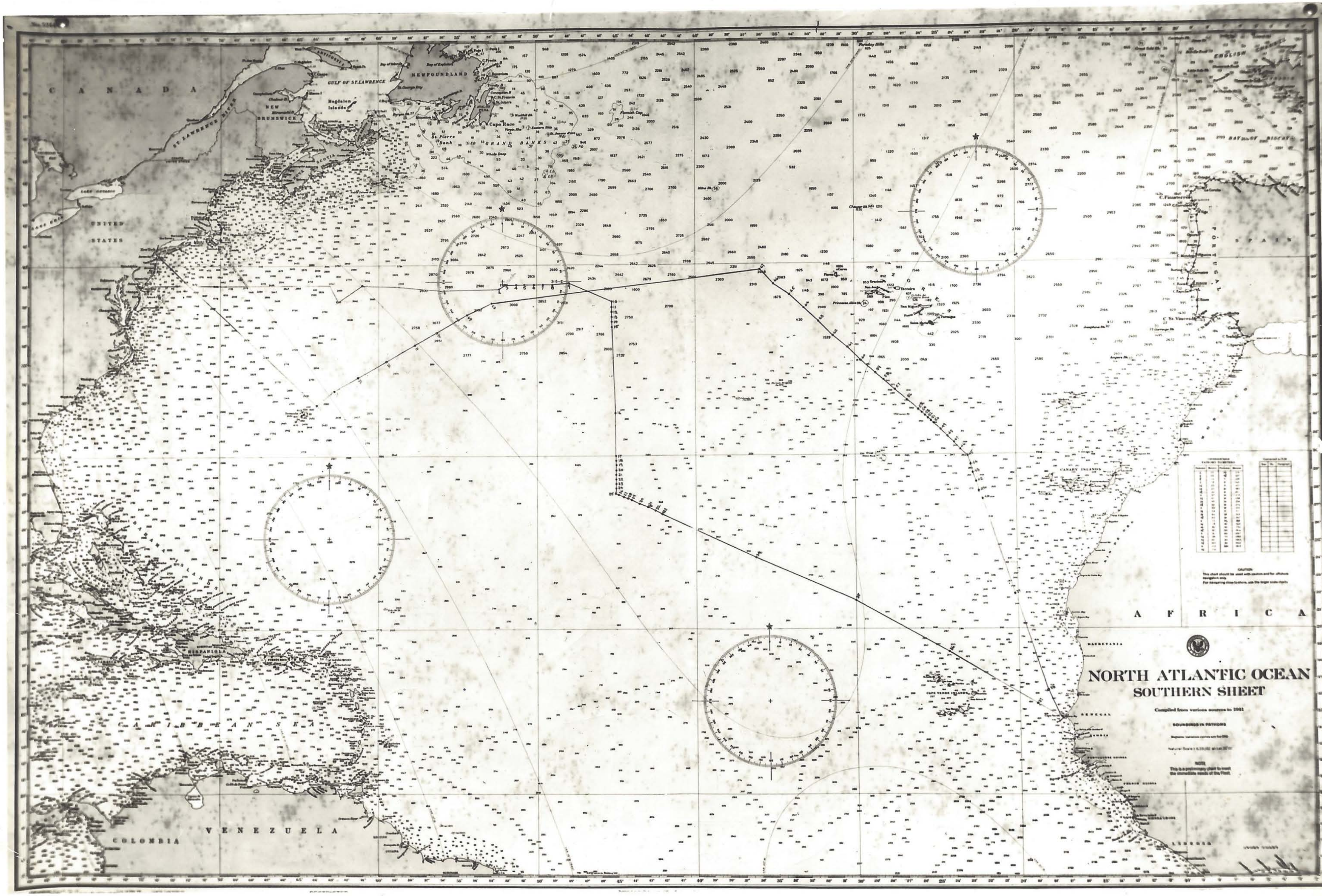
The U. S. S. Muir had a dual recording gear. Whem the sound of the explosion and ~~it's~~ its echoes were picked up by her fathometer they were fed on one ~~cirucit~~ circuit into a wire recorder and on another circuit into a Power Level Recorder. Both these instruments also recorded the time ticks of a break circuit chronometer. On the Muir's return the wire recordings were reproduced on an oscillograph and photographed. From these records it was possible to derive a most accurate record of the time and depth of explosion.

After the U. S. S. Muir had progressed about 1000 miles to sea the Loran fixes became less accurate and her positions had to be determined by astronomical observations when weather permitted and by Dead Reckoning at other times.

The station at Eleuthera was the sole monitor of ~~her~~ signals from the "Muir" so that no SOFAR fixes on her position could be obtained and there was no chance of determining axial sound velocity by the method explained on page ~~10~~. 12.

All events were first plotted at the positions reported by the U. S. S. Muir. The Great Circle distance to Eleuthera was

Plate 2 (Multi-colored)



* - see appendix

calculated by the sin-cos method with seven place tables[4] and a correction introduced to compensate for the earth's ellipticity. See appendix III. The velocities calculated from these distances averaged .80304 m/s (4882.72 f/s) but ranged between a high of .81061 m/s (~~4824.5~~ 4929.09 f/s)- shot ~~511~~ #534; and a low of .79346 m/s (4824.5 f/s) - Shot #511.

Feeling that an error in the Muir's reported position probably caused most of this wide velocity range, an attempt to replot her course with such of her navigational data as was available at Woods Hole, was made. All star fixes and running fixes of the sun were replotted and her course layed down from fix to fix. Shots fired between fixes were layed on that course at such DR positions as were appropriate. The position of each shot was then graded A, B, C or D as to its probable accuracy. For a description as to the method of grading each shot position please refer to Appendix II.

Of the 16 "A" shots, 9 were heard at Eleuthera. The Great Circle distances of these were calculated and compensated for the earth's ellipticity. The velocities, with the exception of #541 (see Appendix II), ranged from .80415 m/s (4889.5 f/s) to .80463 m/s (4892.3 f/s) and averaged .80440 m/s (4891.0 f/s).

All the Muir's reported positions were transferred to the replot sheet and approximate azimuths to Eleuthera were drawn from these positions. Using .80440 m/s as the probable velocity and applying this to the travel time, the distance to Eleuthera was computed. This distance was compared with the elliptical distance to Eleuthera from the Muir's reported position, the difference

4 Seven-Place Values of Trigonometric Functions. Compiled by Dr. J. Peters. D. Van Nostrand Company, Inc. New York 1942.

being the intercept on which a Sofar line of position was plotted.

The results of the plotting were not infallible but showed enough consistency to be encouraging. Some shot positions previously classified as "B" or "C" were found to be accurate. Others were not, but were so consistently in error that the writer believes they were correctly classified. The "D" shot (#537) had an intercept of but 0.277 miles from the Muir's reported position.

If 4891.0 f/s be accepted tentatively as the approximate velocity, the following conclusions may be drawn —

Where differences in position between Muir's plotting and Woods Hole replotting occurred, Woods Hole was nearer correct in 28 cases and the Muir in 11 cases. For computations, see Table V.

The position of shots #505, #506 and #507 obtained from Loran fixes do not check with Sofar.

Sofar places the positions of shots 508 to 518 inclusive well to the east of Muir's reported and Woods Hole replotted positions.

Shots #534, and #539 must have been fired well to the west of Muir's reported and Woods Hole replotted position although the intervening shots were close to the replot positions.

Shot #542 was fired east of reported position and 543 and 548 west of reported and replotted positions.

Shots #577 and #578 were fired east of reported position.

Table IV showing U. S. S. Muir's reported position, the replot position, probable accuracy of replot position, time of firing (corrected for interval between actual discharge and time of reception aboard ship), time of peak cut-off at Eleuthera and comments, follows:

TABLE IV

ATLANTIC RIDGE 11-45:-SHOT LIST

Shot No.	Date 1945	Reported Position Lat. (N)	Reported Position Long. (W)	Replot Position Lat. (N)	Replot Position Long. (W)	Quality of Replot Position	Firing Time	Reception at Eleuthera	Accuracy Tolerance	Comments
505	11/17	38-48.0	53-22.0	38-48.0	53-22.0	B	19-02-26.05	19-31-50.10	±0.1	
506		39-03.0	51-58.6	39-03.5	51-58.3	B	23-02-26.89	23-33-07.80	±0.1	
507	11/18	39-02.2	51-20.0	39-02.5	51-20.0	B	01-02-21.88	01-33-46.70	±0.2	
508		39-02.0	50-42.5	39-02.5 39-02.5	50-41.3 50-36.5	C	03-02-12.77	03-34-22.0	±2.0	
509		39-02.0	50-04.5	39-02.5 39-02.5	50-03.0 49-53.5	C	05-02-16.17	05-35-06.20	±0.1	
510		39-02.0	49-26.5	39-02.5 39-02.5	49-24.0 49-10.5	C	07-02-28.30	07-35-52.50	±0.1	See Appendix II
511		39-02.0	48-49.0	39-02.5 39-02.5	48-47.8 48-27.4	C	09-02-20.61	09-36-19.10	±0.1	
512		39-02.0	48-11.0	39-02.5 39-02.5	48-11.0 47-44.5	C	11-02-47.43	11-37-13.70	±0.15	
513		38-10.0	45-17.0	38-09.2	45-15.5	B	DUD			No Charted Reason
514		37-54.0	45-17.0	37-54.2	45-15.5	A	20-02-34.38	Not monitored		
515		37-39.0	45-17.0	37-39.2	45-15.5	A	21-02-38.90	Not monitored		
516		37-23.5	45-17.0	37-24.2	45-15.5	B	22-02-20.03	Not monitored		
517		37-08.5	45-17.0	37-09.2	45-15.5	B	23-02-32.87	23-38-41.50	±0.1	
518	11/19	36-52.5	45-17.0	36-54.2	45-15.5	B	00-02-36.53	00-38-39.40	±0.2	Geo. Reception
519		32-23.0	45-03.5	32-22.1	45-03.5	B	15-02-39.63	15-37-41.60	±1.0	
520	11/20	30-00.0	45-00.0	31-06.0	45-00.3	C	01-02-37.66	01-37-28.85	±0.01	
521		29-46.0	45-00.5	29-46.2	45-00.3	C	02-02-34.35	02-37-24.95	±0.05	
522		29-29.0	45-01.5	29-31.2	45-00.3	C	DUD			No Charted Reason
523		29-16.5	45-00.5	29-16.2	45-00.3	B	04-02-34.11	04-37-22.70	±0.1	
524		29-02.0	45-00.5	29-01.2	45-00.3	B	05-02-15.77	05-37-03.88	±0.05	
525		28-47.5	45-03.5	28-46.2	45-00.3	B	DUD			No Charted Reason
526		28-32.5	45-00.8	28-31.2	45-00.3	B	07-02-21.90	07-37-08.56	±0.1	
527		28-17.5	45-01.0	28-16.2	45-00.3	A	08-02-26.02	08-37-12.32	±0.1	
528		28-03.0	45-01.0	28-01.2	45-00.3	A	09-02-48.07	09-37-35.74	±0.1	
529		27-55.5	44-45.5	27-54.9	44-46.0	A	10-02-15.85	10-37-18.47	±0.05	
530		27-48.0	44-31.0	27-47.4	44-32.0	A	11-02-25.34	11-37-44.20	±0.1	
531		27-41.0	44-16.0	27-40.2	44-18.0	A	12-02-23.01	12-37-57.95	±0.05	
532		27-34.0	44-01.0	27-33.2	44-03.7	A	13-02-39.41	13-38-30.40	±0.5	

TABLE IV (cont.)

Shot No.	Date 1945	Reported Position Lat.(N)	Reported Position Long.(W)	Replot Position Lat.(N)	Replot Position Long.(W)	Quality of Replot Position	Firing Time	Reception at Eleuthera	Accuracy Tolerance	Comments
533	11/20	27-19.0	43-31.0	27-19.5	43-35.8	B	15-02-31.30	15-38-54.65	±0.05	
534		27-04.5	43-00.5	27-05.8	43-03.5	C	17-02-26.85	17-39-11.68	±0.05	
535		26-50.0	42-30.5	26-51.5	42-39.0	C	19-02-31.64	19-39-58.15	±0.1	
536		26-35.0	42-01.0	26-37.2	42-10.1	A	21-02-18.33	21-40-18.04	±0.1	
537	11/21	24-05.0	36-14.0	23-58.5	36-04.0	D	18-02-20.90	18-47-22.10	±0.1	
538	11/22	21.39.0	30-01.0	21-36.7	30-03.9	B	15-02-20.14	15-54-54.07	±0.05	
539	11/23	18-19.5	24-22.5	18-19.0	24-25.1	B	12-02-26.92	13-02-21.85	+0.1 -0.4	
540	11/24	15-27.0	18-47.0	15-23.0 / 15-19.8	18-43.3 / 18-50.5	B	09-02-42.91	Not Heard		Sal Island (Cape Verde Is.) Shadow
541	11/26	16-26.0	18-04.5	16-17.2	18-17.3	A?	20-02-44.33	21-10-36.40	±0.5	Fix 18 Mi. off course--5 hrs. out of Dakar
542	11/27	22-02.0	20-10.0	22-01.0	20-09.8	B	20-02-37.97	21-06-19.90	±0.2	Geophone Reception
543	11/28	27-28.5	22-06.0	27-28.5	54-00.3	C	20-02-23.09	21-02-12.8	±0.1	
544		27-58.0	22-17.0	27-52.0	22-17.0	D	DUD			No Charted Reason
545	11/29	28-29.5	22-29.0	28-26.7	22-28.2	D	DUD			No Charted Reason
546		28-58.0	22-38.0	28-55.4	22-38.8	D	02-02-42.27	Not Heard		Great Meteor Bank
547		29-27.0	22-49.0	29-25.0	22-50.0	C	04-02-50.28	Not Heard		Shadow
548		29-55.5	22-58.5	29-54.0	23-01.2	B	06-02-41.53	07-01-03.90	±0.1	
549		30-16.0	23-25.0	30-19.9	23-29.0	B	DUD			No Charted Reason
550		30-37.5	23-50.0	30-36.0	23-51.0	B	10-02-47.66	11-00-14.80	±0.2	
551		30-58.5	24-15.5	30-57.0	24-14.4	B	12-02-36.40	Not Heard		Possible Shoal Water at 31°-30' (N)40°-43°(W)
552		31-19.0	24-40.5	31-17.0	24-37.3	A	14-02-50.88	Not Heard		1150 fm @ 31°-41' (N) 41°-20'(W)
553		31-38.0	25-03.0	31-35.1	25-00.7	A	16-10-56.22	Not Heard		Possible Shoal Water at 32°(N)38°-43°(W)
554		31-48.0	25-14.0	31-45.0	25-12.5	B	17-02-32.93	Not Heard		Cruiser Bank Shadow
555		31-57.0	25-26.0	31-54.5	25-34.8	B	DUD			No Charted Reason
556		32-05.5	25-38.0	32-04.0	25-36.8	A	19-02-36.01	Not Heard		Cruiser Bank Shadow

TABLE IV (cont.)

Shot No.	Date 1945	Reported Position Lat.(N)	Reported Position Long.(W)	Replot Position Lat.(N)	Replot Position Long.(W)	Quality of Replot Position	Firing Time	Reception at Eleuthera	Accuracy Tolerance	Comments
557	11/29	32-14.5	25-50.5	32-14.6	25-51.0	A	DUD			No Charted Reason
558		32-25.5	26-05.0	32-25.1	26-04.8	A	21-02-41.27	21-57-32.60	±0.2	
559	NO SHOT FIRED									
560	NO SHOT FIRED									
561	11/30	32-57.0	26-47.0	32-57.1	26-45.8	B	00-02-32.17	Not Heard		Colorado Bank Shadow
562	NO SHOT FIRED									
563	NO SHOT FIRED									
564	11/30	33-29.5	27-29.5	33-28.5	27-26.0	B	03-02-41.54	Not Heard		Colorado Bank Shadow
565	NO SHOT FIRED									
566	NO SHOT FIRED									
567	11/30	34-01.5	28-12.0	34-00.0	28-07.7	B	06-02-36.20	Not Heard		Marsala Bank Shadow
568	NO SHOT FIRED									
569	11/30	34-22.0	28-40.5	34-21.7	28-34.7	B	08-02-57.91	Not Heard		1020 Fms @ 34°-20'(N) 34°-15' (W)
570	NO SHOT FIRED									
571	NO SHOT FIRED									
572	11/30	35-04.0	29-37.5	35-04.4	29-31.8	A	12-02-46.75	Not Heard		Possible Shoal Water at 35°(N)34°- 36°(W)
572a		35-45.0	30-27.0	{35-44.4 {35-44.4	30-27.0} 30-24.5}	B	16-02-39.83	Not Monitored		
573		36-23.0	31-22.5	36-24.2	31-22.5	A	20-02-54.36	Not Heard		Probable Shoal Water at 36°(N)33°-37°(W)
574	12/1	37-05.0	32-18.0	37-03.0	32-19.0	B	00-02-22.80	Not Heard		430 Fm. @ 37°(N) 33°-30'(W)
575		37-46.5	33-11.5	37-43.0	33-13.0	C	04-02-32.32	04-50-21.55	±0.05	
576		38-29.0	34-06.0	38-22.0	34-09.0	C	08-02-46.07	08-49-50.50	±0.2	
577		38-59.5	35-05.5	39-02.0	35-03.3	B	12-03-00.81	12-49-24.30	-0.0 +0.3	
578		39-43.0	35-56.5	{39-39.0 {39-41.0	35-58.5} 35-58.5}	B	16-02-49.86	16-48-32.25	±0.05	May be computed back from fix or forward from fix. No Data on course.

TABLE IV (cont.)

Shot No.	Date 1945	Reported Position Lat. (N)	Reported Position Long. (W)	Replot Position Lat. (N)	Replot Position Long. (W)	Quality of Replot Position	Firing Time	Reception at Eleuthera	Accuracy Tolerance	Comments
579	12/4	38-06.5	52-53.0	38-06.7	52-53.0	B	05-32-47.96	06-02-15.6	±0.5	
580		37-21.5	54-39.5	37-20.9	54-39.0	C	11-02-39.35	Not Heard		Bermuda
581		36-35.0	56-21.0	36-36.0	56-20.0	B	16-32-44.33	Not Heard		Shadow
582		35-56.0	57-45.0	35-56.0	57-47.0	A	22-02-44.62	Not Heard		Zone
583	12/5	35-06.5	59-31.5	35-07.0	59-32.0	B	03-32-50.00	Not Monitored		
584		34-17.0	61-17.5	34-17.0	61-17.5	B	09-02-44.15	Not Heard		Bermuda
585		33-34.0	62-49.0	33-34.0	62-49.0	B	14-32-58.27	Not Heard		Shadow
586		32-46.0	63-54.5	32-46.0	63-54.5	B	20-02-41.98	Not Heard		Zone

TABLE V

Shot No.	Velocity Computed from Muir's Reported Position	Replot Classification	Velocity Computed from Replot Position	Nearest to 4891.0 Ft/Sec.
505	4894.8	B	4894.8	Same
506	4903.1	B	4904.6	Muir
507	4876.9	B	4874.5	Muir
508	4844.2	C	4847.6	----
		C	4858.2	Replot
509	4826.5	C	4830.4	----
		C	4851.1	Replot
510	4825.9	C	4831.9	----
		C	4861.0	Replot
511	4824.5	C	4827.6	----
		C	4871.1	Replot
512	4839.8	C	4840.4	----
		C	4896.4	Replot
517	4870.3	B	4874.1	Replot
518	4871.1	B	4875.6	Replot
519	4889.9	B	4889.6	Muir
520	4887.7	C	4887.1	Muir
521	4886.0	C	4886.5	Replot
523	4887.9	B	4888.3	Replot
524	4888.1	B	4888.6	Replot
526	4890.0	B	4891.3	Replot
527	4882.1	A	4892.3	Replot
528	4887.8	A	4889.7	Replot
529	4893.1	A	4891.8	Replot
530	4893.0	A	4890.5	Replot
531	4895.0	A	4889.5	Replot
532	4896.3	A	4889.6	Replot
533	4900.0	B	4887.9	Replot
534	4929.1	C	4921.5	Replot
535	4912.5	C	4891.8	Replot
536	4914.2	A	4892.1	Replot

TABLE V (cont.)

Shot No.	Velocity Computed from Muir's Reported Position	Replot Classification	Velocity Computed from Replot Position	Nearest to 4891.0 Ft/Sec.
537	4891.6	D	4914.4	Muir
538	4896.4	B	4892.4	Replot
539	4905.1	B	4901.3	Replot
541	4895.4	A ?	4882.4	Muir
542	4878.7	B	4856.5	Muir
543	4900.8	C	4900.8	Same
548	4905.6	B	4901.8	Replot
550	4897.1	B	4895.9	Replot
558	4891.9	A	4892.2	Muir
575	4893.8	C	4892.2	Replot
576	4893.4	C	4886.6	Muir
577	4874.1	B	4880.0	Replot
578	4879.5	B	4874.9	Muir
		B	4875.8	Muir
579	4887.2	B	4887.6	Replot

Nearest to 4891.0 Ft/Sec:-

Replot in 28 cases
Muir in 11 cases
Same in 2 cases
41 cases

E. FUTURE VELOCITY DETERMINATIONS BY EMPIRICAL METHODS

Present plans include the installation of four monitoring stations covering the area between California and Hawaii. The hydrophones ~~of~~ at these stations will be shore connected and their permanent positions can be accurately determined. This will obviate one cause of the errors experienced in determining axial sound velocity by means of a three station fix. (See page 13).

When bombs were fired from the U.S.S. Mason, U.S.S. Muir and other vessels, the method of recording the time and depth of explosion was entirely adequate. In spite of extraordinary efforts to determine positions with more accuracy than is generally required for normal ship's navigation, the reported position for many events was open to question. It is recommended that the navigator of a survey vessel grade his estimates of shot positions ~~as~~ carefully, as was done in the replot of the Muir's ~~crust~~ ~~CRUISE,~~ cruise, so that signals originating at a point open to question may not be accorded the same importance as signals whose origin is positively known.

For instance, let us assume that a vessel in a reported position exactly 1000 miles from a monitoring station drops a bomb that is heard at that station 1252.98 seconds later. Then the average axial sound velocity is 4852.6 f/s. However, the presence of an error of only two miles in the vessel's reported position which is common enough one thousand miles out, would show that the average axial sound velocity might be from 4842.9 f/s to 4862.4 f/s. This is an even greater tolerance than that shown by available hydrographic data mentioned on page 7.

F. DESCRIPTION OF FIRST SOFAR POSITION TRIANGULATION EXPERIMENT

The following account of the first Sofar position triangulation experiment is quoted from excerpts of the notes written by J. L. Worzel, Scientist in charge of the USRV Atlantis.

"General. In a previous report[1], the reception of sound signals from small bombs at ranges up to 1000 miles was described. It was stated in that report that a signal heard by three stations could be triangulated and determine the firing ~~psotion~~ position of a small bomb. The Bureau of Ships, Navy Department, requested that a network of three stations be established to prove the feasibility of this type of triangulation as a preliminary step in establishing a network for locating survivors of plane crashes, and other survivors on the deep oceans.

"With the cooperation of the U. S. Navy Underwater Sound Laboratories and the Bureau of Ships, the Woods Hole Oceanographic Institution arranged such a triangulation experiment to commence about August 1, 1945. Since all details could not be arranged by this date the actual operations did not commence until about August 14, 1945.

"Eleuthera Island had been used as a monitoring station on the earlier work and was still in operation, so it was to be one corner of the reception triangle. The Atlantis, research vessel of the Woods Hole Oceanographic Institution, was to proceed to a point about 600 miles east of Bermuda to anchor and lower listening equipment, in order to establish a second corner of the

1 Long Range Sound Transmission Interim Report No1. March 1, 1944 — January 20, 1945 by Maurice Ewing and J. Lamar Worzel Woods Hole Oceanographic Institution Report No. 9. August 25, 1945.

triangle. The ~~U.S.C.G.~~ USCG Valor was to proceed to a position about 400 miles north of Bermuda, heave to and lower listening equipment, in order to establish the third corner of the triangle. This arrangement provided approximately an equilateral triangle 1000 miles to a side. The ~~U. S. S.~~ USS Galaxy was then to run a series of courses within the listening triangle firing at appropriate intervals. Plane flights from Miami to Bermuda to Porto Rico, and and from New York to Bermuda to the Azores were also arranged to drop additional bombs for the stations to monitor.

"<u>Eleuthera</u>. Of the 155 shots fired during these experiments, Eleuthera monitored for 154 of them. 137 shots were received in good order, 3 were in definite shadow zones, 2 were in probable shadow zones, and 12 were duds, possible duds, probable duds, or detonators only.

"<u>Atlantis</u>. New London had designed and built two sets of prototype gear[2] for Sofar listening stations and one set of this gear was put aboard the Atlantis prior to sailing ......On August 27....the ship was anchored in 3040 fathoms of water (see figure 1) and an AX58A hydrophone attached to the anchor cable so that it was at about the sound channel axis depth. The hydrophone was attached to a boom about 3 feet long, whose other end was lashed to the hydrophone cable. An uphaul and a downhaul were used to keep the boom perpendicular to the cable. The hydrophone cable was then lashed to the anchor cable at 400 foot intervals. This installation was quite noisy. On August 28 an attempt was made

2 Receiving, Recording and Timing Equipment for Ultra Long Distance Sound Ranging by W.B. Watkins and W.F. Saars. USN, USL Completion Report No. 35, January 17, 1946.

to recover the cable and the hydrophone attached to the anchor cable. With a great deal of difficulty about 500 feet of cable were recovered when darkness interfered and the cable and hydrophone were abandoned temporarily. The hydrophone cable had wound itself so tightly around the anchor cable that it could not be removed with any reasonable effort. A new hydrophone and cable were rigged over the stern and the surges largely eliminated with airplane shock cord. On August 29 the airplane flight from Miami to Bermuda to Porto Rico (see Figure 4) was monitored. Just before the last three shots of this flight, the anchor cable parted at the bow sheave from excess wear and the last three shots were monitored while drifting. The cable was then moved amidships and the ship brought about on the opposite tack. After this exchange the cable resistance fell way down and the cable was overhauled and repaired. There was a respite in the firing schedule at this time so the Atlantis moved farther North to Lat. 30°N in hopes the wind would be more moderate at higher latitudes.

"The ship was hove to and drifted for the remainder of the time she was listening. The positions at which she was located for the reception of each shot are shown in Figure 2. The noise level was very high most of the time but seemed to be much less between about 1900 and 2300Z or from 1500 to 1900Q. There was no discernible difference in the wind strength during this period than during the other period.

"During the noisy periods, the cable was pulled up and a preamplifier and hydrophone rig put on to try to raise the signal level above the noise level. This was tried twice, but failed both times as a very low frequency noise was always present. This was attributed to electrolysis in the water as the cable insulation was poor by the time the preamp was tried, and the power for the preamp was fed down the cable. The cable had to be repaired three times during the listening period as the insulation failed.

"Bombs were dropped as shown in Figure 1, while the Atlantis was on drift station, to help give velocity data. All the navigation for the whole trip was the routine ship's navigation. On September 4 the listening station was secured and the return trip begun. Of the 130 shots not fired by the Atlantis, 45 were recorded adequately, 43 were monitored but could not be detected above the background noise, 4 were monitored but were duds or probable duds, 21 occurred during cable repairs, 15 were not monitored due to lack of schedule information, and 2 were fired after the station was secured......

"Valor. A set of the New London prototype gear[2] was put aboard the Valor for additional tests. The Valor than proceeded to her assigned position. A modified AX58C hydrophone attached to a 5000 foot length of demolition cable was lowered down into the water. This rig was suspended from a spherical buoy which was secured to the Valor by a 200 foot manila line. The installation was completed approximately 1200Z on August 24 and monitoring commenced immediately. This rig lasted about a day until the wind shifted suddenly, causing the Valor to jibe and foul the buoy and line. While trying to clear the tangle, the hydrophone line parted and all the underwater gear was lost.

2. Receiving, Recording and Timing Equipment for Ultra Long Distance Sound Ranging by W. B. Watkins and W. F. Sears, USN., USL Completion Report No. 35., January 17, 1946.

"A second installation like the first was prepared, except~~ing~~ that only 3000 feet of cable was used in order to conserve the remaining cable supply. This rig was not put into operation until August 28 at 1400Z, so that there would be the greatest chance of operating during the actual triangulation tests. This rig received well to distances up to 500 miles, but at greater ranges the signal strength was too low compared with the noise level.

"On the morning of August 31 the cable was overhauled to put a shock mounting near the hydrophone and to add the remaining 2000 feet of cable. The hydrophone showed fish teeth marks ~~due to fish~~ on the shiny band around it when it was brought aboard. This band was painted over before the hydrophone was lowered again. Several insulation breakdowns occurred and had to be repaired before the hydrophone was again lowered into position and monitoring resumed at 2200Z.

"At 0730Z on September 3 an~~d~~ insulation failure was found. The cable parted while it was being overhauled, and most of the underwater gear was lost. This caused the Valor to cease operations as they had no more cable.

"The positions of the Valor at the time of each reception was determined with Loran. A plot of the Valor's location for each shot is shown in Figure 3.......

"Of the 100 shots fired while the Valor was on station, 4 were recorded adequately, 40 were received aurally and marked on the level recorder tapes so that their times could be determined quite accurately. One reception was noted at a time which can fit two shots equally well, 26 shots were monitored for which there was no reception, 4 shots were monitored with no reception, but they were probably shadowed by Bermuda, 8 shots were monitored which were duds, 14 shots occurred while cable repairs were being effected, and 1 shot was not monitored due to lack of schedule information.......

"Mason. The U.S.S. Mason, DE529, was obtained.....as a firing ship... Three types of bombs were dropped from the Mason. The one which was used most frequently consisted of a Mark 34 bomb case containing two Mark 4 ½ pound demolition blocks with a Woods Hole detonator inserted in one of them. The second type consisted of a case of Mark 4 demolition blocks, with three blocks removed and two Woods Hole detonators inserted in two demolition blocks. This formed a bomb which weighed about 48½ pounds. The third type consisted of a 300 pound Mark 6 depth charge with the normal firing mechanism replaced with two Mark 4 demolition blocks with a Woods Hole detonator inserted in each. The latter bombs only fired the two demolition blocks when it was tried.....

"The Mason fired 82 Mark 34 1 pound bombs, 4 48½ pound bombs and 1 300 pound Mark 6 depth charge. Only the booster charge fired in the depth charge, and in one case only the detonator fired in a

Mark 34 1 pound bomb. The positions at which each shot was fired are shown in Figure 4.

"New London. The overall control of the triangulation experiments and the communications were controlled from USNUSL at New London. The shooting schedules were arranged and transmitted to all parties from this location. The results from each shot were sent as soon as possible from each receiving station to New London where the rough plots of the receptions were made. These served as a check to control the ensuing work and to organize further tests as they were needed.

"The personnel responsible for the operations were Mrs. M.K. Ewing, Lt. E. L. Newhouse when he was not firing from planes, with Dr. M. Ewing in charge of the scientific work and Comdr. J. B. Knight in charge of communications.

"Planes. The first plane flight was arranged by Commander Loveland, U. S. N., Gulf Sea Frontier, Miami, Florida. The plane, a Liberator (PB4Y), had left Boca Chica, Florida, about 0900Z on August 29 and flew to Bermuda at an air speed of 171 miles per hour. It landed at about 1400Z, refueled, and then took off at about 1430Z, and flew to San Juan, Porto Rico, arriving about 2000Z. All the positions were obtained by dead reckoning and radio bearings. Two bombs were dropped with an interval of ten minutes between them after each hour of flight. The positions at which bombs were dropped are shown in Figure 5.

"The second plane flight was on an ATC flight from New York to Bermuda to the Azores. The flight left New York about 1400Z, August 31, and arrived at Bermuda about 1800Z. The flight left Bermuda about 2000Z, August 31, and arrived at the Azores about 0830Z, September 1. Four celestial fixes were obtained between Bermuda and the Azores. All other positions were obtained by dead reckoning. Bombs were dropped at half hour intervals whenever the plane was clear of the Palmetto Point shadow zone and over deep water. The plane flew at an air speed of 178 miles per hour and at an altitude of 7000 feet. The positions at which bombs were dropped are shown in Figure 5.

"Fixes and Lines of Position

Is Table VI necessary?

"~~General~~.

A list of the shots and receptions is given in Table VI.
In general three stations were not working at the same time due to the difficulties encountered on the boat receiving stations. Most of the time the firing ship was able to determine the instant of explosion (hereafter labelled T.B. after general geophysical practice). For plane flights and at times when the ship did not stop, no T.B. was obtained.

"In order to plot a position it was necessary to determine distance from the various sound travel times. To do this a sound velocity had to be used.......A ....figure was determined.... (4888 ft/sec, ~~(see page 13)~~ and this was used on all calculations in this report.

"To determine the shot position, distances and bearings were calculated to a position near the shot position using the HO214 tables. For those shots where a time break was available,

(continued on page 53)

TABLE ~~III~~ VI

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
A320	Aug. 20			Not received	37° - 28.0 (N) ~~28°-47.9' (N)~~ 66°- 38.0(W) ~~54°-29.1' (W)~~ Fired 14-39-22.1(GCT)
A321	Aug. 20			Not received	37°-07.5' (N) 66°-03.0' (W) Fired 20-13-51.3
A322	Aug. 21			02-44-59	36°-53.7' (N) 65°-28.2' (W) Fired 02-26-17.8
A323	Aug. 21			14-33-38.2	35°-59.6' (N) 64°-04.2' (W) Fired 14-14-56.5
A324	Aug. 21			20-34-13.3	35°-34.5' (N) 63°-46.2' (W) Fired 20-15-29.7
A325	Aug. 22			02-44-39.7	34°-47.8' (N) 64°-07.7' (W) Fired 02-26-53.2
A326	Aug. 22			14-22-07.8	33°-10.9' (N) 64°-41.5' (W) Fired 14-06-06.8
A327	Aug. 22			DUD	

TABLE VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
A328	Aug. 23			20-29-56.0	32°-18.3'(N) 64°-22.0'(W) Fired 20-14-12.7
A329	Aug. 24			02-39-48.5	32°-04.6'(N) 63°-38.7'(W) Fired 02-23-37.6
A330	Aug. 24		Not Received	14-17-36.1	31°-39.0'(N) 62°-11.0'(W) Fired 14-00-15.5
A331	Aug. 24		Not Received	15-46-04.1	31°-35.8'(N) 61°-59.1'(W) Fired 15-28-34.2
A332	Aug. 24		Not Received	20-17-45.6	31°-23.4'(N) 61°-27.3'(W) Fired 19-59-50.6
A333	Aug. 25		Not Received	02-42-11.8	31°-06.2'(N) 60°-44.1'(W) Fired 02-23-37.5
A334	Aug. 25		NOT MONITORED Cable Repair~~ed~~s	Not Received	30°-38.2'(N) 59°-23.1'(W) Fired 14-05-09.6 *

* Detonator Only.

TABLE ~~VII~~ VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
A335	Aug. 25		Not monitored Cable Repair~~ed~~s	15-13-06.7	30°-36.4'(N) 59°-16.8'(W) Fired 14-53-18.5
A336	Aug. 26		Not monitored Cable Repair~~ed~~s	02-44-55.1	30°-08.4'(N) 57°-51.2'(W) Fired 02-23-48.7
A337	Aug. 26		Not monitored Cable Repair~~ed~~s	14-30-51.4	29°-43.1'(N) 56°-40.6'(W) Fired 14-08-36.1
A338	Aug. 26		Not monitored Cable Repair~~ed~~s	20-16-45.1	29°-32.3'(N) 56°-09.6'(W) Fired 19-53-54.1
A339	Aug. 27		Not monitored Cable Repair~~ed~~s	02-47-20.9	29°-13.8'(N) 55°-25.1'(W) Fired 02-23-49.1
A340	Aug. 28		Not monitored Cable Repairs	Not Monitored	28°-38.9'(N) 54°-29.0'(W) Fired 02-34-24.0
M341	Aug. 28	37°-58.5'(N) 64°-54.5'(W) Fired 14-19-27.1	38°-22.8'(N) 64°-02.2'(W) Rec. 14-20-28.8	14-39-27.8	High Noise Level Not Received
M342	Aug. 28	37°-58.5'(N) 64°-54.5'(W) Fired 14-47-47-15.2	38°-22.8'(N) 64°-02.2'(W) Rec. 14-48-15.2	15-07-15.6	High Noise Level Not Received

TABLE VII (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M343	Aug. 28	37°-22.0'(N) 62°-57.0'(W) Fired 20-18-28.1	38°-22.4'(N) 64°-14.4'(W) Rec. 20-20-16.4	20-39-17.2	High Noise Level Not Received
M344	Aug. 28	37°-22.0'(N) 62°-57.0'(W) Fired 20-50-54.0	38°-22.4'(N) 64°-14.4'(W) Rec. 20-52-43.3	21-11-45.1	High Noise Level Not Received
V345	Aug. 29		38°-27.3'(N) 64°-28.7'(W) Fired 01-19-13.7	01-39-58.0*	Cable Repairs Not Monitored

* ± 1sec.

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24^{o}-56.2'(N) 76^{o}-06.3'(W)	Position of USRV Atlantis
M346	Aug. 29	36^{o}-56.0'(N) 60^{o}-55.0'(W) Fired 02-14.39.2	38^{o}-29.1'(N) 64^{o}-30.8'(W) Rec. 02-18-26.8	02-36-29.8	Cable repairs Not ~~Received~~ monitored
M347	Aug. 29	36^{o}-56.0'(N) 60^{o}-55.0'(W) Fired 02-46-06.4	38^{o}-29.1'(N) 64^{o}-30.8'(W) Rec. 02-49-54.6	03-07-56.9	Cable repairs Not ~~Received~~ monitored
M348	Aug. 29	36^{o}-19.5'(N) 59^{o}-16.5'(W) Fired 08-16-47.2	38^{o}-26.2'(N) 64^{o}-31.5'(W) Rec. 08-22-32.4	08-39-35.8	Cable repairs Not ~~Received~~ monitored
M349	Aug. 29	36^{o}-19.5'(N) 59^{o}-16.5'(W) Fired 08-49-17.1	38^{o}-26.2'(N) 64^{o}-31.5'(W) Rec. 08-55-02.9	09-12-06.2	Cable repairs Not ~~Received~~ monitored
M371	Aug. 29	36^{o}-19.9'(N) 59^{o}-13.5'(W) Fired 09-49-08.3	38^{o}-26.2'(N) 64^{o}-31.5'(W) Rec. 09-54-55.0	10-11-59.1	28^{o}-47.9'(N) 54^{o}-29.1'(W) Rec. 09-59-43.8
		Position of Plane			
P350	Aug. 29	26^{o}-03'(N) 74^{o}-12'(W) Fired 14-05-xx	38^{o}-27.8'(N) 64^{o}-37.7'(W) Not Received	Not Received	Not Received

TABLE ~~III~~ VI (cont.)

Event	Date	Position of Plane	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
P351	Aug. 29	26°-19'(N) 73°-46'(W) Fired 14-15-xx	38°-27.8'(N) 64°-37.7'(W) Rec. 14-41-00.0	~~Rec~~. 14-26-18.1	Not Received
P352	Aug. 29	27°-39'(N) 71°-39'(W) Fired 15-05-xx	38°-28.5'(N) 64°-38.3'(W) Not Received	~~Rec~~. 15-14-07.0	Not Received
P353	Aug. 29	27°-55'(N) 71°-13'(W) Fired 15-15-xx	38°-28.5'(N) 64°-38.3'(W) Rec. 15-29-29.2	~~Rec~~. 15-21-07.8	28°-47.9'(N) 54°-29.1'(W) Rec. 15-33-28.9
P354	Aug. 29	29°-18'(N) 69°-05'(W) Fired 16-05-xx	38°-29.3'(N) 64°-39.5'(W) Fired 16-15-24.9	~~Rec~~. 16-12-30.0	28°-47.9'(N) 54°-29.1'(W) Not Received*
P355	Aug. 29	29°-35'(N) 68°-40'(W) Fired 16-15-xx	38°-29.3'(N) 64°-39.5'(W) Rec. 16-26-23.4	16-24-32.4	28°-47.9'(N) 54°-29.1'(W) Rec. 16-30-47.9
P356	Aug. 29	31°-05'(N) 66°-38'(W) Fired 17-05-xx	38°-31.0'(N) 64°-40.8'(W) Rec. 17-15-13.1	17-18-33.4	28°-47.9'(N) 54°-29.1'(W) Rec. 17-19-42.5
P357	Aug. 29	31°-21'(N) 66°-11'(W) Fired 17-15-xx	Not Received	Not Received	Not ~~Received~~ MONITORED
P358	Aug. 29	30°-13'(N) 65°-00'(W) Fired 19-50-xx	38°-35.8'(N) 64°-44.7'(W) Not Received	Not Received	Not ~~Received~~ monitored

* At 1620 Atlantis heard a D. C.

TABLE ~~III~~ VI (cont.)

Event	Date	Position of Plane	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
P359	Aug. 29	29°-26'(N) 65°-13'(W) Fired 20-00-xx	38°-35.8'(N) 64°-44.7'(W) Not Received	20-14-14.3	28°-47.9'(N) 54°-29.1'(W) Rec. 20-11-57.4
P360	Aug. 29	28°-06'(N) 65°-13'(W) Fired 20-35-xx	38°-36.8'(N) 64°-46.1'(W) Not Received	20-46-13.0*	28°-47.9'(N) 54°-29.1'(W) Rec20-45-16.1
P361	Aug. 29	~~29~~ 27°-35'(N) 65°-34'(W) Fired 20-45-xx	38°-36.8'(N) 64°-46.1'(W) Not Received	Not Received	Not ~~Received~~ Monitored
P362	Aug. 29	26°-08'(N) 65°-33'(W) Fired 21-20-xx	38°-38.1'(N) 64°-46.2'(W) Rec. 21-37-45.6	21-33-47.8	28°-47.9'(N) 54°-29.1'(W) Rec. 21-35-05.1
P363	Aug. 29	25°-38'(N) 65°-42'(W) Fired 21-30-xx	38°-38.1'(N) 64°-46.2'(W) Not Received	21-40-52.1	Not ~~Received~~ Monitored
P364	Aug. 29	24°-00'(N) 65°-50'(W) Fired 22-05-xx	38°-39.9'(N) 64°-47.0'(W) Not Received	Not Received	Not ~~Received~~ Monitored
P365	Aug. 29	23°-30'(N) 65°-50'(W) Fired 22-15-xx	38°-39.9'(N) 64°-47.0'(W) Rec. 22-34-23.0	22-27-03.2	28°-49.9'(N) 54°-30.2'(W) Rec. 22-29-55.7

* No sharp cut-off $\pm$2 sec.

TABLE ~~III~~ VI (cont.)

Event	Date	Position of Plane	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
P-366	Aug. 29	21°-42'(N) 65°-50'(W) Fired 22-50-xx	38°-40.3'(N) 64°-48.0'(W) Not Received	22-59-50.3	monitored Not ~~Received~~
P367	Aug. 29	21°-05'(N) 65°-50'(W) Fired 23-00-xx	38°-40.3'(N) 64°-48.0'(W) Not Received	Not Received	28°-50.4'(N) 54°-31.1'(W) Rec. 23-16-08.4
P368	Aug. 29	19°-38'(N) 65°-58'(W) Fired 23-35-xx	38°-43.0'(N) 64°-49.0'(W) Not Received	Not Received	Not Received
		Position of USS Mason			
M372	Aug. 30	33°-27.2'(N) 60°-07.8'(W) Fired 02-03-36.6	Cable repairs Not ~~Received~~ monitored	02-23-59.8	Cable repairs Not ~~Received~~ monitored
M373	Aug. 30	33°-27.2'(N) 60°-07.8'(W) Fired 02-36-32.2	Cable repairs Not ~~Received~~ monitored	02-56-55.5	Cable Repairs Not ~~Received~~ monitored
M374	Aug. 30	32°-51.1'(N) 61°-51.8'(W) Fired 08-09-49.2	39°-10.0'(N) 64°-52.0'(W) Rec. 08-18-05.5	08-28-09.0	Cable Repairs Not ~~Received~~ monitored
M375	Aug. 30	32°-51.1'(N) 61°-51.8'(W) Fired 08-37-39.6	Not Received	08-55-58.9	Not ~~Received~~

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M376	Aug. 30	32°-26.6'(N) 63°-36.0'(W) Fired 14-33-28.4	Not ~~Received~~ Monitored	14-49-56.7	Cable repair Not ~~Received~~ Monitored
M377	Aug. 30	32°-26.6'(N) 63°-36.0'(W) Fired 15-09-09.6	39°-32.9'(N) 64°-43.2'(W) Rec. 15-18-00.6	15-25-36.88	Not Received
M378	Aug. 30	32°-26.6'(N) 63°-36.0'(W) Fired 15-59-48.6	39°-32.9'(N) 64°-43.2'(W) ~~Fired~~ Rec. 16-08-41.3	16-16-15.8	Not Received
M379	Aug. 31	32°-03.8'(N) 64°-05.3'(W) Fired 19-04-20.7	Cable repairs Not ~~Received~~ Monitored	19-20-11.5	29°-56.0'(N) 53°-59.4'(W) Rec. 19-15-25.4
M380	Aug. 31	32°-03.8'(N) 64°-05.3'(W) Fired 19-29-31.8	Cable repairs Not ~~Received~~ Monitored	19-45-20.6	29°-56.1'(N) 54°-00.1'(W) Rec. 19-40-35.3
		Position of Plane			
P381	Aug. 31	36°-11.0'(N) 68°-43.0'(W) Fired 20-34-xx	Cable repairs Not ~~Received~~ Monitored	20-51-49.0*	29°-56.8'(N) 54°-01.3'(W) Rec. 20-52-59.8**
P382	Aug. 31	35°-02.0'(N) 67°-35.0'(W) Fired 21-04-xx	Cable repairs Not ~~Received~~ Monitored	21-21-28.0	29°-57.2'(N) 54°-01.9'(W) Rec. 21-21-43.9

* Long Heavy Rumble No sharp peak 1 sec.
** D.C.? at 20-44-38-3

TABLE ~~III~~ VI (cont.)

Event	Date	Position of Plane	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
P383	Aug. 31	33°-50'(N) 66°-30'(W) Fired 21-34-xx	Cable repairs Not ~~Received~~ Monitored	21-47-37.3	29°-57.4'(N) 54°-02.3'(W) Rec. 21-46-44.3
P384a	Sept. 1	33°-00'(N) 62°-50'(W) Fired 00-00-xx	Not Received	Not Received	Not Received
P384	Sept. 1	33°-30'(N) 61°-10'(W) Fired 00-34-xx	39°-25.9'(N) 63°-08.8'(W) Rec. 00-40-21.2	00-51-51*	29°-58.6'(N) 54°-05.7'(W) Rec. 00-41-30.2
		Position of USS Mason			
M401	Sept. 1	31°-22.5'(N) 62°-43.0'(W) Fired 01-03-33.0	39°-23.5'(N) 63°-07.8'(W) Rec. 01-13-26.8**	01-20-15.3	29°-58.8'(N) 54°-06.2'(W) Not Received
		Position of Plane			
P-385	Sept. 1	34°-05'(N) 59°-25'(W) Fired 01-04-xx	39°-23.5'(N) 63°-07.8'(W) Rec. 01-13-26.8'(**	01-26-58.6	29°-58.8'(N) 54°-06.2'(W) Rec. 01-13-45.6

* Long Heavy rumble no sharp peak ±1 sec.

** This reception fits two events

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M402	Sept. 1	31°-22.5'(N) 62°-43.0'(W) Fired 01-13-55.6	39°-23.5'(N) 63°-07.8'(W) Not Received	01-30-37.9	Not Received
		Position of Plane			
P386	Sept. 1	34°-30'(N) 57°-40'(W) Fired 01-34-xx	39°-22.8'(N) 63°-07.4'(W) Rec. 01-44-08.3	01-59-01.4	29°-59.1'(N) 54°-07.0'(W) Rec. 01-43-06.2
P387	Sept. 1	34°-40'(N) 55°-30'(W) Fired 02-04-xx	Not Received	02-27-40*	29°-59.2'(N) 54°-07.6'(W) Rec. 02-09-30/6
P388	Sept. 1	34°-50'(N) 53°-40'(W) Fired 02-34-xx	Not Received	Not Received	29°-59.5'(N) 54°-08.1'(W) Rec. 02-37-53.1
P389	Sept. 1	35°-00'(N) 51°-45'(W) Fired 03-04-xx	39°-20.3'(N) 63°-06.3'(W) Rec. 03-14-04.9	03-31-02~~.14~~*	Not Received**
P390	Sept. 1	49°-50'(W) 35°-20'(N) Fired 03-34-xx	39°-19.5'(N) 63°-06.2'(W) Rec. 03-49-19.5	04-06-43.7	Not Received**
P391	Sept. 1	35°-40'(N) 48°-00'(W) Fired 04-04-xx	39°-17.4'(N) 63°-06.0'(W) Rec. 04-20-28.6	04-38-16.8	Not Received**

* No sharp peak ±1 sec.

** High Noise Level - See Text page 26.

TABLE ~~III~~ VI (cont.)

Event	Date	Position of Plane	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
P392m	Sept. 1	36°-45'(N) 47°-15'(W) Fired 04-34-xx	39°-17.4'(N) 63°-06.0'(W) Rec. 04-48-47.3	05-07-00*	Not Received**
P393	Sept. 1	37°-10'(N) 45°-40'(W) Fired 05-04-xx	39°-17.1'(N) 63°-05.9'(W) Rec. 05-17-49.9	Not Received	Not Received**
P394	Sept. 1	37°-40'(N) 43°-45'(W) Fired 05-34-xx	Not Received	Not Received	Not Received**
V407a	Sept. 1		39°-17.3'(N) 63°-06.2'(W) Fired 06-00-28.0	06-22-53.2 ±.2	Not Received**
P395	Sept. 1	38°-10'(N) 41°-55'(W) Fired 06-04-xx	39°-16.0'(N) 63°-06.0'(W) Rec. 06-21-51.7	Not Received	Not Received**
P396	Sept. 1	38°-25'(N) 39°-55'(W) Fired 06-34-xx	39°-15.6'(N) 63°-06.2'(W) Rec. 06-59-27.9	07-18-23.6	Not Received**
		Position of USS Mason			
M403	Sept. 1	30°-29.0'(N) 61°-22.0'(W) Fired 07-04-03.0	Not Received	07-21-41.5	Not Received**

* No sharp peak ±1 sec.
** High Noise level see text page 26

TABLE ~~III~~ VI (cont.)

Event	Date	Position of Plane	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
P397	Sept. 1	38°-45'(N) 37°-15'(W) Fired 07-04-xx	39°-15.3'(N) 63°-06.3'(W) Not Received	07-50-01.~~16~~ 6	Not Received*
		Position of USS Mason			
M404	Sept. 1	30°-29.0'(N) 61°-22.0'(W) Fired 07-13-46.0	39°-15.3'(N) 63°-06.3'(W) Rec. 07-24-42.0	07-31-24.4	Not Received*
		Position of Plane			
P398	Sept. 1	38°-50'(N) 35°-15'(W) Fired 07-34-xx	39°-14.1'(N) 63°-06.7'(W) ~~Fired 02~~ Rec 07-59-23.5	08-18-25.8	Not Received*
P399	Sept. 1	38°-50'(N) 33°-40'(W) Fired 08-04-xx	39°-14.1'(N) 63°-06.7'(W) Rec. 08-32-00.5	08-51-01.1	Not Received*

* High noise level. See text page 26.

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera $24^{\circ}-56.2'$(N) $76^{\circ}-06.3'$(W)	Position of USRV Atlantis
M405	Sept. 1	$29^{\circ}-40.0'$(N) $60^{\circ}-06.5'$(W) Fired 13-04-23.5	Not Received	13-23-09.8	Cable repairs Not ~~Received~~ monitored
M406	Sept. 1	$29^{\circ}-40.0'$(N) $60^{\circ}-06.5'$(W) Fired 13-33-24.2	$39^{\circ}-02.1'$(N) $63^{\circ}-10.5'$(W) Rec. 13-45-19.6	13-52-10.2	Cable repair Not ~~Received~~ monitored
V407	Sept. 1		$38^{\circ}-59.0'$(N) $63^{\circ}-12.9'$(W) Fired 14-57-~~33.3~~ 34.0	15-19-38.0	Cable repairs Not ~~Received~~ monitored
M408	Sept. 1	$29^{\circ}-24.5'$(N) $59^{\circ}-11.0'$(W) Fired 19-03-38.2	$38^{\circ}-51.1'$(N) $63^{\circ}-20.2'$(W) Rec. 19-16-41.4	19-23-32.9	Not ~~Received~~ monitored
M409	Sept. 1	$29^{\circ}-24.5'$(N) $59^{\circ}-11.0'$(W) Fired 19-34-41.7	$38^{\circ}-50.4'$(N) $63^{\circ}-21.0'$(W) Rec. 19-47-42.5	19-54-36.1	$30^{\circ}-08.6'$(N) $54^{\circ}-21.8'$(W) Rec. 19-39-42.8
M410	Sept. 1	$29^{\circ}-24.5'$(N) $59^{\circ}-11.0'$(W) Fired 20-22-00.6	$38^{\circ}-48.9'$(N) $63^{\circ}-22.3'$(W) Rec. 20-34-59.6	20-41-54.4	$30^{\circ}-09.4'$(N) $54^{\circ}-22.8'$(W) Rec. 20-27-01.5
M411	Sept. 2	$28^{\circ}-18.0'$(N) $57^{\circ}-32.0'$(W) Fired 01-03-30.3	$38^{\circ}-40.0'$(N) $63^{\circ}-31.0'$(W) Rec. 01-17-39.0	01-24-23.1	Not Received

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M412	Sept. 2	28°-18'(N) 57°-32'(W) Fired 01-33-39.9	38°-39.2'(N) 63°-32.2'(W) Rec. 01-47-47.2	01-54-32.39	30°-12.2'(N) 54°-25.8'(W) Rec. 01-37-59.4
M413	Sept. 2	26°-43.0'(N) 57°-33.0'(W) Fired 07-03-59.7	38°-29.0'(N) 63°-42.8'(W) Rec. 07-19-31.5	07-24-41.7	Not ~~Received~~ Monitored
M414	Sept. 2	26°-43.0'(N) 57°-33.0'(W) Fired 07-33-20.0	38°-29.0'(N) 63°-42.8'(W) Rec. 07-48-51.2	07-54-01.8	Not Received
M415	Sept. 2	26°-29.0'(N) 59°-29.0'(W) Fired 13-04-07.4	38°-13.3'(N) 64°-00.3'(W) Rec. 13-19-16.3	13-23-06.8	Not Received
M416	Sept. 2	26°-29.0'(N) 59°-29.0'(W) Fired 13-32-06.0	38°-13.3'(N) 64°-00.3'(W) Rec. 13-47-14.3	13-51-05.0	Not Received
M417	Sept. 2	26°-45.5'(N) 60°-01.5'(W) Fired 19-03-51.4	Not Received	19-22-06.6	30°-21.7'(N) 54°-35.6'(W) Rec. 19-11-11.0
M418	Sept. 2	26°-45.5'(N) 60°-01.5'(W) Fired 19-34-06.3	Not Received	19-52-21.0	30°-22.1'(N) 54°-35.6'(W) Rec. 19-41-27.9
M419	Sept. 2	26°-45.5'(N) 60°-01.5'(W) Fired 20-13-13.5	38°-09.0'(N) 64°-22.0'(W) Rec. 20-28-05.2	20-31-28.6	30°-22.2'(N) 54°-35.9(W) 20-20-33.9

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M1000	Sept. 2	26°-56.8'(N) 59°-51.7'(W) Fired 21-04-xx	Not Received	21-22-43.21	30°-22.8'(N) 54°-36.1'(W) Rec. 21-11-25.5
V420	Sept. 2		38°-08.7'(N) 64°-24.6'(W) Fired 21-35-05.91	21-55-37.51	Not Received
M1001	Sept. 2	27°-05.0'(N) 59°-35.6'(W) Fired 22-04-xx	Not Received	22-22-43.6	30°-23.1'(N) 54°-36.5'(W) Rec. 22-10-53.8
A423	Sept. 2		Not Received	22-56-46.7	30°-23.9'(N) 54°-36.9'(W) Fired 22-32-07.2
M1002	Sept. 2	27°-13.5'(N) 59°-29.9'(W) Fired 23-04-xx	38°-08.0'(N) 64°-28.2'(W) Rec. 23-18-36.4	23-22-57.7	30°-24.0'(N) 54°-37.2'(W) Rec. 23-10-34.0
M1003	Sept. 3	27°-21.0'(N) 59°-04.3'(W) Fired 00-04-xx	Not Received*	00-23-20.3	30°-25.0'(N) 54°-38.1'(W) Rec. 00-10-19.6
M424	Sept. 3	27°-30.0'(N) 58°-48.6'(W) Fired 01-03-45.0	Not Received*	01-23-14.8	30°-25.7'(N) 54°-39.1'(W) Rec. 01-09-41.7

*See text page 28

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M425	Sept. 3	27°-30.0'(N) 58°-48.6'(W) Fired 01-33-30.6	Not Received*	01-52-59.9	30°-26.1'(N) 54°-39.7'(W) Rec. 01-39-28.1
M1004	Sept. 3	27°-30.8'(N) 58°-48.8'(W) Fired 02-04-xx	Not Received*	02-23-44.9	30°-26.5'(N) 54°-40.1'(W) Rec. 02-10-12.1
M1005	Sept. 3	27°-39.2'(N) 58°-33.0'(W) Fired 03-04-xx	Not Received*	03-23-39.2	30°-27.4'(N) 54°-41.1'(W) Rec. 03-04-24.9
M1006	Sept. 3	27°-48.1'(N) 58°-16.8'(W) Fired 04-04-xx	Not Received*	04-23-21.6	Cable repair Not Received**
M1007	Sept. 3	27°-57.3'(N) 57°-00.8'(W) Fired 05-04-xx	Not Received*	05-24-36.6	Cable repair Not Received**
M1008	Sept. 3	28°-06.6'(N) 57°-44.2'(W) Fired 05-04-xx	Not Received*	06-26-27.8	Cable repair Not ~~Received~~ monitored
M426	Sept. 3	28°-15.0'(N) 57°-30.0'(W) Fired 07-04-01.2	Not Received*	07-25-05.4	Cable repair Not ~~Received~~ monitored
M427	Sept. 3	28°-15.0'(N) 57°-30.0'(W) Fired 07-33-44.3	Cable lost Returned to base ~~Not Received~~	07-54-48.1	Cable repairs Not ~~Received~~ monitored

* See text page 28 ** High noise level. See text page 28

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M1009	Sept. 3	28°-16.2'(N) 57°-27.1'(W) Fired 08-04-xx	Not Received	08-24-48.1	Cable repairs Not ~~Received~~ monitored
M1010	Sept. 3	28°-24.6'(N) 57°-10.9'(W) Fired 09-04-xx	Not Received	09-25-55.0	Cable repairs Not ~~Received~~ monitored
M1011	Sept. 3	29°-08.1'(N) 55°-45.9'(W) Fired 14-04-xx	Not Received	14-26-51.1	30°-33.6'(N) 54°-45.8'(W) Not Received*
M1012	Sept. 3	29°-15.0'(N) 55°-28.2'(W) Fired 15-04-xx	Not Received	15-27-23.8	Not Received*
M1013	Sept. 3	29°-23.0'(N) 55°-10.8'(W) Fired 16-04-xx	Not Received	16-27-49.2	Cable repairs Not ~~Received~~ monitored
M1014	Sept. 3	29°-30.2'(N) 54°-53.4'(W) Fired 17-04-xx	Not Received	17-28-06.5	Cable repairs Not ~~Received~~ monitored
M1015	Sept. 3	29°-37.8'(N) 54°-36.0'(W) Fired 18-04-xx	Not Received	18-28-35.0	Cable repairs Not ~~Received~~ monitored
M436	Sept. 3	29°-45.0'(N) 54°-19.0'(W) Fired 19-03-57.8	Not Received	19-28-40.8	Cable repairs Not ~~Received~~ monitored

* High noise level - See text page 28

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M437	Sept. 3	29°-45.0'(N) 54°-19.0'(W) Fired 19-34-27.0	Not Received	19-59-09.9	Cable repairs Not ~~Received~~ monitored
M1016	Sept. 3	29°-47.9'(N) 54°-34.0'(W) Fired 20-04-xx	Not Received	20-28-00.9	Cable repairs Not ~~Received~~ monitored
M1017	Sept. 3	29°-52.2'(N) 54°-42.7'(W) Fired 21-04-xx	Not Received	21-28-23.2	Cable repairs Not ~~Received~~ monitored
M1018	Sept. 3	29°-56.6'(N) 55°-11.0'(W) Fired 22-04-xx	Not Received	22-24-43.1	Cable repairs Not ~~Received~~ monitored
A428	Sept. 3			23-01-59*	30°-38.9'(N) 54°-48.7'(W) ~~Rec.~~ Fired 22-37-28.2
M1019	Sept. 3	30°-00.7'(N) 55°-28.7'(W) Fired 23-04-xx	Not Received	23-28-23.7	Cable repairs Not ~~Received~~ monitored

* Peak accurate to ±1 sec.

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	~~Position of USCG Valor~~	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M1020	Sept. 4	30°-03.8'(N) 55°-42.6'(W) Fired 00-04-xx	~~Not Received~~	00-27-36.6	Not Received*
M438	Sept. 4	30°-14.0'(N) 56°-05.0'(W) Fired 01-03-55.1	~~Not Received~~	Not Received	Not Received*
M439	Sept. 4	30°-14.0'(N) 56°-05.0'(W) Fired 01-33-49.6	~~Not Received~~	01-56-51.0	Not Received*
M1021	Sept. 4	30°-08.9'(N) 56°-07.9'(W) Fired 02-04.xx	~~Not Received~~	02-26-27.9	Not Received*
M1022	Sept. 4	30°-12.8'(N) 56°-25.1'(W) Fired 03-04-xx	~~Not Received~~	03-26-04.6	30°-40.7'(N) 54°-51.6'(W) Rec. 03-05-13.3
M1023	Sept. 4	30°-16.4'(N) 56°-42.6'(W) Fired 04-04-xx	~~Not Received~~	04-25-57.6	30°-41.2'(N) 54°-52.6'(W) Rec. 04-05-38.5
M1024	Sept. 4	30°-20.2'(N) 56°-59.4'(W) Fired 05-04-xx	~~Not Received~~	05-25-57.1	30°-41.6'(N) 54°-53.0'(W) Rec. 05-06-11.1
M1025	Sept. 4	30°-24.0'(N) 57°-17.0'(W) Fired 06-04-xx	~~Not Received~~	06-26-04.5	30°-42.0'(N) 54°-53.9'(W) Rec. 06-06-51.9

* High noise level. See text page 28

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	Position of USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M440	Sept. 4	30°-26'(N) 57°-28'(W) Fired 07-03-20.5	Not Received	07-24-55.1	Not Received*
M441	Sept. 4	30°-26'(N) 57°-28'(W) Fired 07-33-10.2	Not Received	07-54-44.1	Not Received*
M1026	Sept. 4	30°-26.9'(N) 57°-30.2'(W) Fired 08-04-xx	Not Received	08-29-19.0	Not Received*
M1027	Sept. 4	30°-31.2'(N) 57°-49.2'(W) Fired 09-04-xx	Not Received	09-24-43.6	Not Received*
M1028	Sept. 4	30°-35.5'(N) 58°-07.8'(W) Fired 10-04-xx	Not Received	10-32-05.7	Not Received
A429	Sept. 4			10-58-00.3	30°-43.6'(N) 54°-56.2'(W) Fired 10-33-38.4
M1029	Sept. 4	30°-40.0'(N) 58°-26.7'(W) Fired 11-04-xx	Not Received	11-25-22.6	30°-44.0'(N) 54°-57.0'(W) Rec. 11-08-32.5
M1030	Sept. 4	30°-44.1'(N) 58°-45.7'(W) Fired 12-04-xx	~~Not Received~~	12-24-20.6	30°-44.2'(N) 54°-57.6'(W) Rec. 12-08-02.1

* High Noise level. See Text page 26

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	~~Position of~~ USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M442	Sept. 4	30°-49'(N) 59°-05'(W) Fired 13-03-59.9	Not Received	13-24-10.5	Not ~~Received~~ Monitored
M443	Sept. 4	30°-49'(N) 59°-05'(W) Fired 13-33-30.1	Not Received	13-53-40.5	30°-45.1'(N) 54°-48.8'(W) Rec. 13-37-48.1
M1031	Sept. 4	30°-48.9'(N) 59°-05.6'(W) Fired 14-04-xx	Not Received	14-24-33.1	30°-45.3'(N) 54°-59.2'(W) Rec. 14-08-47.0
M1032	Sept. 4	30°-53.8'(N) 59°-24.2'(W) Fired 15-04-xx	Not Received	15-30-19.5	Not ~~Received~~ Monitored
M444	Sept. 4	31°-08.0'(N) 60°-25.0'(W) Fired 19-03-36.0	Not Received	19-22-37.5	30°-47.1'(N) 55°-02.7'(W) Rec. 19-09-11.7
M445	Sept. 4	31°-08.0'(N) 60°-25.0'(W) Fired 19-33-06.4	Not Received	19-52-07.7	30°-47.5'(N) 55°-03.1'(W) Rec. 19-38-42.5
M446	Sept. 4		Not Received	22-57-59.2	30°-48.6'(N) 55°-05.3'(W) Fired 22-33-43.0
M447	Sept. 5	31°-32'(N) 61°-51.0'(W) Fired 01-02-52.0	Not Received	01-20-29.3	30°-49.7'(N) 55°-06.7'(W) Rec. 01-10-09.1

TABLE ~~III~~ VI (cont.)

Event	Date	Position of USS Mason	~~Position of~~ USCG Valor	Received at Eleuthera 24°-56.2'(N) 76°-06.3'(W)	Position of USRV Atlantis
M448	Sept. 5	31°-32.0'(N) 61°-51.0'(W) Fired 01-33.22.2	Not Received	01-50-59.2	30°-49.9'(N) 55°-07.2'(W) Rec. 01-40-39.6
M449	Sept. 5	32°-00.0'(N) 63°-38.0'(W) Fired 07-03-11.6	Not Received	07-19-29.2	Monitored Not ~~Received~~
M450	Sept. 5	32°-00.0'(N) 63°-38.0'(W) Fired 07-33-16.9	~~Not Received~~	07-49-34.9	Monitored Not ~~Received~~

the difference between the observed distance and this calculated distance was plotted graphically, yielding a line at the correct distance from the receiving station. Where there was no time break, differences in distance from two stations were calculated and observed, and the resulting line of position near the shot was plotted. Since the position of the hydrophone at Eleuthera is in doubt by about 1/4 mile, the relation of the hydrophone to the receiving boat is in doubt to about 1/4 of a mile, and the position of the Atlantis is in doubt by about 3 miles, no effort has been made to correct for the ellipticity of the earth since the calculations for this correction are quite tedious. Calculations were made in the area of these tests to ascertain the magnitude of the ellipticity correction. It was found to vary from about + 2 miles to about -1 mile.

"Three position fix with T.B.

"There were four shots which were received by all three stations and for which a T.B. was known. Thus three separate determinations were available allowing a latitude, longitude and velocity to be determined. Using the velocity which was determined as above, a distance from each station was obtained from the HO214 tables distances and bearings to three points near the shot position. Three points were chosen instead of one in order to use the HO214 tables to the best advantage. The positions for which these calculations were made are marked E, A, and V according to the receiver (Eleuthera, Atlantis, or Valor), the distance given in miles near the station letter was calculated, and the bearings are shown as dashed lines with an arrow pointing toward the receiving station. The difference between the calculated distance and the observed distance was stepped off along the bearing line and a perpendicular to the bearing line erected. This perpendicular is a portion of the circle (isodistance line) at the required distance from the station. If all three positions of the receiving stations and all three observed distances had been accurate, and the ellipticity had not been neglected, these perpendiculars would have intersected in a point where the shot occurred. ~~Since they~~ did not ~~didn't all close properly, bisectors of the~~ (solid lines) ~~angles between respective station distance lines were drawn (long dash lines), and these three bisectors (solid lines), joined at a point which was the most probable location of the shot.~~ The choice of angle which is to be bisected is determined easily by seeing which bisector gets further from both stations or closer to both stations simultaneously as you move along it. This is the correct bisector. It will be found that the other bisector gets nearer to one station and farther from the other so that moving along it could not maintain the correct difference in distance. The isodistance line was drawn as a circle rather than a straight line when the distance to the receiving station was less than 100 miles. The diagrams for these types of fixes are given in Figures 6 to 9 inclusive.

"The method of determining velocity from these fixes is described on page 13.

"Three position fix with no T.B.

"These were plotted by assuming a distance to one station, usually Eleuthera, as correct at a point near the shot position given by the firing plane or boat. Using this distance and the difference in distance (difference in travel time x velocity) between Eleuthera and each of the other stations the distances from the other stations were calculated. These were plotted as above. If the triangle formed by these isodistance lines was too large, a new distance was chosen until a suitably sized triangle was obtained. The proper angular bisectors were then drawn and the most probable location of the shot determined. The plots of these fixes are given in Figures 10 to 18 inclusive.

"If the shot location were not known, curves of equal difference in distance or time between each pair of stations could be constructed. Application to these curves on a small scale chart approximately 1:8,000,000 will give a trial location which is within about 50 miles of the true position. From this point calculations as above can be made.

"Two position fix with T.B.

"Isodistance lines from the two stations are drawn as described above. Where these two cross is the most probable shot position. The plots of this type of fix are given in Figures 19 to 50 inclusive.

"Line of position from two receptions with no T.B.

"A distance to one of the stations is chosen for a point near the shot location. By applying the difference in distance to the other station, a cross of the isodistance lines is obtained, and the appropriate bisector is the line of position along which the shot most probably occurred. These lines of position are shown in Figures 51 to 77 inclusive.

"If the shot location were unknown, this plot could be made in the appropriate region by estimating the distance from each station from the duration of the sound reception. More details about signal durations will be given in a later section. From this data a line of position about 100 miles long would be drawn along which the shot would have occurred.

"<u>Line of Position from One Reception and T.B.</u>

"With one reception and T. B., an~~d~~ isodistance line from the station is determined. The most probable location of the shot is on this line; there the isodistance line becomes a line of position. These lines of position are shown in diagrams 78 to 118 inclusive.

"<u>Other Data</u>

"There were 27 shots for which there was a single reception and no T. B. No plots could be made from these. 14 shots were received by station and were either known duds, no station monitoring, or they were concluded to be duds.

"<u>Summary of Position finding</u>.

"The various fixes and the firing positions given by the ship or plane are given in Figures 1 to 5 inclusive. The firing positions are marked as lines perpendicular to the cruise line with the shot number alongside. In part of the work the Mason

Separate here into another report

gave two sets of shot points and courses. The first was given at the time of firing and the second revised after other data was obtained. The second set is plotted. In some instances Loran data was available, and these points are circled as well as having the route crossed. A chart showing the combined results of all events appears as figure 119."

G. METHODS OF TRIANGULATI~~ON BY MONITORING STATIONS OF KNOWN LOCATION WHEN AXIAL SOUND VELOCITY HAS BEEN DETERMINED~~ NG APPROXIMATE POSITION OF SIGNAL SOURCE

In air-sea rescue work, with which Sofar is primarily concerned, there will be no time break and no approximate location in which the signal may be expected to have had its origin~~al~~. It may have occurred at any point (outside the shadow zone of ~~that is not known to be shadowed~~ (generally by shoal water) from) the station at which it is heard. The time difference between the receipt of the peak cu~~t~~-off at ~~the~~ three or more recording stations and a rough knowledge of average axial sound velocity is the only information available. There are a number of methods of computing and plotting the position of a signal. The accuracy of these methods depends upon the precision ~~of~~ with which the axial sound velocity and the position of the listening hydrophones are known.

Mr. Charles A Spoerl of Hartford, Connecticut has presented two methods of determining an approximate position. When this is known, corrections for the earth's ellipticity and any known axial sound velocity variations for the area involved may be introduced into a further formula, also devised by ~~time~~ him, to fix the position precisely.

The first method is strictly mathematical:-

Let X be the position of the origin of the signal and ΦX its latitude. Let the first station at which the signal is heard be known as A, the second B, and the third C.

Let ΦA equal the latitude of A, ΦB equal the latitude of B and ΦC the latitude of C.

Let AB equal the degrees of arc from A to B; BC the degrees of arc from B to C; AC the degrees of arc from A to C and AX the degrees of arc from A to X.

~~This~~ get values may be computed from the formula:-

(1) $\cos AB = \cancel{(\sin \Phi A)(\sin \Phi B) + (\cos DLoAB)(\cos \Phi A)(\cos \Phi B)}$ $\cos(\phi_B - \phi_A) - [\cos\phi_A \cos\phi_B (1 - \cos DLoAB)]$

where DLoAB equals the difference in longitude between A and B.

3 spaces here

Let ψAB equal the initial Great Circle course from A to B, and ψAC the initial Great Circle course from A to C. ~~This~~ are values may be computed from the formula:-

(2) $$\sin \psi AB = \frac{(\sin DLoAB)(\cos \Phi B)}{\sin AB}$$

When DLo is West the sin is negative.

If $\sin \psi AB$ is negative subtract from 360° in north Pacific area - See Fig 1

Let p equal the approximate number of degrees of arc a signal would travel in the time interval between its receipt at A and B.

Let q equal the approximate number of degrees of arc a signal would travel in the time interval between its receipt at A and C. See Fig 1 ~~This~~ are values may be computed as follows:-

$p = \Delta t \times V$ where

(3) Δt = Interval in seconds between receipt at A and B ~~multiplied by approximate axial sound~~ V = velocity of sound transmission at the axis of the sound channel in nautical miles per second.

Other quantities needed are as follows:

(4) $\psi AB - \psi AC = \psi a$

Fig 1

(5) $$P = \frac{\cos p - \cos AB}{\sin AB}$$

(6) $$P' = \frac{\sin p}{\sin AB}$$

(7) $$R = \frac{\cos (q-p) - \cos BC}{(\sin AB)(\sin AC)}$$

(8) $$Q = \frac{\cos q - \cos AC}{\sin AC}$$

(9) $$Q' = \frac{\sin q}{\sin AC}$$

(10) $$\cot AX = \frac{(PP' + QQ') - (PQ' + P'Q) \cos\psi a \pm \sqrt{2PQR} \sin\psi a}{P^2 + Q^2 - 2PQ \cos\psi a}$$

Discard all negative results. If there are two positive ~~ones~~ answers continue separately with each.

(11) $\cos \measuredangle BAX = P \cot AX - P'$

(12) Initial Great Circle course from A to X = $\psi AX = \measuredangle BAX + \psi AB$

(13) $\sin \Phi X = (\cos AX)(\sin \Phi A) + (\sin AX)(\cos \Phi A)(\cos \psi AX)$

(14) Sin difference in longitude from A to X = $\frac{(\sin AX)(\sin \psi AX)}{\cos \Phi X}$

(15) Longitude X = Difference in longitude A to X applied to Longitude A

NOTE: To check, compute distances from X to B and C. They should equal AX + p and AX + q respectively.

Three spaces here

Stet

· The derivation of this equation is given in Appendix IV.

Mr. Spoerl's second method consists of plotting three circles on a stereographic chart. For construction of this project~~or~~ion see Appendix V.

The following page is a standard work sheet devised at the Woods Hole Oceanographic Institution for directing the necessary steps required for locating the approximate shot position by this method. The Woollard target, mentioned thereon, was designed by Dr. G. P. Woollard and consists of a series of concentric circles, whose radii increase by uniform distances, drawn on transparent Kodatrace paper. There is a small hole in the center of the target through which a pencil mark, indicating the center of that circle which fulfills the required conditions, may be drawn on the stereographic chart.

METHOD FOR TRIANGULATING POSITION OF SHOT

LTR.	STATION	ARRIVAL TIME OF PEAK H M S
A		— — .
B		— — .
C		— — .

First arrival station is A; second arrival B; etc.

INTERVAL	X sound velocity m/s
s	. ' = p = °- . '
s	. ' = q = °- . '

	Long.	Lat.	Ltr.		Latitude	Longitude
Pt. Arena	123-56.0(W)	38-51.0(N)		$B_1=\phi_B+p=$	°- . '( )	°- . '
Monterey	122-02.0(W)	36-38.0(N)		$B_2=\phi_B-p=$	°- . '( )	°- . '
Hilo Bay	154-58.0(W)	19-57.0(N)		$C_1=\phi_C+q=$	°- . '( )	°- . '
Kaneohe	157-40.0(W)	21-35.0(N)		$C_2=\phi_C-q=$	°- . '( )	°- . '

On stereographic chart:-

Plot B_1, B_2, C_1, C_2. Draw circle B on B_1B_2 as diameter; circle C on C_1C_2 as diameter.

By means of the Woollard Target ascertain the center of a circle which passes through point A and is tangent to circles B and C. (Note that if A lies between circles B and C and inside their common external tangent, there are two possible circles.) Draw circle X (and, if necessary, Circle X') on this center with the distance from center to A as radius. Draw the meridian through the center of circle X (and X').

This meridian intersects it at X_1 and X_2 (and X_1' and X_2').

Read off Lat. X_1 =	°- . '( )	Lat X_1' =	°- . '( )
Lat. X_2 =	°- . '( )	Lat X_2' =	°- . '( )
Sum =	°- . '( )	Sum =	°- . '( )
÷2 =	°- . '( )	÷2 =	°- . '( )

Approx. position Lat. X =	°- . '( )	or Lat X' =	°- . '( )
LO_X =	°- . '( )	or $LO_{X'}$ =	°- . '( )

A third method of determining the approximate position of a signal's origin is by the use of Sofar Position Plotting Sheets: A small section of one of these sheets is reproduced in Figure 120. These plotting sheets are similar to Loran charts.

Each curve on a Sofar Position Plotting Sheet is the locus of all points whose difference in distance between two monitoring stations is constant. The difference of peak cut-off arrival times, at any two monitoring stations, when multiplied by the approximate axial sound velocity, indicates the curve, or an interpolated curve between two curves as drawn on the plotting sheet, which is a line of position of the signal source. The crossing of three such lines of position, as may be determined from the times of signal receipt at three stations, would indicate an approximate fix. Or, an approximate fix may be ascertained by a two station reception by that point, on the curve, which corresponds to the signal's duration. see pps 1+2

It may be here noted that an erroneous position may be properly plotted on a three station reception if an incorrect axial sound velocity is used to compute the isodifference in distance curves. If a signal is heard by four stations this error is apparent as there is no point on the plotting sheet which will satisfy the inaccurate data.

H. METHOD OF OBTAINING ~~EXACT~~ More Precise POSITION OF SIGNAL SOURCE.

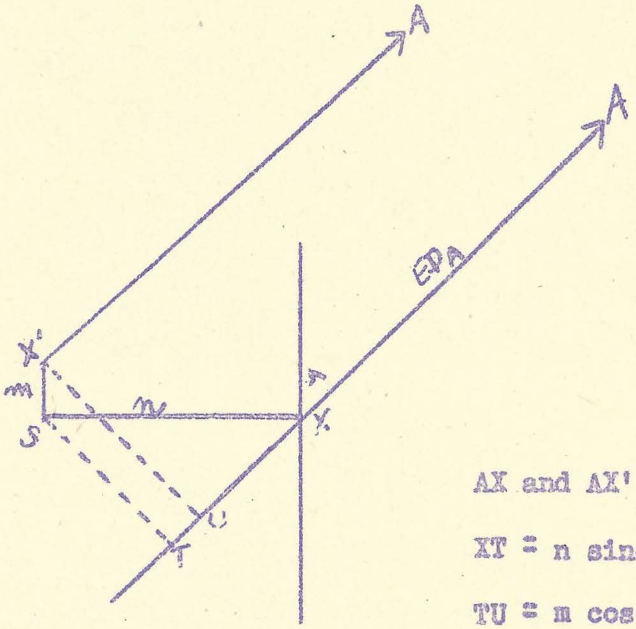
~~When~~ After the approximate ~~(hereafter referred to as the assumed)~~ position of the signal source has been determined by any of the above described methods, ~~an ingenious~~ a correction formula, devised by Mr. C. A. Spoerl, may be used for the elimination of any residual errors due to the use of incorrect axial sound velocity or incomplete allowances for the earth's ellipticity.

omit

or the neglect of allowance for the earth's ellipticity. (The isodifference in distance curves on the Sofar Position Plotting Sheets were computed with compensation for ellipticity).

The first step is to determine the elliptical distance and initial Great Circle course from the assumed signal source to each of three monitoring stations. See Appendix ~~III~~ for formulae. Reference to data to be obtained by the approximate methods described above, ~~in Sections C, D, and E above~~, should permit an accurate determination of travel time from the assumed signal source to each of these stations. The observed interval between signal receipt at Stations A and B is then added to the travel time from the assumed signal source to Station A. The travel time from the assumed signal source to Station B is then subtracted from this sum. The difference, when multiplied by the axial sound velocity known to be correct for the position of the assumed signal source, gives a residual D. A similar computation for Stations A and C gives residual E.

The formula follows:



Let X = position of shot as found by the stereographic chart.

Let X'= true position of shot which is m miles north and n miles west of X.

If α is the bearing of station A from X measured east of north, we have, approximately:

AX and AX' nearly parallel.

$XT = n \text{ sine} \alpha$

$TU = m \cos \alpha$

$AX' = AX \quad n \text{ sine} \alpha - m \cos \alpha$

and similarly for stations B and C (with approximate bearings β and γ respectively.

But we must have:

$AX' + p = BX'$ and $AX' + q = CX'$; where p equals velocity interval between signal receipt at stations A & B and q = velocity interval between signal receipt at stations A & C.

or: $-n (\text{sine}\alpha - \text{sine}\beta) + m (\cos\alpha - \cos\beta) = (AX - AX') - (BX - BX')$

$= AX - BX + BX' - AX'$

$= AX - BX + p$

and $\quad -n (\text{sine}\alpha - \text{sine}\gamma) + m (\cos\alpha - \cos\gamma) = AX - CX + q$

Let us call Diff. D and Diff. E the residual errors.

Diff. D = AX + p - BX (~~Form shows $ED_A + p - ED_B$~~)

Diff. E = AX + q - CX (~~" " $ED_A + q - ED_C$~~)

Then $m (\cos\alpha - \cos\beta) - n (\text{sine}\alpha - \text{sine}\beta$ = Diff. D.

$n (\cos\alpha - \cos\gamma) - n (\text{sine}\alpha - \text{sine}\gamma$ = Diff. E.

The solution of these simultaneous equations gives:

$$m = d\,(\text{Diff. D}\,(\text{sine}\,\gamma - \text{sine}\,\delta)\quad \text{Diff E}\,(\text{sine}\,\delta - \text{sine}\,\beta))$$

$$n = d\,(\text{Diff. D}\,(\cos\gamma - \cos\delta)\quad \text{Diff E}\,(\cos\delta - \cos\beta))$$

where
$$d = \frac{1}{4\,\text{sine}\,\frac{\beta-\gamma}{2}\cdot\text{sine}\,\frac{\beta-\alpha}{2}\cdot\text{sine}\,\frac{\gamma-\delta}{2}}$$

Hence the approximate correction in latitude and longitude are m minutes and $\frac{n}{\cos\phi_x}$ minutes respectively, or allowing for ellipticity.

$$\text{Correction in latitude} = \left(1 - \frac{4 - 9\cos^2\phi_x}{900}\right) m'$$

$$\text{Correction in longitude} = \left(1 - \frac{4 - 3\cos^2\phi_x}{900}\right) \frac{n'}{\cos\phi_x}.$$

Caps.

I. Hypothetical Problem in Triangulation:

To illustrate the application of the procedures explained in Sections G and H, it is proposed to do a problem in triangulation using each of the three above described methods for finding an approximate position and then applying the correction formula to each of these assumed positions. To add further complications, it will be assumed that the axial sound velocity is not constant throughout the area, but that its varying velocities may be ascertained by the triangulator through hydrographic and empirical data at his disposal. The problem follows:

short there

Station	Latitude	Longitude	Time of Peak Cut-off
Monterey	36°-38.0'(N)	122°-02.0'(W)	12"-12^{m}-50.38^{s}
Point Arena	38°-51.0'(N)	123°-56.0'(W)	12"-14^{m}-15.03^{s}
Hilo Bay	19°-57.0'(N)	154°-58.0'(W)	12"-31^{m}-16.57^{s}
Kaneohe	21°-35.0'(N)	157°-40.0'(W)	12"-33^{m}-25.05^{s}

~~Since the position of the shot is unknown until this data is analyzed we must arbitrarily choose an~~ The axial sound velocity based on the ~~available~~ hydrographic measurements is ~~On page 7~~ an average ~~axial sound velocity of~~ 4852.5 ft/sec (.7980731 m/s) ~~was suggested and this will be used.~~ Using the same notations and equation numbers given on pp ~~To illustrate Mr. Spoerl's mathematical formula~~ equations ~~for finding~~ the approximate ~~signal source~~ we will designate Monterey as Station A; Point Arena as Station B; and Hilo Bay as Station C. ~~Equation Numbers refer to description of the formula on pages 58 and 59.~~

Then: from Eq (1) from Eq (2)

Great Circle Arc AB = 2.67775° ψAB = 326.4462°
AC = 33.2077° ψAC = 248.9237°
BC = 32.7219° ~~AC = 248.9237°~~

~~Arrival Time Arrival Time Interval x .7980731 m/s~~

~~A 12-12-50.38 12-12-50.38 84.65^{s} 67.577' = p = 1.1263°~~

$p =$ Time ~~OBSERVED~~

Arrival Time ~~Arrival Time~~ Interval x.7980731 m/s

A 12-12-50.38 ~~12-12-50.38 84.65^s x 67.577' = p = 1.1263°~~

B 12-14-15.03 (3)

= 84.65s $p = 84.65 \times .7980731 = 67.577' = 1.1263°$

$\Delta t_{AB} = 0-1-24.65$ ~~12-31-16.57 1106.19^s 882,820' = q = 14.7137°~~

$\Delta t_{AC} = 18-26.14 = 1106.19s$ $p = 1106.19 \times .7980731 = 882,820' = 14.7137°$

$$\begin{array}{ll} \psi\ AB & 326.4462° \\ \psi\ AC & \underline{248.92370} \\ \psi\ a = & 77.5225° \end{array} \qquad (4)$$

$$P = \frac{.998068 - .9989081}{.0467186} = .0192364 \qquad (5)$$

$$P' = \frac{.0196564}{.0467186} = .4207403 \qquad (6)$$

$$R = \frac{.9720127 - .8413046}{.0467186 \times .5476757} = 5.1084591 \qquad (7)$$

$$Q = \frac{.9672070 - .8366908}{.5476757} = .2383092 \qquad (8)$$

$$Q' = \frac{.2539892}{.5476757} = .4637583 \qquad (9)$$

For substitution in Eq(10) values are as follows:

$$\begin{array}{lll} PP' = .0192364 \times .4207403 = & .0080935 \\ QQ' = .2383092 \times .4637583 = & \underline{.1105179} \\ \quad PP' + QQ' & = .1186114 \end{array}$$

$$\begin{array}{ll} PQ' = .0192364 \times .4637583 = & .0089210 \\ P'Q = .4207402 \times .2383092 = & \underline{.1002663} \\ \quad PQ' + P'Q & = .1091873 \\ \quad \cos\psi a & = .2160562 \\ \quad (PQ' + P'Q)\cos\psi a & = .0235906 \end{array}$$

$2PQR = 2 \times .0192364 \times .2383092 \times 5.1084591 = .0468364$

$\sqrt{2PQR} =$ .2164172

$\sin\psi a =$ .2113056 .9763809 $\sqrt{2PQR}\sin\psi a = .2113056$

$P^2 = (.0192364)^2 = .0003700$

$Q^2 = (.2383092)^2 = \underline{.0567913}$

$P^2 + Q^2 = .0571613$

$2PQ \cos \psi a = 2 \times .0192364 \times .2383092 \times .2160562 = .0019809$

$$\text{Cot } Ax = \frac{.1186114 - .0235906 \pm .2113056}{.0571613 - .0019809} = \frac{.3063264}{.0551804} \qquad (10)$$

$$= 5.551362$$

$$Ax = 10.2115^\circ \quad \sin Ax = .1772823 \quad \cos Ax = .9841600$$

$$\cos \measuredangle BAX = (.0192364)(5.551362) - .4207403 = -.3145039 \qquad (11)$$

$$\begin{aligned} \measuredangle BAX &= 251.6691^\circ \\ \psi AB &= \underline{326.4462^\circ} \\ \psi AX &= 578.1153^\circ = 218.1153^\circ \end{aligned}$$

$$\sin \psi AX = -.6172460 \quad \cos \psi AX = -.7867703$$

$$\begin{aligned} \sin \Phi x &= (.9841600)(.5966914) + (.1772823)(.8024708)(-.7867703) \\ &= .5872398 - .1119290 = .4753108 \qquad (13) \\ \Phi x &= 28.3796^\circ = 28^\circ\text{-}22.8'(N) \quad \cos \Phi x = .8798178 \end{aligned}$$

$$\text{Sin Difference in Longitude A to X} = \frac{(.1772823)(-.6172460)}{.8798178} \qquad (14)$$

$$= -.1243743$$

$$\text{DLo A to X} = 7.1446^\circ(W) \quad = \underline{7^\circ\text{-}08.7'(W)} \qquad (15)$$

$$\text{Longitude A} \quad = 122^\circ\text{-}02.0'(W)$$

$$\text{Longitude X} \quad = 129^\circ\text{-}10.7'(W)$$

Therefore the ~~assumed~~ approximate position is 28°-22.8(N); 129°-10.7'(W) end.

To correct this position to compensate for the earth's ellipticity and for variations in axial sound velocity within the area, we must use the formulae discussed in Section H. Calculations to determine the Great Circle distances compensated for the earth's ellipticity, and the initial Great Circle courses from this assumed signal source to stations A, B, and C follow on the next two pages:-

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~~METHOD FOR TRIANGULATING POSITION OF SHOT~~

Work Sheet (II) Spheroid Correction Formula — Great Circle Distance and Bearing of Stations from Approximate Signal Position

~~ϕ_s, L_s, etc., NOTE THAT s REFERS TO THE APPROPRIATE STATION ϕ_A, ϕ_B or ϕ_C; L_A, L_B or L_C~~

	COL. 1 = COS ϕ	STA. LTR.	COL. 2 $\cos\phi_x \cdot \cos\phi_s$	COL. 3 2 X COL. 2	COL. 4 $\phi_x - \phi_s$	COL. 5 COS COL. 4	COL. 6 1 - COL. 5
APPROX. ϕ_x = 28° 22.8' (N)							
ϕ_x = 28.3800° (N)	.879 8145						
LAT. PT. ARENA = 38.8500 (N)	.778 7909	B	.685 1915	1.370 3830	10.4700° (N)	.983 3502	.016 6498
LAT. MONTEREY = 36.6333 (N)	.802 4708	A	.706 0254	1.412 0508	8.2533° (N)	.989 6431	.010 3569
LAT. HILO BAY = 19.9500 (N)	.939 9907	C	.827 0174	1.654 0348	8.4300° (S)	.989 1957	.010 8043
~~LAT. KANEOHE = 21.5833 (N)~~	~~.929 8837~~				° ( )		

	DL_o X-S		DL_o X-S	COL. 7 E = (+) W = (−) SIN DL_o X-S	COL. 8 COS DL_o X-S	COL. 9 1 - COS DL_o X-S
APPROX. ϕ_x = 129° - 10.7' (W)						
" PT. ARENA = 123° - 56.0' (W)	5° - 14.7' (E)	B	5.2450° (E)	(+).091 4147	.995 8129	.004 1871
" MONTEREY = 122° - 02.0' (W)	7° - 08.7' (E)	A	7.1450° (E)	(+).124 3808	.992 2346	.007 7654
" HILO BAY = 154° - 58.0' (W)	25° - 47.3' (W)	C	25.7883° (W)	(−).435 0472	.900 4076	.099 5924
~~" KANEOHE = 157° - 40.0' (W)~~			° ( )	( ).		

Great Circle Distance Equation

	INSERT COL. 5	INSERT COL. 2	INSERT COL. 9	COL. 10 COS X-S	COL. 11 G.C.D. X-S	COL. 12 COL. 11 X 60	COL. 13 COL. 11 X .0174532
A	(.989 6431)	−(.706 0254)	(.007 7654)	= .984 1605	= 10.2114°	612.684'	.178 2225
B	(.983 3502)	−(.685 1915)	(.004 1871)	= .980 4812	= 11.3390°	680.340'	.197 9029
C	(.989 1957)	−(.827 0174)	(.099 5924)	= .906 8311	= 24.9290°	1495.740'	.435 0931

	COL. 14 SINE COL. 11	COL. 15 3 X COL. 14	COL. 16 COL. 15 - COL. 13	COL. 17 COL. 15 + COL. 13	COL. 18 1.001197 X COL. 12	COL. 19 1C - COL. 10
A	.177 2806	.531 8418	.353 6193	.710 0643	613.370	.015 8395
B	.196 6136	.589 8408	.391 9378	.787 7437	681.102	.019 5188
C	.421 4949	1.264 4847	.829 3916	1.699 5778	1497.415	.093 1689

Great Circle Bearing Equation

	INSERT COL. 7	INSERT COL. 1	INSERT COL. 14	COL. 20 SIN G.C. BEARING	COL. 21 G.C. Bearing ϕ_s from ϕ_x	COL. 22 W = (−) E = (+) COS COL. 21
A	(+.124 3808)	(.802 4708)	÷ (.177 2806)	= (+).563 0168	α = (N) 34.2647° (E)	(+).826 4453
B	(+.091 4147)	(.778 7909)	÷ (.196 6136)	= (+).362 0956	β = (N) 21.2290° (E)	(+).932 1406
C	(−.435 0472)	(.939 9907)	÷ (.421 4949)	= (−).970 2141	γ = (S) 75.9807° (W)	(−).242 2481

CONVERT COL. 21 TO Degrees from N: α = 34.2647°; β = 21.2290°; γ = 255.9807°

~~ALL WEST DLo SINES AND BEARINGS ARE NEGATIVE~~

~~255.9807~~

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Work Sheet (2) Spherical Correction Formula 70

Elliptical Correction $e = \frac{1}{4} a f\, dZ$

~~METHOD FOR TRIANGULATING POSITION OF SHOT~~

$$dZ = \frac{[3 \sin XA - .017453^3 XA][1 + \cos(\phi_X - \phi_A)][1 + \cos(\phi_X - \phi_A) - 2\cos\phi_X \cos\phi_A]}{1 + \cos XA}$$

	For Station "A"	For Station "B"	For Station "C"
Column 6 $(1-\cos(\phi_x-\phi_s))$	.010 3569	.016 6498	.010 8043
Column 3 $(2\cos\phi_x \cos\phi_s)$	1.412 0508	1.370 3830	1.654 0348
Sum	1.422 4077	1.387 0328	1.664 8391
Column 6 $(1-\cos(\phi_x-\phi_s))$	.010 3569	.016 6498	.010 8043
Product	.014 7317	.023 0938	.017 9874
Column 17	.710 0643	.787 7437	1.699 5778
Product	.010 4605	.018 1922	.0305710
Column 19	.015 8395	.019 5188	.093 1689
Dividend B	.660 4059	.932 0245	.328 1245
1.0 + Column 5 $2(\cos\phi_x \cos\phi_s)$	1.989 6431	1.983 3502	1.989 1957
Column 3	1.412 0508	1.370 3830	1.654 0348
Difference	.577 5923	.612 9672	.335 1609
1.0 + Column 5	1.989 6431	1.983 3502	1.989 1957
Product	1.149 2025	1.215 7286	.666 7006
Column 16	.353 6193	.391 9378	.829 3916
Product	.406 3802	.476 4900	.552 9559
1.0 + Column 10	1.984 1605	1.980 4812	1.906 8311
Dividend A	.204 8121	.240 5930	.289 9868
Dividend B	.660 4059	.932 0245	.328 1245
Dividend A- Div.B	(-) .455 5938	(-) .691 4315	(-) .038 1377
	2.9168	2.9168	2.9168
Product	(-) 1.329	(-) 2.017	(-) .111
Column 18	613.370	681.102	1497.415
Sum = E. D. =	612.041	679.085	1497.304

Work Sheet 3
Spoerl Correction Formula

~~Let us assume that data has been computed showing that the~~

Mean axial sound velocity from ~~this assumed~~ approximate signal source ~~average~~ to receiving station :-
determined from axial velocity chart of area

4846 f/s (.7970040 m/s) to Station A (Monterey)

4846 f/s (.7970040 m/s) to Station B (Point Arena)

4851.2 f/s (.7978593 m/s) to Station C (Hilo Bay)

4852.8 f/s (.7981224 m/s) to Kaneoke

At ~~assumed~~ approximate signal source 4846 f/s (.7970040 m/s).

Therefore:-

	To A	To B	To C
Elliptical distance from assumed signal source	612.041 mi	679.085 mi	1497.304 mi
Average axial sound velocity m/s	.7970040	.7970040	.7978593
	767.93^s	852.05^s	1876.65^s

Travel Time to Station A	767.93^s	767.93^s
Observed Interval (p.67) A to B	84.65^s	
" " (p.67) A to C		1106.19^s
Sum	852.58^s	1874.12^s
Travel Time to Station B	852.05	
" " " C		1876.65^s
Difference	+0.53	-2.53^s
Axial sound velocity at assumed signal source m/s	.7970040	.7970040

Residuals D = +.422 E = -2.016

Work Sheet (4) Second Correction Formula

~~METHOD FOR~~ TRIANGULATING ~~POSITION OF SHOT~~

CN = Bearing rel North

~~ED_A =~~	~~ED_A =~~	$C_N B$ = 21.2290	$C_N B$ = 21.2290°	$C_N Y$ = 255.9807°
~~P =~~ See page 71	~~Q =~~ See page 71.	$C_N Y$ = 255.9807°	$C_N \alpha$ = 34.2647	$C_N \alpha$ = 34.2647°
~~SUM =~~	~~SUM =~~	DIFF. F = (−) 234.7517	DIFF. G = (−) 13.0357°	DIFF. H = (+) 221.7160°
~~ED_B =~~	~~ED_C =~~	÷ 2 = (−) 117.37585°	÷ 2 = (−) 6.51785°	÷ 2 = (+) 110.8580°
DIFF. D = (+) .422	DIFF. E = (−) 2.016	# SINE F/2 = (−) −62.62415 .888 0093	# SINE G/2 = (−) .113 5127	# SINE H/2 = (+) +69.1420 .934 4657

SINE F/2 = (−) .888 0093	COL. 20Y = (−) .970 2141	COL. 22Y = (−) .242 2487	COL. 1 = COS ϕ_x = .879 8145	
SINE G/2 = (−) .113 5127	COL. 20α = (+) .563 0168	COL. 22α = (+) .826 4453	$\cos^2 \phi_x$ .774 0736	$\cos^2 \phi_x$ .774 0736
PRODUCT = (+) .100 8003	DIFFERENCE = (−) 1.533 2309	DIFFERENCE = (−) 1.068 6940	3.0000000	9.0000000
SINE H/2 = (+) .934 4657	DIFF. D = (+) .422	DIFF. D = (+) .422	PRODUCT = R = 2.322 2208	PRODUCT = S = 6.966 6624
PRODUCT = (+) .094 1944	PRODUCT = J = (−) .647 0234	PRODUCT = K = (−) .450 5669	4.000	4.000
.25/PROD. = .2500000	COL. 20α = (+) .563 0168	COL. 22α = (+) .826 4453	4.0 − PROD. R = (+) 1.677 7792	4.0 − PROD. S = (−) 2.966 6624
DIVIDEND = d = (+) 2.654 0855	COL. 20B = (+) .362 0956	COL. 22B = (+) .932 1406	900.00	900.00
	DIFFERENCE = (+) .200 9212	DIFFERENCE = (−) .105 6953	DIV. = T = (+) .001 8642	DIV. = V = (−) .003 2963
	DIFF. E = (−) 2.016	DIFF. E = (−) 2.016	1.000 0000	1.000 0000
	PRODUCT = (−) .405 0571	PRODUCT = (+) .213 0817	1.0 − DIV. T = (+) .998 1358	1.0 − DIV. V = 1.003 2963
	PRODUCT J = (−) .647 0234	PRODUCT K = (−) .450 5669	PRODUCT M = (−) .630 3060	PRODUCT M = (−) 2.792 3116
	SUM = (−) 1.052 0805	SUM = (−) .237 4852	PRODUCT = (−) .629 1310	PRODUCT W = (−) 2.802
	DIVIDEND = d = (+) 2.654 0855	DIVIDEND d = (+) 2.654 0855	cos ϕ_x = .879 8145	
	PRODUCT = M = (−) 2.792 3116	PRODUCT = M = (−) .630 3060	DIV. V = (−) .715	

8 / .080

APPROX. LAT. OF SHOT = 28° − 22.8′ (N)
*PRODUCT W = 0° − 2.80′ (S)
POSITION OF SHOT = SUM = 28° − 20.0′ (N)

APPROX. LONG. OF SHOT = 129° − 10.7′ (W)
*DIVIDEND Y = 0° − 00.72′ (E)
SUM = 129° 09.98′ (W)

* IF EITHER OF THESE ITEMS IS GREATER THAN 15′, REPEAT PAGES 2 TO 4 INCL., USING THE NEW SHOT POSITION.

SINES F/2, G/2, H/2 CARRY SIGNS OF THEIR ANGLES MODIFIED AS FOLLOWS:–
SINE 0.0000° TO 90.0000° = SINE AS GIVEN
SINE 90.0001° TO 180.0000° = SINE (180° − ANGLE)
~~SINE 180.0001° TO 270.0000° = SINE (180° + ANGLE)~~
~~SINE 270.0001° TO 359.9999° = SINE (360° − ANGLE)~~

The correct position, as calculated on the preceding pages is 28°-20.0(N); 129°-10.0(W).

The same calculations, but using the stereographic chart for finding the assumed position, follow on the next ~~four~~ five pages. The stereographic chart is plotted in figure 121. Note that the assumed axial velocity 4852.5 f/s (.7980731 m/s) remains unchanged until the approximate position is determined.

A = time of signal arrival at the first station receiving
B, C, etc = time of arrival at the other stations

ρ = equivalent distance of the time lag between the signal arrival at stations A and B $\rho = \underline{B-A} \times$ trans. vel in mi/sec expressed in degrees

q = equivalent distance of time lag between ~~station~~ signal arrival at stations A and C

Construction center for time lag circle at B on meridian of B but half way between latitude $\phi_B + \rho$ and latitude $\phi_B - \rho$.

Similarly the construction point for time lag circle at C will be on the meridian of C half way between latitude $\phi_C + q$ and latitude $\phi_C - q$.

To establish the circle tangent to the time lag circles for B and C and passing through A it is easiest to use a transparent overlay on which ~~are~~ ~~is~~ drawn a series of concentric circles at intervals of about 1/4 inch. This template is then ~~slid~~ superimposed on the stereographic chart plot and adjusted so that one circle fullfills the above conditions. The ~~shot construction center of~~ mean of the latitudes covered by this circle ~~define~~, and the meridian of the center point define the shot location. See Fig 2

Note: If station A lies between the time lag circles for stations B and C and inside their common external tangent, there are two possible circles ~~of~~ defining the shot position. The correct one will normally be self indicative.
~~ascertained from the signal~~ dur

METHOD FOR TRIANGULATING POSITION OF SHOT

LTR.	STATION	ARRIVAL TIME OF PEAK H - M - S
A	Monterey	12 - 12 - 50.38
B	Pt. arena	12 - 14 - 15.03
C	Hilo Bay	12 - 31 - 16.57

First arrival station is A; second arrival B; etc.

INTERVAL	X sound velocity .798073 m/s		
84.65s	67.579'	= p =	1° - 07.6'
1106.19s	882.820'	= q =	14° - 42.8'

	Long.	Lat.	Ltr.
Pt. Arena	123-56.0(W)	38-51.0(N)	B
Monterey	122-02.0(W)	36-38.0(N)	A
Hilo Bay	154-58.0(W)	19-57.0(N)	C
Kaneohe	157-40.0(W)	21-35.0(N)	

	Latitude	Longitude
$B_1 = \phi_B + p =$	39° - 58.6'(N)	123° - 56.0' W
$B_2 = \phi_B - p =$	37° - 43.4'(N)	123° - 56.0' W
$C_1 = \phi_C + q =$	34° - 39.8'(N)	154° - 58.0' W
$C_2 = \phi_C - q =$	5° - 14.2'(N)	154° - 58.0' W

On stereographic chart:-

Plot B_1, B_2, C_1, C_2. Draw circle B on B_1B_2 as diameter; circle C on C_1C_2 as diameter.

By means of the Woollard Target ascertain the center of a circle which passes through point A and is tangent to circles B and C. (Note that if A lies between circles B and C and inside their common external tangent, there are two possible circles.) Draw circle X (and, if necessary, Circle X') on this center with the distance from center to A as radius. Draw the meridian through the center of circle X (and X').

This meridian intersects it at X_1 and X_2 (and X_1' and X_2').

Read off Lat. X_1 =	38° - 45.0'(N)	Lat X_1' =	° - . '( )
Lat. X_2 =	18° - 25.0'(N)	Lat X_2' =	° - . '( )
Sum =	57° - 10.0'(N)	Sum =	° - . '( )
÷2 =	28° - 35.0'(N)	÷2 =	° - . '( )

Approx. position Lat. X =	28° - 35.0'(N)	or Lat X' =	° - . '( )
LO_X =	129° - 10.0'(W)	or LO_X' =	° - . '( )

METHOD FOR TRIANGULATING POSITION OF SHOT

ϕ_S; Lo_S; etc. NOTE THAT $_S$ REFERS TO THE APPROPRIATE STATION ϕ_A, ϕ_B OR ϕ_C; Lo_A; Lo_B OR Lo_C.

APPROX $-\phi_X$ = 28° - 35.0'(N)	COL. 1 = COS φ		COL. 2	COL. 3	COL. 4	COL. 5	COL. 6
$-\phi_X$ = 28.5833° (N)	.8781225	STA. LTR.	$\cos\phi_X \cdot \cos\phi_S$	2 X COL. 2	$\phi_X - \phi_S$	COS COL. 4	1 − COL. 5
LAT. PT. ARENA = 38.8500 (N)	.7787909	B	.6838738	1.3677476	10.2667 ° (N)	.9839888	.0160112
LAT. MONTEREY = 36.6333 (N)	.8024708	A	.7046677	1.4093354	8.0500 ° (N)	.9901462	.0098538
LAT. HILO BAY = 19.9500 (N)	.9399907	C	.8254270	1.6508540	8.6333 ° (S)	.9886693	.0113307
LAT. KANEOHE = 21.5833 (N)	.9298837	—	.	.	. ° ( )	.	.

APPROX Lo_X = 129° - 10.0'(W)	DLo_{X-S}		DLo_{X-S}	COL. 7* E = (+) W = (−) SIN DLo_{X-S}	COL. 8 COS DLo_{X-S}	COL. 9 1 − COS DLo_{X-S}
PT. ARENA = 123° - 56.0'(W)	5° - 14.0'(E)	B	5.2333 °(E)	(+).0912114	.9958315	.0041685
MONTEREY = 122° - 02.0'(W)	7° - 08.0'(E)	A	7.1333 °(E)	(+).1241782	.9922599	.0077401
HILO BAY = 154° - 58.0'(W)	28° - 48.0'(W)	C	25.8000 °(W)	(−).4352311	.9003188	.0996812
KANEOHE = 157° - 40.0'(W)	° - '( )	—	. °( )	( ).		

INSERT COL. 5 − INSERT COL. 2 INSERT COL. 9	COL. 10 COS X-S	COL. 11 G.C.D X-S	COL. 12 COL. 11 X 60	COL. 13 COL. 11 X .01745329
(.9901462) − (.7046677)(.0077401) =	.9846920	= 10.0381 °	602.286 °	.1751979
(.9839888) − (.6838738)(.0041685) =	.9811381	= 11.1459 °	668.754 °	.1945326
(.9886693) − (.8254270)(.0996812) =	.9063897	= 24.9889 °	1499.334 °	.4361385

COL. 14 SINE COL. 11	COL. 15 3 X COL. 14	COL. 16 COL. 15 − COL. 13	COL. 17 COL. 15 + COL. 13	COL. 18 1.004197 X COL. 12	COL. 19 1.0 − COL. 10
.1743030	.5229090	.3477111	.6981069	602.960	.0153080
.1933080	.5799240	.3853914	.7744566	669.503	.0188619
.4224427	1.2673281	.8311896	1.7034666	1501.013	.0936103

	INSERT COL. 7	INSERT COL. 1	INSERT COL. 14	COL. 20 SIN G.C. BEARING	# COL. 21 #	COL. 22 COS COL. 21 E = (+) W = (−)
A	(+.1241782)	(.8024708) ÷	(.1751979) =	(+).5687818	α = (N) 34.6653 °(E)	(+).8224886
B	(+.0912114)	(.7787909) ÷	(.1933080) =	(+).3674685	β = (N) 21.5596 °(E)	(+).9300359
C	(−.4352311)	(.9399907) ÷	(.4224427) =	(−).9684465	γ = (S) 75.5685 (W)	(−).2492223

CONVERT COL. 21 TO C_N: $-\alpha$ = 34.6653°; β = 21.5596°;
γ = 255.5685°

* ALL WEST DLo SINES ~~[illegible]~~ ARE NEGATIVE.
N OR S FROM $\phi_X - \phi_S$ COL. 4; E OR W FROM DLo_{X-S} COL.

METHOD FOR TRIANGULATING POSITION OF SHOT

	For Station "A"	For Station "B"	For Station "C"
Column 6	.009 8538	.016 0112	.011 3307
Column 3	1.409 3354	1.367 7476	1.650 8540
Sum	1.419 1892	1.383 7588	1.662 1847
Column 6	.009 8538	.016 0112	.011 3307
Product	.013 9844	.022 1556	.018 8337
Column 17	.698 1069	.774 4566	1.703 4666
Product	.009 7626	.017 1586	.032 0826
Column 19	.015 3080	.018 8619	.093 6103
Dividend B	.637 7449	.909 6936	.342 7251
1.0 + Column 5	1.990 1462	1.983 9888	1.988 6693
Column 3	1.409 3354	1.367 7476	1.650 8540
Difference	.580 8108	.616 2412	.337 8153
1.0 + Column 5	1.990 1462	1.983 9888	1.988 6693
Product	1.155 8984	1.222 6156	.671 8029
Column 16	.347 7111	.385 3914	.831 1896
Product	.401 9187	.471 1855	.558 3955
1.0 + Column 10	1.984 6920	1.981 1381	1.906 3897
Dividend A	.202 5093	.237 8357	.292 9073
Dividend B	.637 7449	.909 6936	.342 7251
Dividend A- Div.B	(-) .435 2356	(-) .671 8578	(-) .049 8178
	2.9168	2.9168	2.9168
Product	(-) 1.269	(-) 1.960	(-) .145
Column 18	602.960	669.503	1501.013
Sum = E D. =	601.691	667.543	1500.868

The assumed signal source is but 12.5 miles from that ascertained by the mathematical method above. It is reasonable to suppose that the same axial sound velocities apply for this small change in distance.

Therefore:

	To A	To B	To C
Elliptical distance from assumed signal source	601.691	667.543	1500.868
Average axial sound velocity m/s	.7970040	.7970040	.7978593
	754.94^{s}	837.51^{s}	1881.12^{s}
Travel time to Station A	754.94^{s}		754.94^{s}
Observed interval (p. 67) A to B	84.65^{s}		
Observed interval (p. 67) A to C			1106.19^{s}
Sum	839.59^{s}		1861.13^{s}
Travel time to Station B	837.57^{s}		
" " " C			1881.12^{s}
Difference	+ 2.02		-19.99
Axial sound velocity at assumed signal source m/s	.7970040		.7970040
Residuals D =	+1.610	E =	15.932

METHOD FOR TRIANGULATING POSITION OF SHOT

D_A = ~~see page 77~~ '	ED_A = '	$C_N B$ = 21.5596°	$C_N B$ = 21.5596°	$C_N Y$ = 255.5685°
P = '	g = '	$C_N Y$ = 255.5685°	$C_N \alpha$ = 34.6653	$C_N \alpha$ = 34.6653°
SUM ~~see page 77~~ '	SUM ~~see page 77~~ '	DIFF F = (–) 234.0089°	DIFF G = (–) 13.1057°	DIFF H = (+) 220.9032°
D_B = '	ED_C = '	÷ 2 = (–) 117.00445°	÷ 2 = (–) 6.55285°	÷ 2 = (+) 110.4516°
DIFF D = (+) 1.610 '	DIFF E = (–) 15.932 '	# SINE F/2 = (–) 62.99555 .890 9712	# SINE G/2 = (–) .114 1196	# SINE H/2 = (+) 69.5484 .936 9676
SINE F/2 = (–) .890 9712	SIN COL 20 Y = (–) .968 4465	COS COL 22 Y = (–) .249 2223	COL 1 = cos ϕ_x = .878 1225	
SINE G/2 = (–) .114 1196	COL 20 α = (+) .568 7818	COL 22 α = (+) .822 4886	$\cos^2 \phi_x$ .771 0991	$\cos^2 \phi_x$ .771 0991
PRODUCT = (+) .101 6773	DIFFERENCE = (–) 1.537 2283	DIFFERENCE = (–) 1.071 7109	3.0000000	9.0000000
SINE H/2 = (+) .936 9676	DIFF D = (+) 1.610	* DIFF D = (+) 1.610	PRODUCT = R = 2.3132973	PRODUCT = S = 6.9398919
PRODUCT = (+) .095 2683	PRODUCT = J = (–) 2.4749376	PRODUCT = K = (–) 1.7254545	4.000	4.000
.25/PROD = .2500000	COL 20 α = (+) .568 7818	COL 22 α = (+) .822 4886	4.0 - PROD R = (+) 1.6867027	4.0 - PROD S = (–) 2.9398919
DIVIDEND = d = (+) 2.624 1677	COL 20 B = (+) .367 4685	COL 22 B = (+) .930 0359	900.00	900.00
	DIFFERENCE = (+) .201 3123	DIFFERENCE = (–) .107 5473	DIV = T = (+) .0018741	DIV = V = (–) .0032665
	DIFF E = (–) 15.932	* DIFF E = (–) 15.932	1.0000000	1.0000000
	PRODUCT = (–) 3.2073235	PRODUCT = (+) 1.713 4436	1.0 - DIV T = (+) .9981259	1.0 - DIV V = 1.0032665
	PRODUCT J = (–) 2.4749376	PRODUCT K = (–) 1.7254545	PRODUCT m = (–) .0315186	PRODUCT m = (–) 14.9112060
	SUM = (–) 5.6822611	SUM = (–) .012 0109	PRODUCT = (–) .0314595	PRODUCT W = (–) 14.960'
	DIVIDEND = d = (+) 2.6241677	* DIVIDEND = d = (+) 2.6241677	cos ϕ_x = .8781225	Jd
	PRODUCT = m = (–) 14.911 2060	PRODUCT = m = (–) .0315186	DIV. V = (–) .036	

APPROX LAT. OF SHOT = 28° - 35.0' (N)
* PRODUCT W = 0° - 14.96' (N)
POSITION OF SHOT = SUM = 28° - 20.04' (N)

APPROX. LONG. OF SHOT = 129° - 10.0' (W)
* DIVIDEND V = 0° - .04' (E)
SUM = 129° 09.96' (W)

* IF EITHER OF THESE ITEMS IS GREATER THAN 15', REPEAT PAGES 2 TO 4 INCL., USING THE NEW SHOT POSITION.

\# SINES F/2, G/2, H/2 CARRY SIGNS OF THEIR ANGLES MODIFIED AS FOLLOWS:—
SINE 0.0000° TO 90.0000° = SINE AS GIVEN
SINE 90.0001° TO 180.0000° = SINE (180° - ANGLE)
SINE 180.0001° TO 270.0000° = -SINE (180° + ANGLE)
SINE 270.0001° TO 359.9999° = -SINE (360° - ANGLE)

(3) Lines — Position Plotting Sheet Method — Establishing a Fix

The third method, and by far the easiest and quickest one, to use ~~is by use of a~~ is the Sofar Position Plotting sheet (see figure ~~120~~ 3) and the Sofar Position Plotting Tables, a portion of which are reproduced in Tables VIIa, VIIb, VIIc and VIId.

Start An example of the use of the plotting Sofar plotting sheet is as follows

Using the same assumed axial sound velocity, 4852.5 f/s (.7980731 m/s) as previously, the computations ~~follows~~ are:

Station	Time of Peak Cut-Off				Station	Time of Peak Cut-Off		
	H	M	S			H	M	S
Monterey	12	12	50.38		Kaneohe	12	33	25.05
Point Arena	12	14	15.03		Hilo Bay	12	31	16.57
Difference	(-)	1 -	24.65				2 -	08.48
Difference in seconds	(-)		84.65^s					128.48^s
Assumed axial sound velocity			.7980731 m/s					.7980731 m/s
Distance to ~~Monter~~ Monterey less distance to Point Arena	(-)		67.557 Mi		Distance to Kaneohe less distance to Hilo Bay			102.536 Mi.

Station	Time of Peak Cut-Off			Station	Time of Peak Cut-Off		
	H	M	S		H	M	S
Kaneohe	12	33	25.05	Hilo Bay	12	31	16.57
Point Arena	12	14	15.03	Point Arena	12	14	15.03
Difference		19-	10.02			17-	01.54
Difference in seconds			1150.02^s				1021.54^s
Assumed axial sound velocity			.7980731 m/s				.7980731 m/s
Distance to Kaneohe less distance to Point Arena			917.800 Mi	Distance to Kaneohe less distance to Hilo Bay			815.264 Mi.

these values
Plotting on figure 120 gives a large triangle, about 7 miles to a side. which This proves, as stated on page 62, that the assumed average axial sound velocity was in error.

Had the signal been heard by only three stations and the same assumed axial sound velocity chosen, we would have had reasonable fixes at the following points:-

If not heard at Kaneohe the apparent position is
28^{o}-19.2(N) 129^{o}-09.5 (W)

If not heard at Hilo Bay the apparent position is
28^{o}-19.2(N) 129^{o}-09.5 (W)

If not heard at Monterey the apparent position is
28^{o}-24.2(N) 129^{o}-14.7 (W)

If not heard at Point Arena, Sofar Plotting Position sheet V-M would have to have been used as it contains similar curves showing distances to Kaneohe and Hilo Bay less distance to Monterey.

stop However, the ~~trial~~ assumed axial sound velocity was expected to be somewhat in error. Now that the area of the signal source is known, accurate velocities may be incorporated to ascertain the travel times to each of three stations and Mr. Spoerl's formula may be applied.

To construct the Sofar Position Plotting Sheets it was necessary to compute the elliptical distances from random points to each station. These computations form the basis for the Sofar Position Plotting Tables. By using one of these random points as the assumed signal source much of the long computation may be omitted as it has already been done.

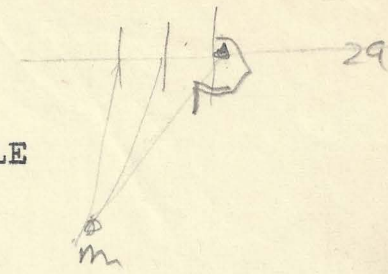


TABLE VIIa

SOFAR POSITION PLOTTING TABLE

Monterey 36°-38.0'(N)
122°-02.0'(W)

For Lat. 29°(N) cos = .8746197 Long. Const.= .9981054 Lat.Const= 1.0032052

From:-

Long.	Elliptical Distance (z_{X-A}) Miles	ψ_{X-A} Course	$\sin \psi_{XA}$ Sine Course	$\cos \psi_{XA}$ Cosine Course
128°-00W	547.011	031.7234°	+ 0.5258194	+0.8505964
-30	561.277	033.4209	.5554375	.8315579
129°-00	576.296	035.6494	.5828232	.8125986
-30	591.998	037.4534	.6081159	.7938482
130°-00	608.351	039.1550	.6314199	.7754406

For 28°(N) cos = .8829476 Long. Const.= .9981542 Lat. Const.=1.0033515

From:-

	Miles	Course	Sine Course	Cosine Course
128°-00W	598.632	028.7339°	+ 0.4807420	+ 0.8768618
-30	611.824	030.6519	.5098214	.8602806
129°-00	625.772	032.4817	.5370301	.8435630
-20	640.414	034.2245	.5624364	.8268401
130°-00	655.709	035.8822	.5861201	.8102238

For 27°(N) cos =.8910065 Long. Const. =.9982019 Lat.Const. =1.0034945

From:-

	Miles	Course	Sine Course	Cosine Course
128°-00W	651.632	026.2282°	+ 0.4419470	+ 0.8970410
-30	663.895	028.0445	.4701577	.8825827
129°-00	676.898	029.7886	.4968020	.8678643
-30	690.588	031.4612	.5219209	.8529938
130°-00	704.939	033.0621	.5455483	.8380797

TABLE ~~VIIIb~~ 2

SOFAR POSITION PLOTTING TABLE

Point Arena 38°-51.0'(N)
123°-56.0'(W)

Lat Φ — 1-Div T* — 1-Div V*

For 29°(N) cos Φ = .8746197 ~~Long Const.~~ = .9981054 ~~Lat Const.~~ = 1.0032052

From:- Long	Elliptical Distance (2XA) Miles	ψXA Course	Sin ψXA Sine Course	Cos ψXA Cosine Course
128°-00W	623.273	017.8002°	+ 0.3056985	+ 0.9521283
-30	631.780	019.7962	.3386757	.9409032
129°-00	641.143	021.7331	.3702839	.9289188
-30	651.320	023.6084	.4004828	.9163040
130°-00	662.286	025.4199	.4292487	.9031862

Lat Φ — 1-Div T — 1-Div V

For 28°(N) cos Φ =8829476 ~~Long Const.~~ = .9981542 ~~Lat. Const.~~ = 1.0033515

From:-	Miles	Course	Sine Course	Cosine Course
128°-00W	680.423	016.2762°	+ 0.2802774	+ 0.9599218
-30	688.314	018.1278	.3111379	.9503649
129°-00	697.003	019.9323	.3409096	.9400961
-30	706.475	021.6873	.3695405	.9292145
130°-00	716.695	023.3907	.3969986	.9178191

Lat Φ — 1-Div T — 1-Div V

For 27°(N) cos Φ = .8910065 ~~Long Const.~~ = .9982019 ~~Lat. Const.~~ = 1.0034945

From:-	Miles	Course	Sine Course	Cosine Course
128°-00W	737.991	014.9917°	+ 0.2586795	+ 0.9659633
-30	745.340	016.7158	.2876253	.9577432
129°-00	753.456	018.4022	.3156856	.9488639
-30	762.313	020.0484	.3428143	.9394034
130°-00	771.889	021.6525	.3689764	.9294388

* See Work Sheet 4

TABLE V~~II~~c

SOFAR POSITION PLOTTING TABLE

Hilo Bay 19°-57.0'(N)
154°-58.0'(W)

For 29°(N) cos = .8746197 Long Const.=.9981054Lat.Const. = 1.0032052

From:-

	Miles	Course °	Sine Course	Cosine Course
128°-00W	1566.558	255.8146	-0.9695077	-0.2450604
-30	1541.095	255.3473	.9674769	.2529593
129°-00	1515.678	254.8686	.9653296	.2610336
-30	1490.314	254.3782	.9630600	.2692862
130°-00	1465.016	253.8311	.9604451	.2784698

For 28°(N)cos =.8829476 Long Const.=.9981542 Lat. Const.= 1.0033515

From:-

	Miles	Course °	Sine Course	Cosine Course
128°-00W	1553.028	257.8133	-0.9774648	-0.2110979
-30	1527.098	257.3813	.9758454	.2184618
129°-00	1501.216	256.9378	.9741253	.2260087
-30	1475.375	256.4837	.9723033	.2337220
130°-00	1449.587	256.0168	.9703667	.2416374

For 27°(N)cos =.8910065 Long Const.=.9982019 Lat.Const.=1.0034945

From:-

	Miles	Course °	Sine Course	Cosine Course
128°-00W	1541.534	259.8472	-0.9843412	-0.1762739
-30	1515.187	259.4513	.9830997	.1830712
129°-00	1488.870	259.0471	.9817836	.1900020
-30	1462.588	258.6310	.9803780	.1971269
130°-00	1436.347	258.2039	.9788814	.2044294

TABLE V~~II~~d

SOFAR POSITION PLOTTING TABLE

Kaneohe 21°-35.0(N)
157°-40.0(W)

For 29°(N) cos = .8746197 Long.Const.= .9981054 Lat.Const.= 1.0032052

From:-

	Miles	Course	Sine Course	Cosine Course
128°-00W	1666.950	261.3535°	-0.9886347	-0.1503377
-30	1640.968	260.9819	.9876389	.1567464
129°-00	1615.010	260.6031	.9865810	.1632726
-30	1589.082	260.2156	.9854543	.1699412
130°-00	1563.190	259.8184	.9842524	.1767687

For 28°(N) cos =.8829476 Long.Const.= .9981542 Lat.Const.= 1.0033515

From:-

	Miles	Course	Sine Course	Cosine Course
128°-00W	1659.017	263.2291°	-0.9930526	-0.1178997
-30	1632.666	262.9056	.9923440	.1235045
129°-00	1606.338	262.5607	.9915825	.1294758
-30	1580.029	262.2090	.9907690	.1355599
130°-00	1553.742	261.8490	.9898979	.1417824

For 27°(N) cos =.8910065 Long. Const.=.9982019 Lat.Const.= 1.0034945

From:-

	Miles	Course	Sine Course	Cosine Course
128°-00W	1653.039	265.1486°	-0.9964174	-0.0845718
-30	1626.360	264.8462	.9959571	.0898295
129°-00	1599.697	264.5369	.9954578	.0952046
-30	1573.~~697~~041	264.2228	.9949209	.1006604
130°-00	1546.403	263.9010	.9943398	.1062467

The nearest point given in the tables to the approximate signal source is 28°-00(N) 129°-00(W). This is in a region where the same average ~~signal~~ axial sound velocities to each station as were previously used, again apply. (See page 71).

Therefore:

	To A	To B	To C
Elliptical distance from 28°-00(N) 129°-00(W) from Table VII	625.772	697.003	1501.216
Average axial sound velocity m/s Travel Time	.7970040	.7970040	.7978593
~~Sum~~	785.16^s	874.53^s	1881.55^s

Travel time to Station A	785.16^s	785.16^s
Observed interval (p. 67) A to B	84.65^s	
Observed interval (p. 67) A to C		1106.19^s
Sum	869.81^s	1891.35
Travel time to Station B	874.53^s	
Travel time to Station C		1881.55
Difference	(-) 4.72^s	(+) 9.80^s
Axial sound velocity at assumed signal source	.7970040 m/s	.7970040 m/s
Residuals	D = (-) 3.762	E = (+) 7.811

METHOD FOR TRIANGULATING POSITION OF SHOT

ED_A = . ' (See page 85)	ED_A = . ' (See page 85)	$C_N\beta$ = 19.9323°	$C_N\beta$ = 19.9323°	$C_N\gamma$ = 256.9378°
P = '	q = '	$C_N\gamma$ = 256.9378°	$C_N\alpha$ = 32.4817°	$C_N\alpha$ = 32.4817°
SUM = . '	SUM = . '	DIFF. F =(−) 237.0055°	DIFF. G =(−) 12.5494°	DIFF. H =(+) 224.4561°
ED_B = . '	ED_C = . '	÷ 2 =(−) 118.50275°	÷ 2 =(−) 6.2747°	÷ 2 =(+) 112.22805°
DIFF. D =(−) 3.762 '	DIFF. E =(+) 7.811 '	# SINE F/2 =(−) .878 7942 (61.49725)	# SINE G/2 =(−) .109 2954	# SINE H/2 =(+) .925 6855 (67.77195)

SINE F/2 =(−) .878 7942	COL. 20γ =(−) .974 1252	COL. 22γ =(−) .226 0087	COL. 1 = cos ϕ_x = .	
SINE G/2 =(−) .109 2954	COL. 20α =(+) .537 0301	COL. 22α =(+) .843 5630	$\cos^2 \phi_x$.	$\cos^2 \phi_x$.
PRODUCT T =(+) .096 0482	DIFFERENCE =(−) 1.511 1554	DIFFERENCE =(−) 1.069 5717	9.0000000	9.0000000
SINE H/2 =(+) .925 6855	DIFF. D = (−) 3.762 →	DIFF. D = (−) 3.762	PRODUCT = R = .	PRODUCT = S = .
PRODUCT =(+) .088 9104	PRODUCT = J =(+) 5.684 9166	PRODUCT = K =(+) 4.023 7287	4.000	4.00
25/PROD = .250 0000	COL. 20α =(+) .537 0301	COL. 22α =(+) .843 5630	4.0 − PROD. R =( ) .	4.0 − PROD. S =( ) .
DIVIDEND = d =(+) 2.811 8195	COL. 20β =(+) .340 9096	COL. 22β =(+) .940 0961	900.00	900.00
	DIFFERENCE =(+) .196 1205	DIFFERENCE =(−) .096 5331	DIV. = T =( ) .	DIV. = V =( ) .
	DIFF. E = (+) 7.811 →	DIFF. E = (+) 7.811	1.0000000	1.0000000
	PRODUCT = (+) 1.531 8972	PRODUCT = (−) .754 0200	Long. Corr. ~~1.0 − DIV. T~~ =(+) .998 1542	Lat. Corr. ~~1.0 − DIV. V~~ = 1.003 3515
	PRODUCT J =(+) 5.684 9666	PRODUCT K =(+) 4.023 7287	PRODUCT N =(+) 9.193 8307	PRODUCT M =(+) 20.292 5184
	SUM =(+) 7.216 8638	SUM = (+) 3.269 7087	PRODUCT T =(+) 9.176 8607	PRODUCT W =(+) 20.361'
	DIVIDEND = D =(+) 2.811 8195 →	DIVIDEND = D =(+) 2.811 8195	cos ϕ_x = .882 9476	
	PRODUCT = M = (+) 20.292 5184	PRODUCT = N =(+) 9.193 8307	DIV. V =(+) 10.393	

APPROX. LAT. OF SHOT = 28° − 00.0' (N)
* PRODUCT W = 0° − 20.36' (N)
POSITION OF SHOT = SUM = 28° − 20.36' (N)

APPROX. LONG. OF SHOT = 129° − 00.0' (W)
* DIVIDEND V = 0° − 10.39' (W)
SUM = 129° − 10.39' (W)

* IF EITHER OF THESE ITEMS IS GREATER THAN 15', REPEAT PAGES 2 TO 4 INCL., USING THE NEW SHOT POSITION.

\# SINES F/2, G/2, H/2 CARRY SIGNS OF THEIR ANGLES MODIFIED AS FOLLOWS:—
SINE 0.0000° TO 90.0000° = SINE AS GIVEN
SINE 90.0001° TO 180.0000° = SINE (180° − ANGLE)
SINE 180.0001° TO 270.0000° = −SINE (180° + ANGLE)
SINE 270.0001° TO 359.9999° = −SINE (360° − ANGLE)

The correction formula, as computed on the preceding page, shows a product W of 20.36'(N). If either product W or dividend V is greater than 15' a recalculation is necessary, the new position being used as a basis for the assumed position.

This recalculation is not always necessary, and would not have been in this case except for the fact that the Sofar Tables, at present, are not computed for positions at sufficiently close intervals.

Pages 88 and 89 show the preliminary computations for this new position.

Therefore:

	To A	To B	To C
Elliptical distance from 28^{o}-20.4(N) 129^{o}-10.4(W)	613.867	681.217	1496.986
Average axial sound velocity m/s	.7970040	.7970040	.7978593
Travel time	770.22^{s}	854.72^{s}	1876.25

Travel time to Station A	770.22^{s}	770.22^{s}
Observed interval (p. 67) A to B	84.65^{s}	
Observed interval (p. 67) A to C		1106.19^{s}
Sum	854.87^{s}	1876.41^{s}
Travel time to Station B	854.72^{s}	
Travel time to Station C		1876.25^{s}
Difference	(+) 0.15^{s}	(+) 0.16^{s}
Axial sound velocity at m/s assumed signal source	.7970040 m/s	.7970040 m/s
Residuals	D = (+) .120	E = (+) .128

The correction formula is shown on page 90.

METHOD FOR TRIANGULATING POSITION OF SHOT

ϕ_S; Lo_S; etc. Note that $_S$ refers to the appropriate station ϕ_A, ϕ_B or ϕ_C; Lo_A, Lo_B or Lo_C

	Col. 1 = cos φ	Sta Ltr.	Col. 2 cos φx · cos φs	Col. 3 2 × Col. 2	Col. 4 φx − φs	Col. 5 cos Col. 4	Col. 6 1 − Col. 5
Approx. φx = 28° − 20.4′ (N)							
φx = 28.3400° ( )	.8801462						
Lat. Pt. Arena = 38.8500 (N)	.7787909	B	.685 4499	1.370 8998	10.5100° (N)	.983 2231	.0167769
Lat. Monterey = 36.6533 (N)	.8024708	A	.706 2916	1.412 5832	8.2933° (N)	.989 5426	.010 4574
Lat. Hilo Bay = 19.9500 (N)	.9399907	C	.827 3292	1.654 6584	8.3900° (S)	.989 2978	.010 7022
Lat. Kaneohe = 21.5833 (N)	.9298837				° ( )		

	DLo X−S		DLo X−S	Col. 7 E = (+) W = (−) Sin DLo X−S	Col. 8 Cos DLo X−S	Col. 9 1 − cos DLo X−S
Approx. Lox = 129° − 10.4′ (W)						
" Pt. Arena = 123° − 56.0′ (W)	5° − 14.4′ (E)	B	5.2400° (E)	(+).091 3278	.995 8209	.004 1791
" Monterey = 122° − 02.0′ (W)	7° − 08.4′ (E)	A	7.1400° (E)	(+).124 2942	.992 2454	.007 7546
" Hilo Bay = 154° − 58.0′ (W)	25° − 47.6′ (W)	C	25.7933° ( )	(−).435 1258	.900 3696	.099 6304
" Kaneohe = 157° − 40.0′ (W)	° − ′ ( )		° ( )	( )		

	Insert Col. 5	Insert Col. 2	Insert Col. 9	Col. 10 cos X−S	Col. 11 G.C.D X−S	Col. 12 Col. 11 × 60	Col. 13 Col. 11 × .0174533
A	(.989 5426)	−(.706 2916)	(.007 7546)	= .984 0656	= 10.2420°	614.520′	.178 7566
B	(.983 2231)	−(.685 4499)	(.004 1791)	= .980 3585	= 11.3747°	682.482′	.198 5259
C	(.989 2978)	−(.827 3292)	(.099 6304)	= .906 8707	= 24.9236°	1495.416′	.434 9988

	Col. 14 Sine Col. 11	Col. 15 3 × Col. 14	Col. 16 Col. 15 − Col. 13	Col. 17 Col. 15 + Col. 13	Col. 18 1.00117 × Col. 12	Col. 19 Col. 18 − Col. 12
A	.177 8061	.5334183	.354 6617	.712 1749	615.208	.015 9344
B	.197 2245	.591 6735	.393 1476	.790 1994	683.246	.019 6415
C	.421 4094	1.264 2282	.829 2294	1.699 2270	1497.090	.093 1293

	Insert Col. 7	Insert Col. 1	Insert Col. 14	Col. 20 Sin G.C. Bearing	Col. 21 #	Col. 22 # cos Col. 21 E = (+) W = (−)
A	(+.124 2942)	(.802 4708)	÷ (.177 8061)	= (+).560 9620	α = (N) 34.1224° (E)	(+).827 8411
B	(+.091 3278)	(.778 7909)	÷ (.197 2245)	= (+).360 6309	β = (N) 21.1389° (E)	(+).932 7089
C	(−.435 1258)	(.939 9907)	÷ (.421 4094)	= (−).970 5863	γ = (S) 76.0690 (W)	(−).240 7532

Convert Col. 21 to C_N: − α = 34.1224°; β = 21.1389°;

γ = 256.0690°

All west DLo sines are negative.
N or S from $\phi_X - \phi_S$ Col. 4; E or W from DLo_{X-S} Col.

METHOD FOR TRIANGULATING POSITION OF SHOT

	For Station "A"	For Station "B"	For Station "C"
Column 6	.010 4574	.016 7769	.010 7022
Column 3	1.412 5832	1.370 8998	1.654 6584
Sum	1.423 0406	1.387 6767	1.665 3606
Column 6	.010 4574	.016 7769	.010 7022
Product	.014 8813	.023 2809	.017 8230
Column 17	.712 1749	.790 1994	1.699 2270
Product	.010 5981	.018 3966	.030 2853
Column 19	.015 9344	.019 6415	.093 1293
Dividend B	.665 1074	.936 6165	.325 1962
1.0 + Column 5	1.989 5426	1.983 2231	1.989 2978
Column 3	1.412 5832	1.370 8998	1.654 6584
Difference	.576 9594	.612 3233	.334 6394
1.0 + Column 5	1.989 5426	1.983 2231	1.989 2978
Product	1.147 8853	1.214 3737	.665 6974
Column 16	.354 6617	.393 1476	.829 2294
Product	.407 1110	.477 4281	1.552 0159
1.0 + Column 10	1.984 0656	1.980 3585	1.906 8707
Dividend A	.205 1902	.241 0816	.289 4878
Dividend B	.665 1074	.936 6165	.325 1962
Dividend A- Div.B	(-) .459 9172	(-) .695 5349	(-) .035 7084
	2.9168	2.9168	2.9168
Product	(-) 1.341	(-) 2.029	(-) .104
Column 18	615.208	683.246	1497.090
Sum = E D. =	613.867	681.217	1496.986

METHOD FOR TRIANGULATING POSITION OF SHOT

D_A = ~~See page 87~~ '	ED_A = ~~See page 87~~ '	$C_N B$ = 21.1389°	$C_N B$ = 21.1389°	$C_N Y$ = 256.0690°
P = '	g = '	$C_N Y$ = 256.0690°	$C_N \alpha$ = 34.1224°	$C_N \alpha$ = 34.1224°
SUM = '	SUM = '	DIFF F = (−) 234.9301	DIFF. G = (−) 12.9835°	DIFF. H = (+) 221.9466°
D_B = '	ED_C = '	÷ 2 = (−) 117.46505°	÷ 2 = (−) 6.49175°	÷ 2 = (+) 110.9733°
DIFF. D = (+) .120	DIFF. E = (+) .128	# SINE F/2 = (−) 62.53495 .8872923	# SINE G/2 = (−) .113 0602	# SINE H/2 = (+) 69.0267 .933 7473

SINE F/2 = (−) .887 2923	COL. 20Y = (−) .970 5863	COL. 22Y = (−) .240 7532	COL. 1 = cos ϕ_x = .880 1462	
SINE G/2 = (−) .113 0602	COL. 20α = (+) .560 9620	COL. 22α = (+) .827 8411	$\cos^2 \phi_x$ .774 6573	$\cos^2 \phi_x$ .774 6573
PRODUCT = (+) .100 3174	DIFFERENCE = (−) 1.531 5483	DIFFERENCE = (−) 1.068 5943	3.0000000	9.0000000
SINE H/2 = (+) .933 7473	DIFF. D = (+) .120	DIFF. D = (+) .120	PRODUCT = R = 2.3239719	PRODUCT = S = 6.9719157
PRODUCT = (+) .093 6711	PRODUCT = J = (−) .183 7858	PRODUCT = K = (−) .128 2313	4.000	4.000
.25/PROD. = .2500000	COL. 20α = (+) .560 9620	COL. 22α = (+) .827 8411	4.0 − PROD. R = (+) 1.6760281	4.0 − PROD. S = (−) 2.9719157
DIVIDEND = d = (+) 2.668 9128	COL. 20β = (+) .360 6309	COL. 22β = (+) .932 7089	900.00	900.00
	DIFFERENCE = (+) .200 3311	DIFFERENCE = (−) .104 8678	DIV. = T = (+) .0018623	DIV. = V = (−) .0033021
	DIFF. E = (+) .128	DIFF E = (+) .128	1.0000000	1.0000000
	PRODUCT = (+) .025 6424	PRODUCT = (−) .013 4231	1.0 − DIV. T = (+) .9981377	1.0 − DIV. V = 1.0033021
	PRODUCT J = (−) .183 7858	PRODUCT K = (−) .128 2313	PRODUCT M = (−) .3780632	PRODUCT M = (−) .4220709
	SUM = (−) .158 1434	SUM = (−) .141 6544	PRODUCT T = (−) .3773591	PRODUCT W = (−) .423
	DIVIDEND = d = (+) 2.668 9128	DIVIDEND = d = (+) 2.668 9128	cos ϕ_x = .880 1462	
	PRODUCT = M = (−) .422 0709	PRODUCT = M = (−) .3780632	DIV. V = (−) .429	

.25/Prod

APPROX. LAT. OF SHOT = 28° − 20.4′ (N)
*PRODUCT W = 0° − .42′ (S)
POSITION OF SHOT = SUM = 28° − 19.98′ (N)

APPROX. LONG. OF SHOT = 129° − 10.4′ (W)
*DIVIDEND V = 0° − .42′ (E)
SUM = 129° − 09.98′ (W)

* IF EITHER OF THESE ITEMS IS GREATER THAN 15′, REPEAT PAGES 2 TO 4 INCL., USING THE NEW SHOT POSITION.

SINES F/2, G/2, H/2 CARRY SIGNS OF THEIR ANGLES MODIFIED AS FOLLOWS:—
SINE 0.0000° TO 90.0000° = SINE AS GIVEN
SINE 90.0001° TO 180.0000° = SINE (180° − ANGLE)
SINE 180.0001° TO 270.0000° = −SINE (~~180° − ANGLE~~) ANGLE − 180°
SINE 270.0001° TO 359.9999° = −SINE (360° − ANGLE)

Summarizing the preceding computations we found that the following positions were ascertained:-

Method Used	Approximate Position	Corrected Position
Mathematical formula	28°-22.8'(N)129°-10.7(W)	28°-20.00(N)~~129°-~~ 129°- 09.98'(W)
Stereographic Chart	28°-35.0'(N)129°-10.0'(W)	28°-20.04'(N)~~129°-~~ 129°- 09.96'(W)
Sofar Position Plotting Sheet	28°-19.2'(N)129°-09.5'(W) or 28°-24.2'(N)129°-14.7'(W)	
	using28°-00.0(N)129°-00.0'(W)	28°-20.36'(N)~~129°-~~ 129°- 10.39'(W)
	using 28°-20.4(N)129°-10.4'(W)	28°-19.98'(N) 129°-09.98'(W)

In presenting the problem, the peak cut-offs were computed from a signal source of 28°-20.0'(N); 129-10.0'(W) and an explosion time of 12^{h}-00^{m}-00.00^{s}.

J. PROPOSED IMPROVEMENTS IN SOFAR POSITION PLOTTING TABLES.

The tables, at present covering positions 1° of latitude and 30' of longitude apart, should be enlarged to cover points 20' of latitude and 30' of longitude apart. This would mean that product W or dividend V in the correction formula would rarely exceed 15' if a reasonable correct assumed signal source were chosen.

As soon as enough data on axial sound velocity throughout an ocean is available, contour charts showing its variation should be constructed. From these the axial sound velocity at each random station should be estimated and incorporated in the tables. This is necessary for use in computing residuals D and E.

The travel times from the random points to each monitoring station should be computed and substituted for the present mileage column in the tables. This would shorten the steps necessary to find residuals D and E.

K. PROPOSED IMPROVEMENTS IN SOFAR POSITION PLOTTING SHEETS.

Using the information computed as described in Section J, Sofar Position Plotting Sheets should be constructed showing isodifference of travel time curves instead of the present isodifference of distance curves. This would insure greater accuracy in determining the approximate position of signal source.

APPENDIX I

LOCATION OF HYDROPHONES

In order to locate ~~definitely~~ the exact position and depth of a hydrophone, a searching vessel may fire bombs as below described, from KNOWN positions in the area in which the hydrophone is situated.

It is assumed that the hydrophone will be located so far off shore that horizontal sextant angles (such as were used at Eleuthera) for determining the search vessel's position would be inaccurate. Therefore Shoran, Radar, and Loran methods must be used.

To ~~gain~~ obtain the greatest possible accuracy, it is necessary that the search vessel remain as nearly stationary as weather conditions permit during the firing and recording of the shots, hence the experiment should be carried ~~on~~ out in calm weather so as to cut ~~down~~ to ~~the~~ a minimum all errors caused by drift, surface noises, etc.

It is proposed that all signals heard at the receiving station be fed directly into a radio (preferably an FM) and relayed practically instantaneously to the search vessel where all recording equipment will be housed.

The recording equipment shall consist of a hydrophone dropped over the side with a suitable amplification unit and loud speaker attached, a break circuit chronometer with attached signal key, a radio to receive signals from the receiving station and a suitable recording device similar to the Brush instrument as modified by Dr. J. A. Peoples. The radio, break circuit chronometer, key, hydrophone amplifier will be connected in parallel, all feeding into the modified Brush instrument.

The routine of the experiment shall consist of the search vessel proceeding to a point where the hydrophone is supposedly located. When she has LOST ALL WAY a BT lowering should be made, fathometer reading recorded and ship's position determined by the most accurate method available. After a suitable keying signal has identified the event on the modified Brush recorder tape, it is proposed to drop two bombs over the side simultaneously:- One bomb shall consist of a surface cap shot similar to those used at Eleuthera. The difference of time between the its explosion and its receipt at the receiving station (as relayed by radio to the search vessel) will be recorded on the modified Brush instrument. The other bomb shall consist of a Woods Hole Mark 34 bomb so designed that the charge is large enough to be heard, together with its echoes, through the search ship's hydrophone but SMALL enough so that it will not injure the searched hydrophone if the explosion takes place in ~~at~~ close proximity to it. The difference of time between the explosion and echoes as heard through the search vessel's hydrophone and ~~its~~ their receipt at the receiving station shall be recorded on the modified Brush instrument.

From the time difference between explosion and echoes of the Woods Hole Mark 34 bomb, calculations can be made to determine the water depth and the explosion depth. Any variation between the water depth thus determined and that shown by the fathometer might be due to:

1. Ship's leeway between dropping time and explosion time.

2. Failure of bomb to drop vertically due to underwater current.

3. Slope of bottom.

Shots should be dropped and recorded as above explained at different positions. As the ~~searched~~ Sought hydrophone is neared the time-distance interval between the explosion of the surface cap shot and its receipt at the receiving station will diminish until it coincides with the water depth. Similarly the receiving station will receive the modified Mark 34 bomb signal before it is heard on the search vessel by an amount equal to the difference in time-distance values between explosion depth and water depth less explosion depth.

Of course it is not probable that the search vessel will actually fire a bomb directly over the searched hydrophone. However, the data from the modified Brush instrument will provide two solutions for determining the hydrophone's position.

The BT lowering should provide data in respect to the sound velocity from the point of explosion to the bottom and to the surface. The various BT lowerings at each position should provide a reasonable accurate means of judging the velocity from each explosion to the searched hydrophone. From there on, it will be a problem in triangulation, using surface cap shots data for one approach and modified Woods Hole 34 bomb data as a second.

APPENDIX II

ATLANTIC RIDGE 11-45.

Loran fixes determined the reported positions of shots 505, 506 and 507. Two previous Loran fixes seemed so far from the Muir's course that these positions were viewed with some question and were given a "B" rating. The apparent velocities of these shots were 4894.8 f/s, 4903.5 f/s and 4876.9 f/s respectively.

The positions of shots #508 to #512 inclusive were figured on a DR basis using the position of shot 507 as a point of departure. All the apparent velocities of these shots were low (ranging from 4824.5 f/s to 4844.2 f/s, and it was felt that the Muir was considerably east of her reported positions. This was ~~somewhat~~ partially confirmed by a running fix of the sun using lines of position obtained at GCT 1324, 1404, 1453 and 1557. This 1557 running fix (shot 512 was dropped at 1100) plotted with a radius of about 1.8 miles with its center at a point 14.5 miles east of her DR position. On this information the Muir's position for shots 508 to 512 inclusive was replotted and the shot position classified as "C". The velocities calculated from the first four replot positions ranged low from 4851.1 f/s to 4871.1 f/s; but ~~position~~ the velocity of #512 was high at 4896.4 f/s. Incidentally, though the Muir was proceeding east at a constant speed, the travel time of these shots would indicate that, though they were dropped exactly two hours apart, the Muir's progress over the bottom was not constant!

As of 2020/18 a star fix was plotted consisting of two sights of Fomalhaut, two sights of Vega and a sight each of Marfak and Capella. Ignoring one of Vega's lines of position, the balance gave a fix with a possible radius of two miles. (Parenthetically a + 8.6' sextant correction would put this thrown-out Vega line of position in with the other lines of position. This is mentioned here as a similar correction on certain other lines of position in later fixes would tend to indicate that one sextant required an instrument correction factor.) The positions of shots #514 and #515, fired at each side of this fix, were rated as "A" despite the fact that their positions as plotted at Woods Hole were 1.9 miles west of the Muir's reported positions. These (nor was shot #516-rated "B") were not monitored at Eleuthera.

The positions of shots 517 and 518 were rated "B" as they were fired 2^h-40^m and 3^h-40^m respectively after the 2020/18 fix. The position of shot 518 was confirmed within a distance of two miles on Muir's DR course, by a moon line of position taken at 0021/19. The moon line of position however crosses the Muir's course at an angle which may indicates that she may have been as much as 7 miles east of her reported position and 5 miles east of the replotted position. Velocities computed from both positions for both shots are low, ranging from 4870.3 f/s to 4875.6 f/s.

The position of shot 519 was replotted 0.9 miles south of the Muir's reported position. There was a considerable gap in navigational data available at Woods Hole between this shot and an excellent running fix plotted as of 1½ hours earlier. Hence this position was rate "B". It is, however, confirmed by an LAN observation taken 12 minutes earlier. The velocity for Muir's reported position was 4889.9 f/s from the replot 4889.6 f/s.

A star fix, consisting of a sight each of seven stars was plotted as of 2040/19. The Hamal and Rasalague lines of position were out of line but the balance made a fix of 1 mile radius.

A star fix consisting of a sight each of Jupiter, Mars and 4 stars was plotted as of 0907/20. The line of position for Arcturus was disregarded as away out of line but the balance gave a fix of 1½ mile possible radius. The position of shots #520 to #528 inclusive were spaced between these fixes. Shots #520, #521 and Dud #522 were rated ~~at~~ as "C", Shots #523, #524, Dud #525 and Shot #526 were rated as "B" and shots #527 and #528 were rated as "A".

The next star fix, plotted as of 2033/20, consisted of four stars, three of which made a small triangle. The fourth, Kochab, had a line of position nearly parallel with Fomalhaut ~~put~~ but about 3 miles away from the small triangle and was disregarded. This fix was used to determine course and speed from the 0907/20 fix. Various sun sights were taken between these fixes and the shot positions were graded as to accuracy with these observations in mind. Shots #529, #530, #531 and #532 were rated "A", Shot #533 as "B", Shot #535 as "C", and Shot #536 as "A".

The next star fix of 0835/21 of 2 stars and Jupiter was of 1¼ miles possible radius. This was confirmed at 1414 (LAN) by a running fix with sun lines advanced from observations at 1121 and 1221. Soon after this the Muir maneuvered to carry out gunnery practice. The DR record was confused through lack of competent record.

colloquial

~~8~~

Two sun observations, one at 1753, the other at 1815 indicated that she was at 23-58.5 (N) 36-04.0 (W) for shot #537 dropped at 1800. A star fix as of 2000/21, consisting of eight stars gave a psotion of 23-52.0 (N), 34-39.4 (W). This fix plotted with 5 lines of position crossing at a single point, the other three varying from one to three miles away. This was the best and most reliable looking fix on the whole cruise from a replot standpoint. but was obviously in error as it meant a run of 78 miles in 2 hours. The Muir evidently plotted this fix at 23-50 (N) 35-37.0 (W), making her DR position for shot #537 at 24-05 (N) 36-14.0 (W). This apparently ignored the sun observations of 1753 and 1815. The replot position of shot #537 was therefore classified as "D". It is interesting to note that the velocity on this shot was 4891.6 from Muir's reported position and 4914.4 from the replot, indicating that the Muir's position was more accurate.

A running fix was plotted as of 1349 of about one mile radius and consisting of sun observations at 1042, 1235 and LAN at 1349. The position of shot #538, fired at 1500, was computed by DR from this position on a course to the next star fix plotted as of 2000/22. This was rated "B" as the 2000/22 star fix had a radius of over 3 miles and was plotted as of 5 hours later.

The replot position of shot #539, fired at 1200, was rated "B" as it was plotted back from a highly questionable running fix as of 1325. Of the four sun observations two were obviously at fault. The other two crossed the DR course at approximately the DR position. The two faulty sun lines would have been more probable with the + 8.6' sextant connection noted above in the 2020/18 fix.

h

The replot position of shot 540 (not heard at Eleuthera-as blanketed by the Cape Verde Islands) was rated "B" as it was fired 2 hours after a 0700/24 star fix.

After a short stop at Dakar the Muir left on a course of 347° T. Speed 18 knots. This would place the position of shot 541, as reported by the Muir, at 16-26.0 (N) 18-04.5 (W). However, a star fix consisting of 4 stars as of 1900/26, one hour before firing time, gave a fix of 16-03.5 (N) 18-11.6 (W) about 18 miles off her course though less than 5 hours out of Dakar. Each of three of the stars in this fix had lines of position crossing at a single point. The fourth was away out but by applying +8.6′ sextant correction, its line of position would confirm the others within 0.8 miles. This shot position was rated "A" on the basis of the fix but later discarded as the fix was apparently questionable on the DR basis and the travel time to Eleuthera.

An attempt to arrive at a star fix as of 1900/27 by the Muir gave a large triangle formed by lines of position of Altair, Deneb and Vega. The line of position of a fourth star, Alpheratz, was nearly parallel to that of Altair but lay as far to the west of their DR position as did Altair's line of position lay to the east of it. This caused the suspicion of a +8.6′ instrument error mentioned above, which, if applied to the Alpheratz and Altair observations whose azimuths lay abeam on opposite sides, would place the vessel on her DR course. Whether to apply this same correction to Deneb and Vega was questionable as there was no data available as to the number of officers taking observations. If Deneb's observation were not so corrected, it would be wide of Vega's uncorrected observation. If corrected, and Vega's line of position were left uncorrected, a fix of less than one mile radius could be plotted. If Vega's observation were corrected, Deneb's observation, whether corrected or not, would be out. On the basis of a later fix, 0727/28, the one mile fix seemed more probable. Shot #542 was located from this fix and was rated "D".

A star fix of seven stars plotted as of 1914/28 showed a large triangle of 3½ miles possible radius consisting of lines of position of Altair, Marfak and Fomalhaut. Hamal's line of position was wide of this triangle unless the +8.6′ sextant correction was applied. A small triangle of less than ½ mile possible radius was formed close to the center of the larger one by the lines of position of Polaris, Deneb and Vega.

The next star fix was plotted as of 0744/29. This made a fix of 1½ mile radius by using three stars, the fourth star's line of position being about 6 miles away from the DR position.

The positions of shots #543, #546, #547, #548 and Duds #544 and #545 were layed out on the course between the two above fixes. Their ratings were #548 as "B", #543 and #547 as "C" and #544, #545, and #546 as "D".

From the 0744/29 fix the Muir's course was replotted through a running fix as of LAN 1320/29 to a 1917/29 star fix of 1 mile possible radius and thence to an excellent running fix as of LAN at 1353/30. Positions for shots fired on this run were layed on this course on an average speed basis. Those rated "A" were #552, #553, #556, #558, #572, and Dud #557. The rest of the positions were rated "B" and comprised Duds #549, #555 and shots #550, #551, #554, #561, #564, #567 and #569.

J

The six star fix plotted as of 1926/30 had a possible radius of 1 mile and shot #572A fired between this and the 1353/30 running fix was classified as "B" as a sun observation taken at 1607 placed the Muir 2½ miles east of her course.

A running fix at LAN 1433/1 had about a 1½ radius of possible positions. Position of shots fired between this and the 1926/30 fix were classified as "A" for #573, "D" for #574 and #577 and "C" for #575 and #576.

A fix of 3 mile radius was plotted as of 2000/1 on observations of 4 stars. During the interim between this fix and the 1433/1 fix a change of course had been made and no record is available at Woods Hole as to when this occurred. A 1626 ~~sin~~ SUN line of position further confuses the issue. Therefore there seem to be two equally logical positions where shot #578 may have been fired. These positions, both computed, were rated "B".

Shot #579's position was determined by a Loran fix and rated as "B". (See remarks on shots #505, #506, and #507).

A five star fix of 1½ mile radius was plotted as of 2100/4. This, together with two sun observations taken at 1213 and 1419, was used to determine and classify the positions of shot #580, rated "C", shot #581 rated "B", and shot #582 rated "A".

"The position of shots #583 to #586 inclusive were determined by Loran fixes and classified as "B"."

APPENDIX III

FORMULA FOR GREAT CIRCLE DISTANCE:

Let Z be the spherical distance between two points in latitude Φ_1 and Φ_2

$$\cos Z = (\sin \Phi_1)(\sin \Phi_2) + (\cos \Phi_1)(\cos \Phi_2)(\cos \text{difference in longitude})$$

$$\cos Z = \cos(\phi_2 - \phi_1) - (\cos\phi_1)(\cos\phi_2)(1 - \cos \text{difference in longitude})$$

FORMULA FOR CORRECTION FOR ELLIPTICITY:- Based on the Clarke spheroid of 1866

Compute: $x = \cos(\Phi_2 - \Phi_1)$

$y = \cos \Phi_1 \cos \Phi_2$

$\text{arc } Z = Z \text{ in minutes} \times .01745329$

$$\delta Z \doteq \frac{(3 \sin Z - \text{arc } Z)(1+x)(1+X-2y)}{1+\cos Z} - \frac{(3 \sin Z + \text{arc } Z)(1-x)(1-x+2y)}{1 - \cos Z}$$

Elliptical distance (in miles) = 1.0011197 times Z in minutes + 2.9168 times δZ

FORMULA FOR INITIAL GREAT CIRCLE COURSE from point in latitude Φ_1 to point in latitude Φ_2

$$\text{Sin course} = \frac{(\sin \text{difference in longitude})(\cos \Phi_2)}{\sin Z}$$

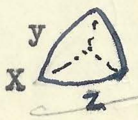
When ~~they~~ difference in longitude is West the sin is negative.

When the sin of course is negative the bearing is west of the elevated pole.

APPENDIX IV

Derivation of the Fundamental Equation

Most of spherical trigonometry can be handled by the use of the basic formula for spherical triangles:

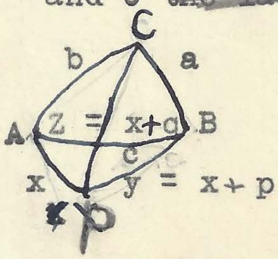


$$\cos x = (\cos y)(\cos z) + (\sin y)(\sin z)(\cos X). \qquad (1)$$

In ~~this~~ the above form, ~~it gives~~ a side is given in terms of the other two sides and their included angle; transposed, it gives ~~an~~ the angle in terms of the three sides:

$$\cos X = \frac{\cos x - (\cos y)(\cos z)}{(\sin y)(\sin z)} \qquad (2)$$

In the problem ~~we are dealing with we will~~ used for illustration let A be the first station to ~~record~~ receive the ~~explosion set off~~ signal originating at X, and B ~~the next~~, and C ~~the last~~ two other stations ~~to do so~~.



~~The great circle distances will be referred to by the letters in the diagram.~~ In Fig 16 three receiving stations at A, B, and C are known points; a, b, and c are known great circle distances between receiving stations. p and q are also known: they are the differences in the times of arrival of the signal reduced to distances.

Start

In order to ~~get a relationship that is easily handled, we~~ easily use the relationship expressed in Eq(2) ~~require~~ a formula expressed in terms of the cosines of the angles between the stations is required ~~rather than sines,~~ Let $\angle CAB = \alpha$, $\angle BAP = \beta$ and $\angle CAP = \gamma$. From the Fig 16 it is seen $\angle CAP = \angle CAP + \angle BAP$ ~~for otherwise (2) is not applicable. For example, if α, β and γ are any three angles for which α + β =~~ or $\alpha + \beta = \gamma$ ~~, this relationship may be reduced to cosine form as follows:~~ The cosine form for this relationship is

$$\cos\gamma = (\cos\alpha)(\cos\beta) - (\sin\alpha)(\sin\beta) \qquad (3)$$

transposing

$$\cos\gamma - (\cos\alpha)(\cos\beta) = -(\sin\alpha)(\sin\beta);$$

squaring, $\cos^2\gamma - 2(\cos\alpha)(\cos\beta)(\cos\gamma) + (\cos^2\alpha)(\cos^2\beta) = (\sin^2\alpha)(\sin^2\beta)$

Since ~~But~~ $\sin^2 x = 1 - \cos^2 x$; ~~hence this may be written~~

$$\cos^2\gamma - 2(\cos\alpha)(\cos\beta)(\cos\gamma) + (\cos^2\alpha)(\cos^2\beta) = (1 - \cos^2\alpha)(1 - \cos^2\beta)$$

$$= 1 - \cos^2\alpha - \cos^2\beta + (\cos^2\alpha)(\cos^2\beta),$$

or $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 + 2(\cos\alpha)(\cos\beta)(\cos\gamma)$. (4) ~~(3)~~

From Fig 16 it is seen that another ~~Now our figure features several~~ set~~s~~ of angles at A ~~which have the property in question~~. For example, at the point A we must have either $\measuredangle CAB + \measuredangle BAP = \measuredangle CAP$

or

$$\measuredangle CAP + \measuredangle PAB = \measuredangle CAB$$

In either case, (3) holds, and from the first ~~this~~ relationship ~~may be derived what we want~~.

Let $\alpha = \measuredangle A = \measuredangle CAB$, $\beta = \measuredangle BAP$, $\gamma = \measuredangle CAP$.

Then, from ~~by~~ (2), with ~~if~~ p ~~is~~ the equivalent distance for the time lag in signal at B relative to A ~~corresponding to the first difference in signal times~~,

$$\cos\beta = \frac{\cos(x+p) - (\cos x)(\cos c)}{(\sin x)(\sin c)} = \frac{(\cos x)(\cos p) - (\sin x)(\sin p) - (\cos x)(\cos c)}{(\sin x)(\sin c)}$$

$$= \frac{\cos x\,(\cos p - \cos c) - (\sin x)(\sin p)}{(\sin x)(\sin c)} = \cancel{\cot x \frac{\cos p - \cos c}{\sin c}}$$

$$= \cot x \frac{\cos p - \cos c}{\sin c} - \frac{\sin p}{\sin c} \qquad (5)$$

Abbreviating expressions so that ~~If we abbreviate~~ $\cot x = w$, $\dfrac{\cos p - \cos c}{\sin c} = P$, $\dfrac{\sin p}{\sin c} = P'$

Eq (5) ~~this~~ may be written

$$\cos\beta = wP - P'$$

In ~~exactly~~ similar fashion, the ~~we may derive~~

$$\cos\gamma = wQ - Q',$$

where Q ~~means~~ $\equiv \dfrac{\cos q - \cos b}{\sin b}$ and Q' ~~means~~ $\equiv \dfrac{\sin q}{\sin b}$

From ~~By~~ (3), it is found ~~we find~~

$$\cos^2\alpha + (wP-P')^2 + (wQ-Q')^2 = 1 + 2(\cos A)(wP-P')(wQ-Q'), \qquad (6)$$

which is a ~~ordinary~~ quadratic equation in w. Rearranging it, it becomes

$$w^2(P^2-2PQ\cos A + Q^2) - 2w\,(PP' - [PQ' + P'Q]\cos A + QQ')$$
$$+ ([P']^2 - 2P'Q'\cos A + [Q']^2) - \sin^2 A = 0,$$

$$\text{whence } w = \frac{(PP' + QQ') - (PQ' + P'Q)\cos A \pm \sqrt{D}}{(P^2 + Q^2) - 2PQ\cos A}, \qquad (7)$$

close up

$$\text{where } D = \left[(PP' + QQ') - (PQ' + P'Q)\cos A\right]^2 - (P^2 - 2PQ\cos A + Q^2)\left([P']^2 - 2P'Q'\cos A + [Q']^2 - \sin^2 A\right).$$

This reduces to

$$D = 2\,PQ\sin^2 A\left(\frac{\cos(q-p) - \cos a}{(\sin b)(\sin c)}\right)$$

~~which suggests defining~~ Let $R = \dfrac{\cos(q-p) - \cos a}{(\sin b)(\sin c)}$;

whereupon

$$w = \cot x = \frac{(PP' + QQ') - (PQ' + P'Q)\cos A \pm \sqrt{2\,PQR}\,\sin A}{(P^2 + Q^2) - 2\,PQ\cos A} \qquad (8)$$

This determines the great circle distance x between A and P. ~~We have seen that~~ The bearing of P from A was defined by (4)

$$\cos\beta = \cos\angle BAP = wP - P',$$

~~and thus the great circle bearing of X at point A may be found.~~

Knowing both bearing and distance, ~~one may find~~ the latitude and longitude of ~~X~~ P can be determined by any of the standard methods.

end here

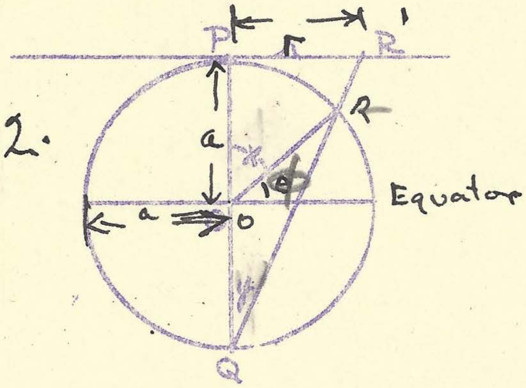
APPENDIX V.

This is too brief. If you don't want to expand it, refer to a Textbook which explains it.

Polar STEREOGRAPHIC PROJECTION

1. Polar Basis of Stereographic Projection.

2\.



Since the central angle is double the inscribed angle:

$x = 2y$

The stereographic scale PR' is thus given by

$r = 2a \tan y = 2a \tan x/2$

but x = ~~North~~ polar distance ~~if the projection is from the North pole,~~ then:

$= \tan y = \frac{r}{2a}$ where a = radius of earth

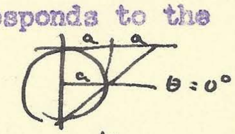
$x = 90^\circ - \text{latitude} = 90^\circ - \emptyset$

and,

$r = 2a \tan \left(\frac{90^\circ - \emptyset}{2}\right)$

Stereographic scale

When $\emptyset = 0$, $r = 2a \tan 45^\circ = 2a$ so that a radius of 2a corresponds to the equator.



If a 20 cm stereographic scale is desired we have: Diam earth = 20 cm

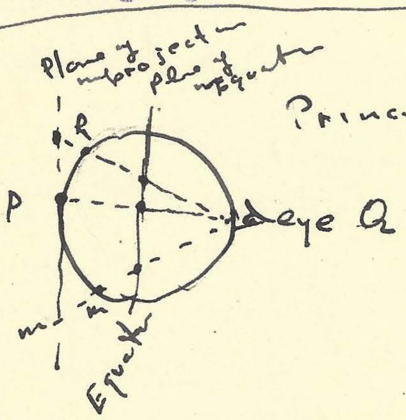
$r = 20 \tan \left(\frac{90^\circ - \emptyset}{2}\right)$ centimeters

2\. ~~Sample~~ Stereographic Chart.

In the following figure line PA is longitude 30°W. Point X is latitude 30°N.

Over

Principal



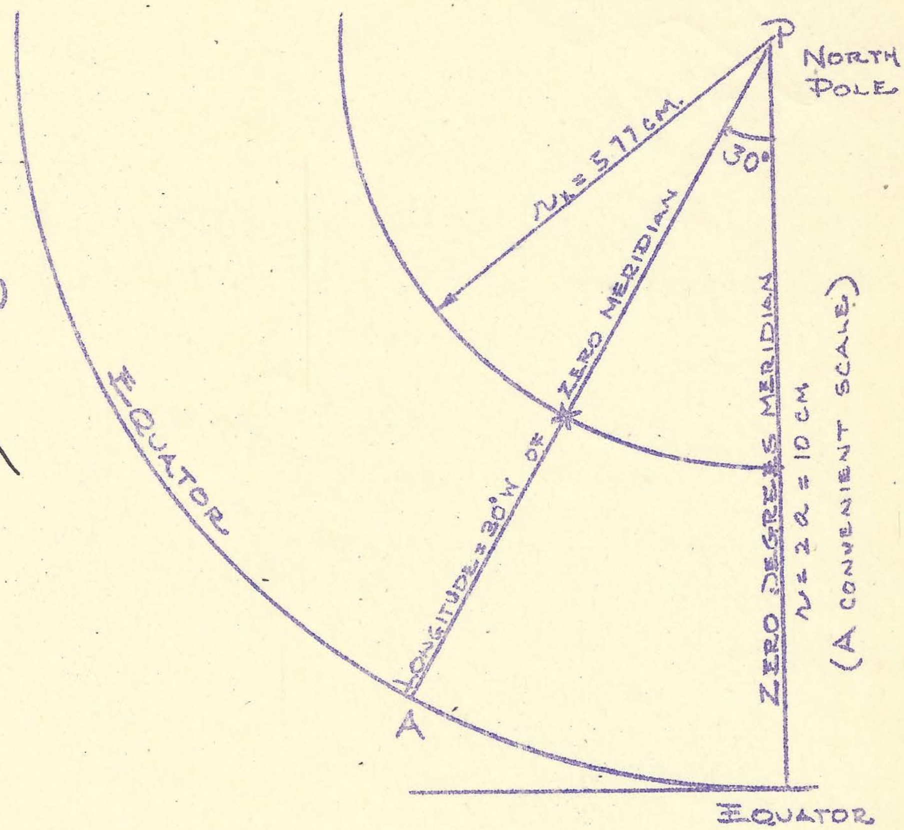
Use scale of earth diameter =
2a : 10 cm

$$r_x = 10 \tan \frac{(90^\circ - 30^\circ)}{2}$$

$$r_x = 10 \tan 30^\circ$$

$$r_x = 10 \,(.577)$$

$$r_x = 5.77 \text{ cm.}$$



Therefore, any point on the global surface can be reduced to a plane area projection for triangulation purposes.

A twenty centimeter scale (2a = 20 cm.) gives results which are accurate to approximately 15 miles.

Steps (1) set up Pole position and draw 30°W Long as locus of point to be plotted

(2) Compute distance from pole on basis 2a = 10 cm

$$r = 2a \tan \frac{(90^\circ - \theta)}{2}$$

$$r = 10 \tan \frac{(90^\circ - 30^\circ)}{2}$$

$$r = 10 \tan 30^\circ = 5.77 \text{ cm}$$

3 position on polar chart for line 1 at 5.77 cm from pole

APPENDIX VI

CONSTRUCTION OF SOFAR POSITION PLOTTING SHEETS

Sofar Position Plotting Sheets are Mercator ~~proportion~~ POSITION plotting sheets on which have been drawn curves showing iso-difference in distance between monitoring stations.

Start

The construction of these plotting sheets requires computations of distances to each monitoring station from a number of random points. For areas greater than 300 miles distant ~~for~~ from each monitoring station, adequate curves may be plotted from distances computed from random points selected on each whole degree of latitude and each half degree of longitude. For areas less than 300 miles from any station, more random points are required as the resultant hyperbola changes its curvature more rapidly.

The Great Circle distance, compensated for the earth's ellipticity, must be computed for each random point to each monitoring station. For Formulae used, see Appendix III. The distance~~s~~ from each random point to station A ~~are~~ is subtracted from the distance from the same point to station B. These differences in distance form the basis for the Elliptical Distance Difference Curve:- In this case, Distance to B less distance to A.

The Elliptical Distance Difference Curves are plotted by graphic interpolation which consists of laying a sheet of standard cross section paper across the plotting sheet so that its horizontal lines are parallel to a parallel of latitude. The differences in distance between a pair of stations to the random points which lie on this parallel of latitude are pointed off on the vertical lines of the cross section paper. A curve connecting these points will intersect an horizontal cross section paper line for any required quantity. In making the SOFAR Position Plotting Sheets an interval of ten miles was chosen and these points were plotted on the various parallels of latitude. Curves connecting these points are the Elliptical Distance Difference Curves.

For an area covered by four monitoring stations a plotting sheet with six curves may be drawn. To simplify the SOFAR Position Plotting sheets now being drawn for the Pacific, but four lines appear on each sheet. The sheets to be drawn will be numbered and lettered as follows:-

	Area	Curves
I-PA	30°-35°N 115°-122°W	Distance to Kaneohe less distance to Point Arena
		" " Hilo Bay " " " " "
		" " Kaneohe " " " Hilo Bay
II-M	40°- 45°N 122°-132°W	Distance to Kaneohe less distance to Hilo Bay
		" " Monterey " " " Pt. Arena
		" " Hilo Bay " " " Monterey
		" " Kaneohe " " " "
II-PA	40°- 45°N 122°-132°W	Distance to Kaneohe less distance to Hilo Bay
		" " Monterey " " " Pt. Arena
		" " Hilo Bay " " " " "
		" " Kaneohe " " " " "
III-M & III-PA	35°- 40°N 122°- 132°W	Same as II-M & II-PA
IV-M & IV- PA	30°- 35°N 122°-132°W	" " " " " " "
V-M & V-PA	24°- 30°N 122°-132°W	" " " " " " "
VI-M & VI-PA	35°-40°N 132°-142°W	" " " " " " "
VII-M & VII-PA	30°-35°N 132°-142°W	" " " " " " "
VIII-M & VIII-PA	24°-30°N 132°-142°W	" " " " " " "
IX-M & IX-PA	18°-24°N 132°-142°W	" " " " " " "
X-M & X-PA	35°-30°N 142°-152°W	" " " " " " "
XI-M & XI-PA	24°-30°N 142o-152°W	" " " " " " "

111

XII-K	18°-24°N 142°-152 W	Distance to Monterey less distance to Pt.A. " " Kaneohe " " " Hilo Bay " " " " " " Monterey " " " " " " "
XII-HB	18°-24°N 142°-152°W	Distance to Monterey less distance to Pt. Arena " " Kaneohe " " " Hilo Bay " " Hilo Bay " " " Monterey " " " " " " " Pt. Arena
XIII-K & XIII-HB	12°-18°N 142°-152°W	Same as XII-K and XII- HB
XIV-M & XIV-PA	24°-30°N 152°-162°W	Same as II-M and II-PA
XV-K & XV-HB	18°-24°N 152°-162°W	Same as XII-K and XII-HB
XVI-K & XVI- HB	12°-18°N 152°-162°W	Same as XII-K and XIII-HB

stop to p 79

It has been found, as a check on the accuracy of the computations and plotting , that the following three curves if drawn on a ten mile interval basis, should intersect:-

A crossing of a Kaneohe less Monterey curve and an Hilo Bay less Monterey curve should fall on a Kaneohe less Hilo Bay curve.

A crossing of a Kaneohe less Point Arena curve and an Hilo Bay less Point Arena curve should fall on a Kaneohe less Hilo Bay curve.

A crossing of a Kaneohe less Monterey curve and a Kaneohe less Point Arena curve should fall on a Monterey less Point Arena curve.

A crossing of an Hilo Bay less Monterey curve and an Hilo Bay less Point Arena curve should fall on a Monterey less Point Arena Curve.

INDEX OF FIGURES

Figure

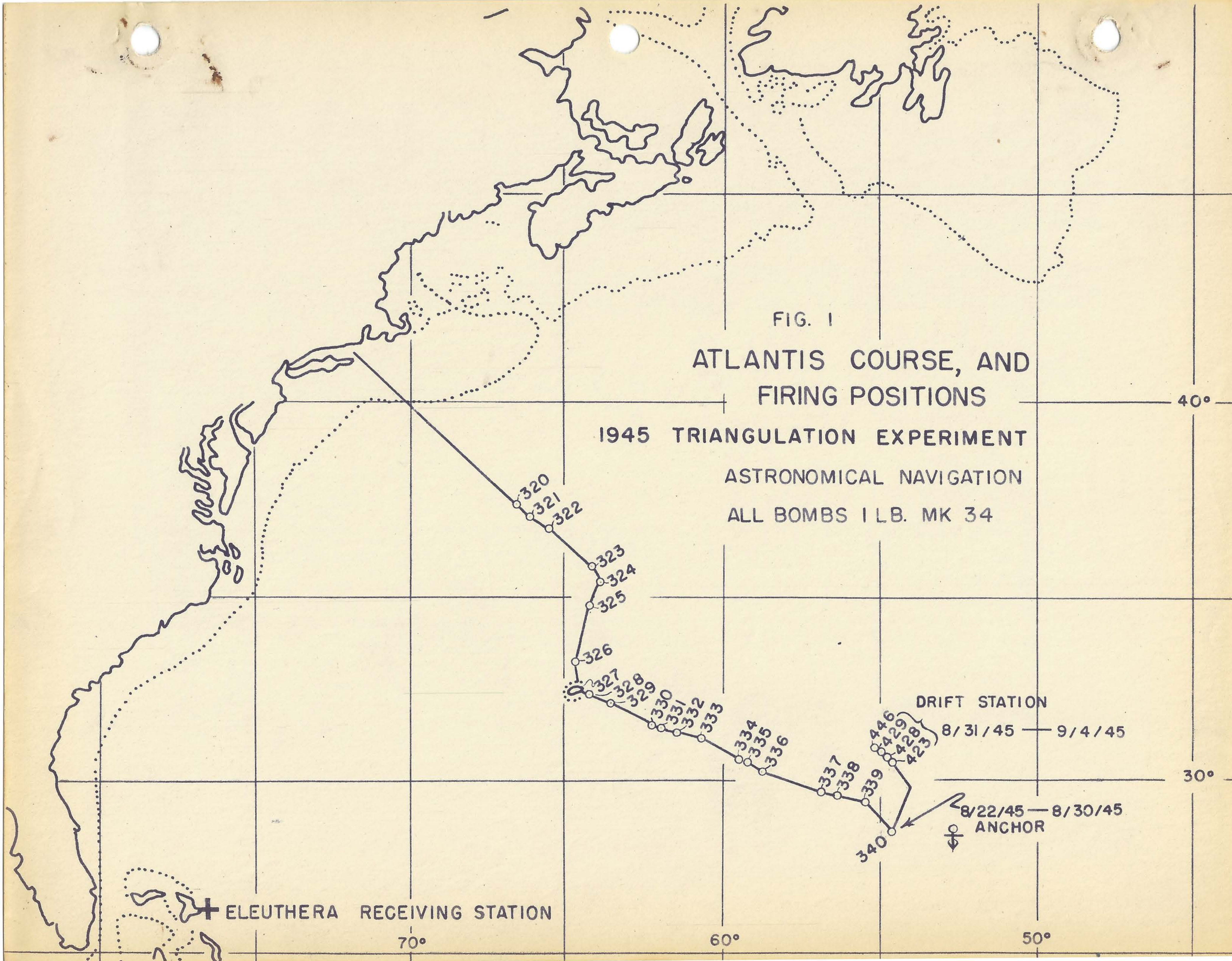


FIG. I
ATLANTIS COURSE, AND
FIRING POSITIONS
1945 TRIANGULATION EXPERIMENT
ASTRONOMICAL NAVIGATION
ALL BOMBS I LB. MK 34
320
321
322
323
324
325
326
327
328
329
330
331
332
333
334
335
336
337
338
339
340
DRIFT STATION
446
429
428
423
8/31/45 — 9/4/45
8/22/45 — 8/30/45
ANCHOR
ELEUTHERA RECEIVING STATION
40°
30°
70°
60°
50°

FIG. 2

ATLANTIS POSITION FOR SHOTS RECEIVED

1945 TRIANGULATION EXPERIMENT

+ ASTRONOMICAL FIX

990 991 992 ATLANTIS RECEIVING POSITION

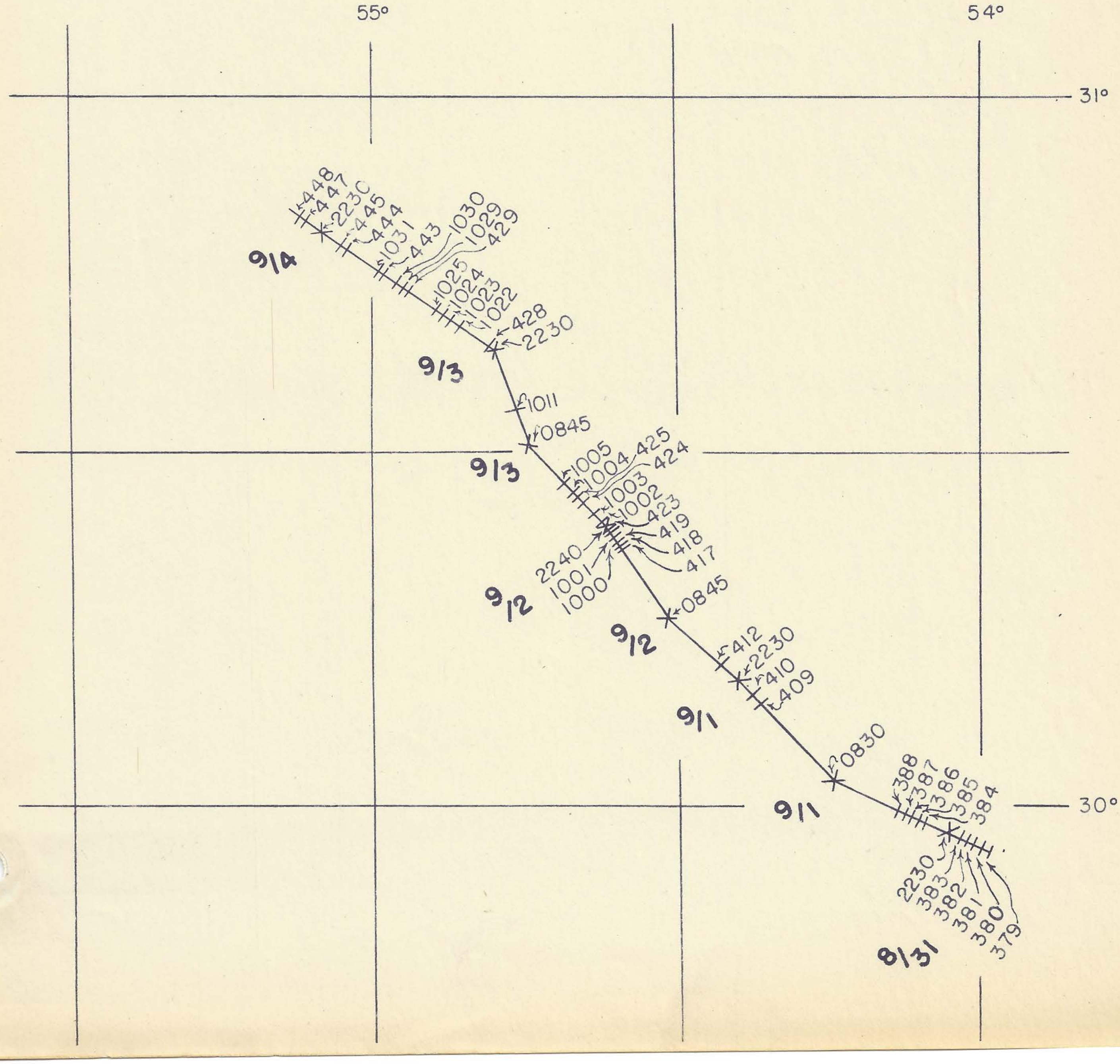


FIG. 3

VALOR POSITIONS

1945 TRIANGULATION EXPERIMENT

ALL POSITIONS DETERMINED BY LORAN

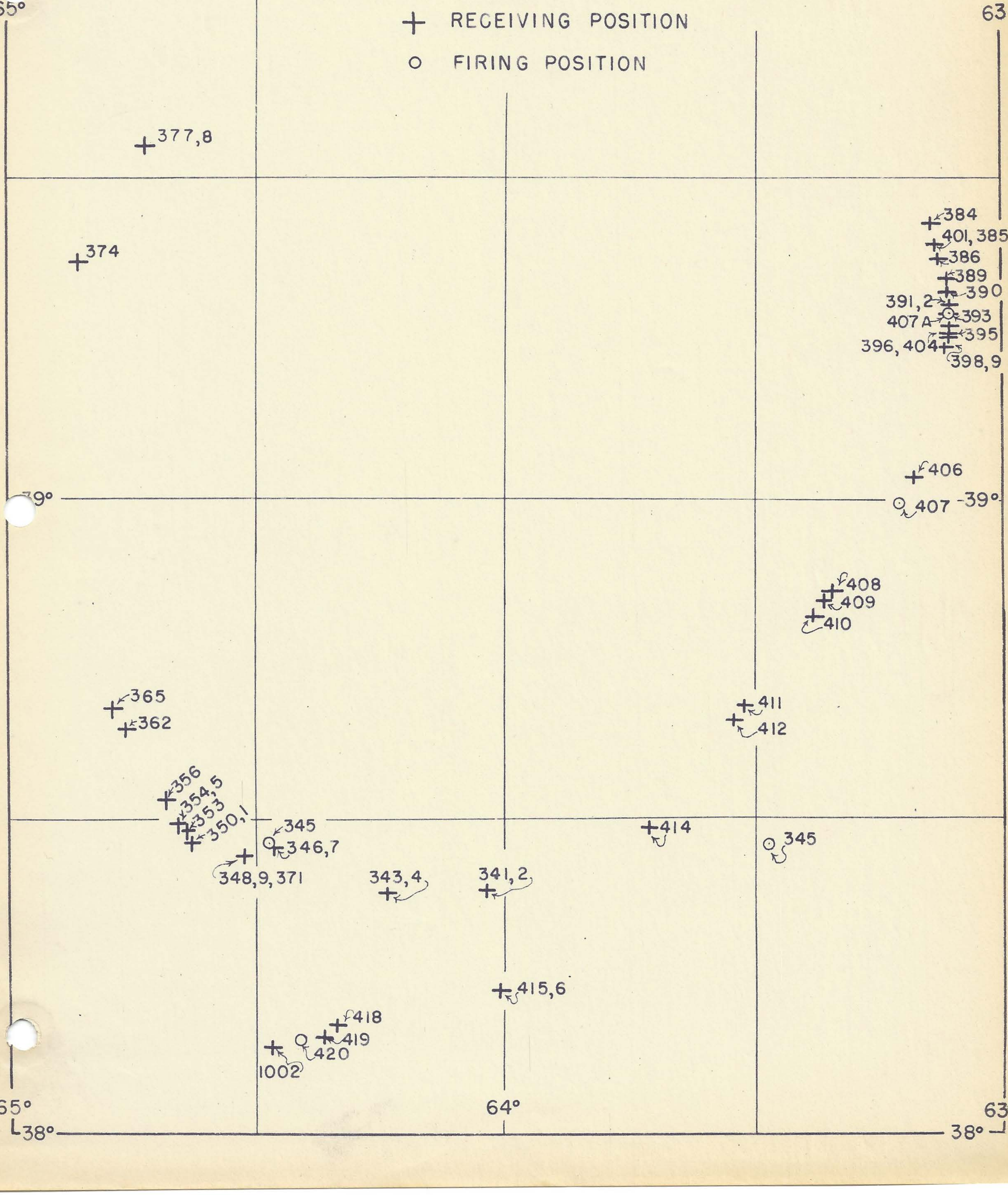


FIG. 4

U.S.S. MASON FIRING POSITIONS
1945 TRIANGULATION EXPERIMENT
ASTRONOMICAL AND LORAN
NAVIGATION

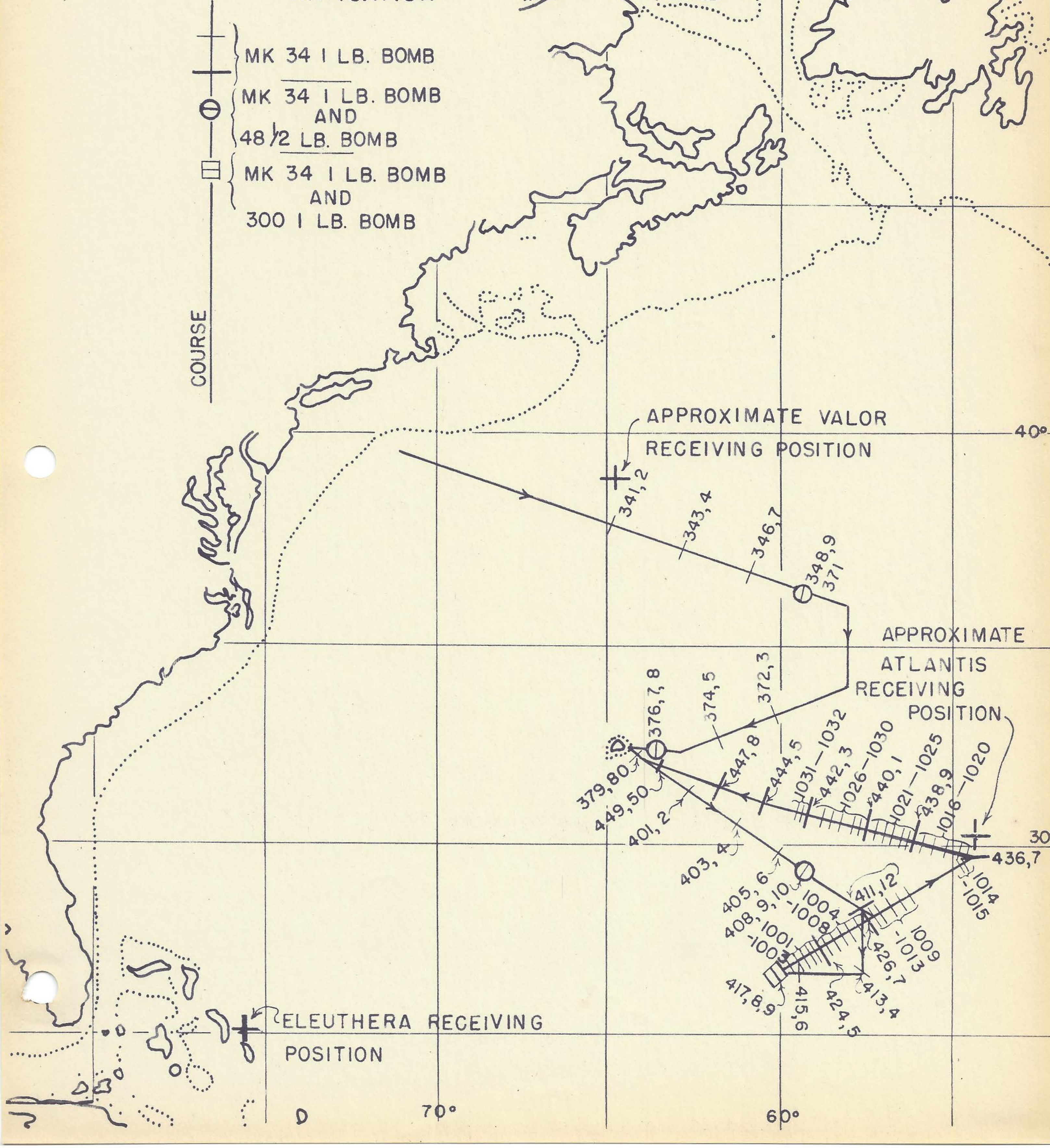
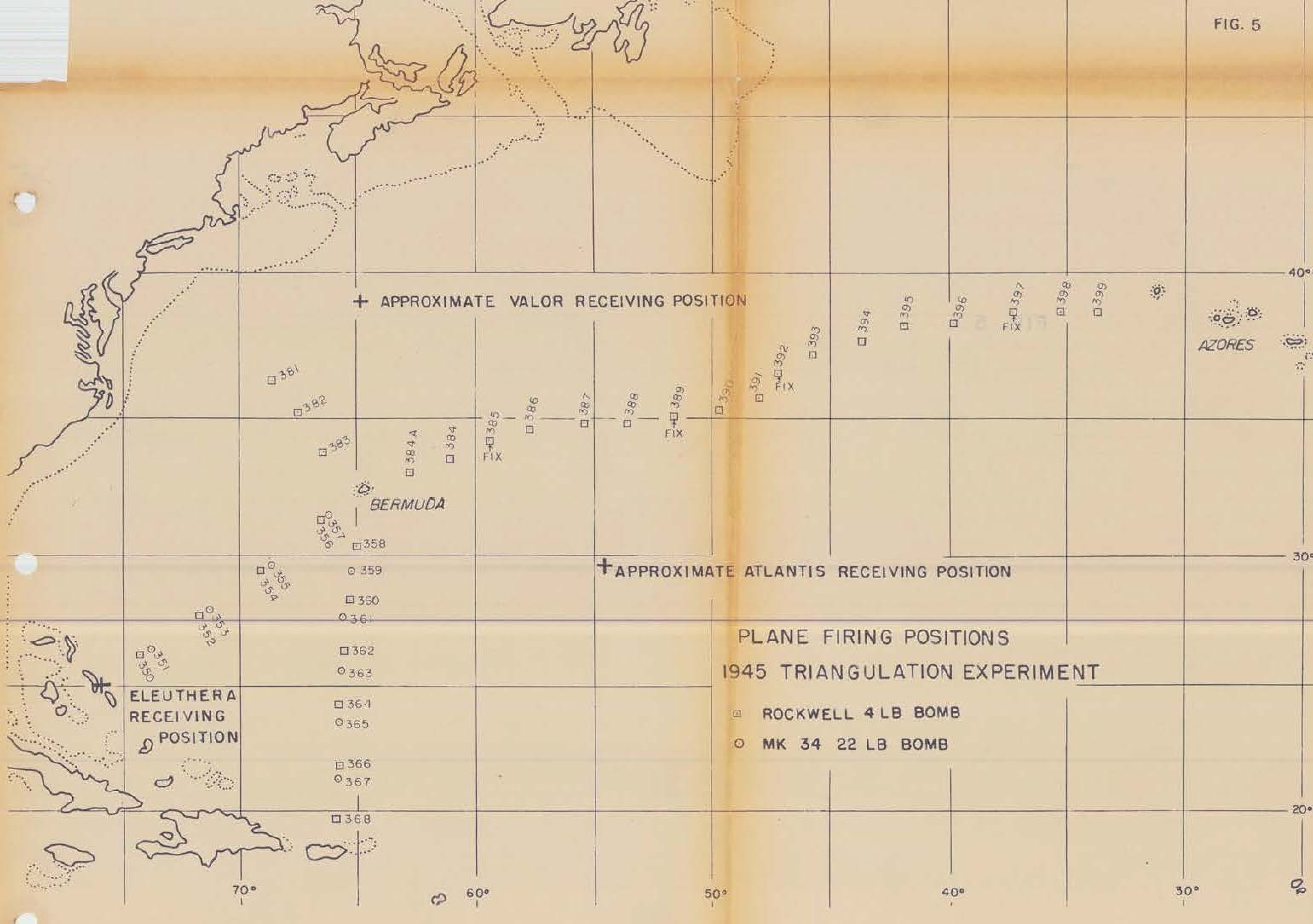


FIG. 5



+ APPROXIMATE VALOR RECEIVING POSITION
+ APPROXIMATE ATLANTIS RECEIVING POSITION
ELEUTHERA RECEIVING POSITION
BERMUDA
AZORES
381
382
383
384 A
384
385
FIX
386
387
388
389
FIX
390
391
392
FIX
393
394
395
396
397
FIX
398
399
350
351
352
353
354
355
356
357
358
359
360
361
362
363
364
365
366
367
368
40°
30°
20°
70°
60°
50°
40°
30°
PLANE FIRING POSITIONS
1945 TRIANGULATION EXPERIMENT
ROCKWELL 4 LB BOMB
MK 34 22 LB BOMB

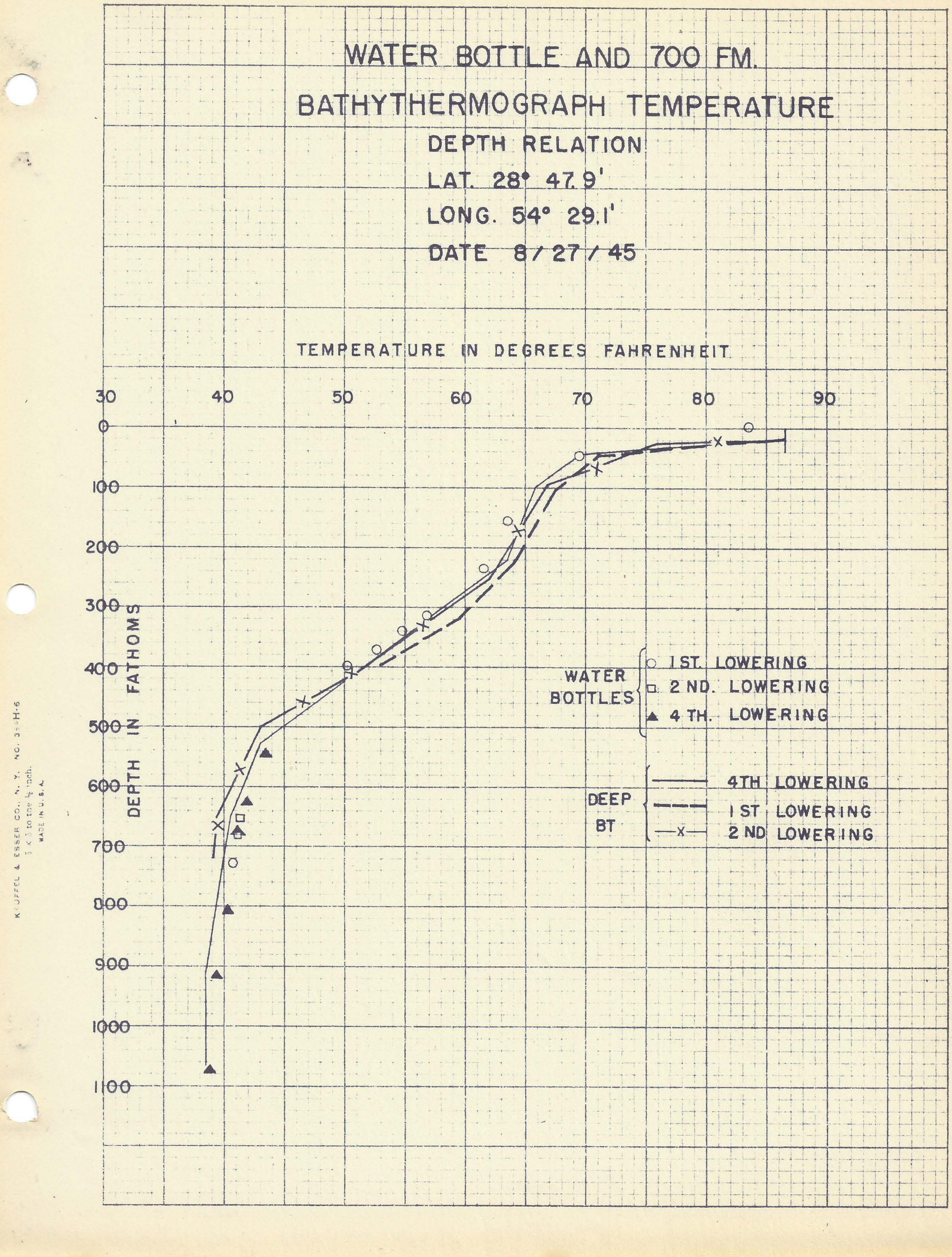
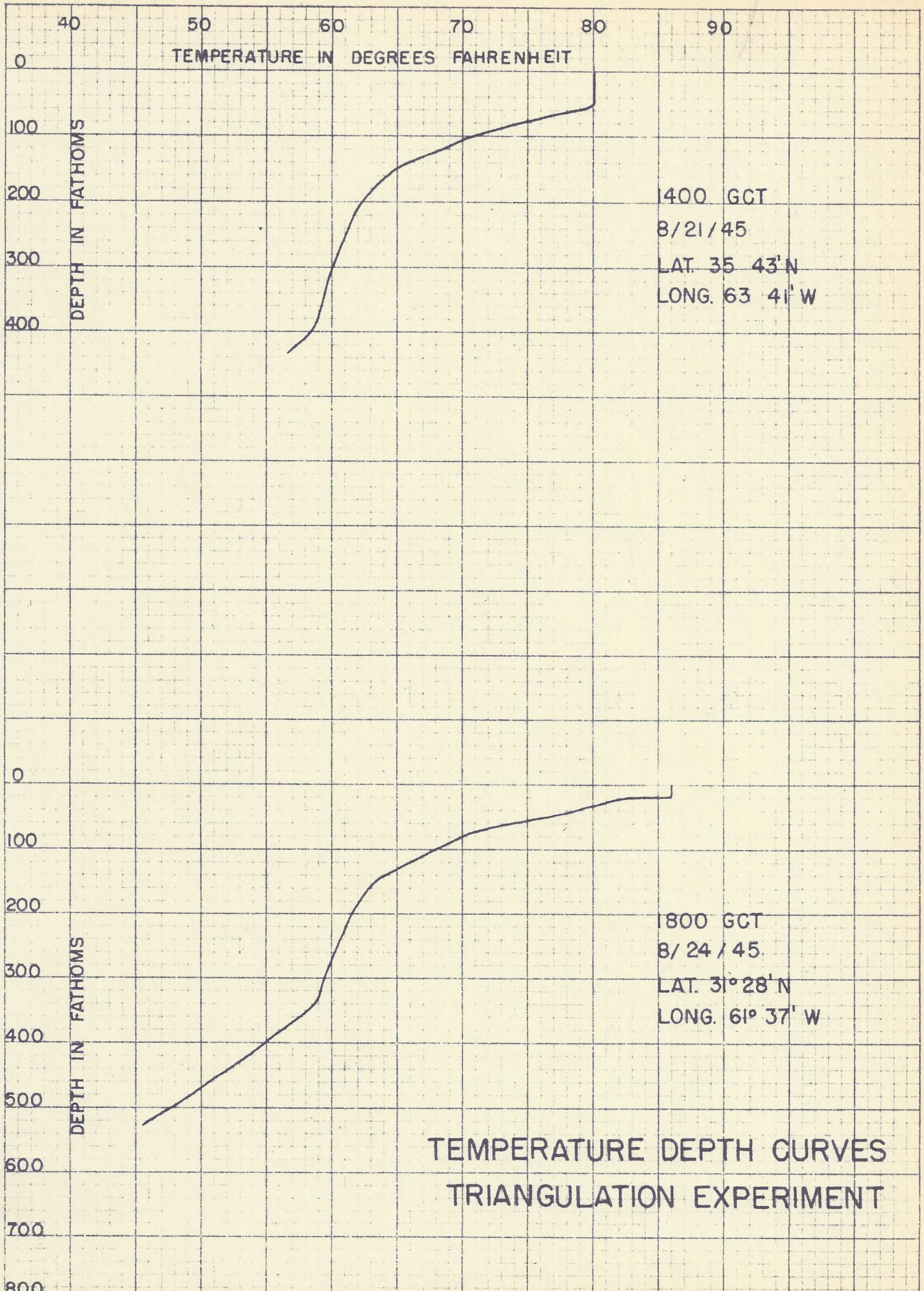

WATER BOTTLE AND 700 FM.
BATHYTHERMOGRAPH TEMPERATURE
DEPTH RELATION
LAT. 28° 47.9'
LONG. 54° 29.1'
DATE 8/27/45
TEMPERATURE IN DEGREES FAHRENHEIT
30
40
50
60
70
80
90
0
100
200
300
400
500
600
700
800
900
1000
1100
DEPTH IN FATHOMS
WATER BOTTLES
1 ST. LOWERING
2 ND. LOWERING
4 TH. LOWERING
DEEP BT
4TH LOWERING
1 ST LOWERING
2 ND LOWERING
KEUFFEL & ESSER CO., N. Y. NO. 359-H-6
5 X 5 to the ¼ inch.
MADE IN U.S.A.

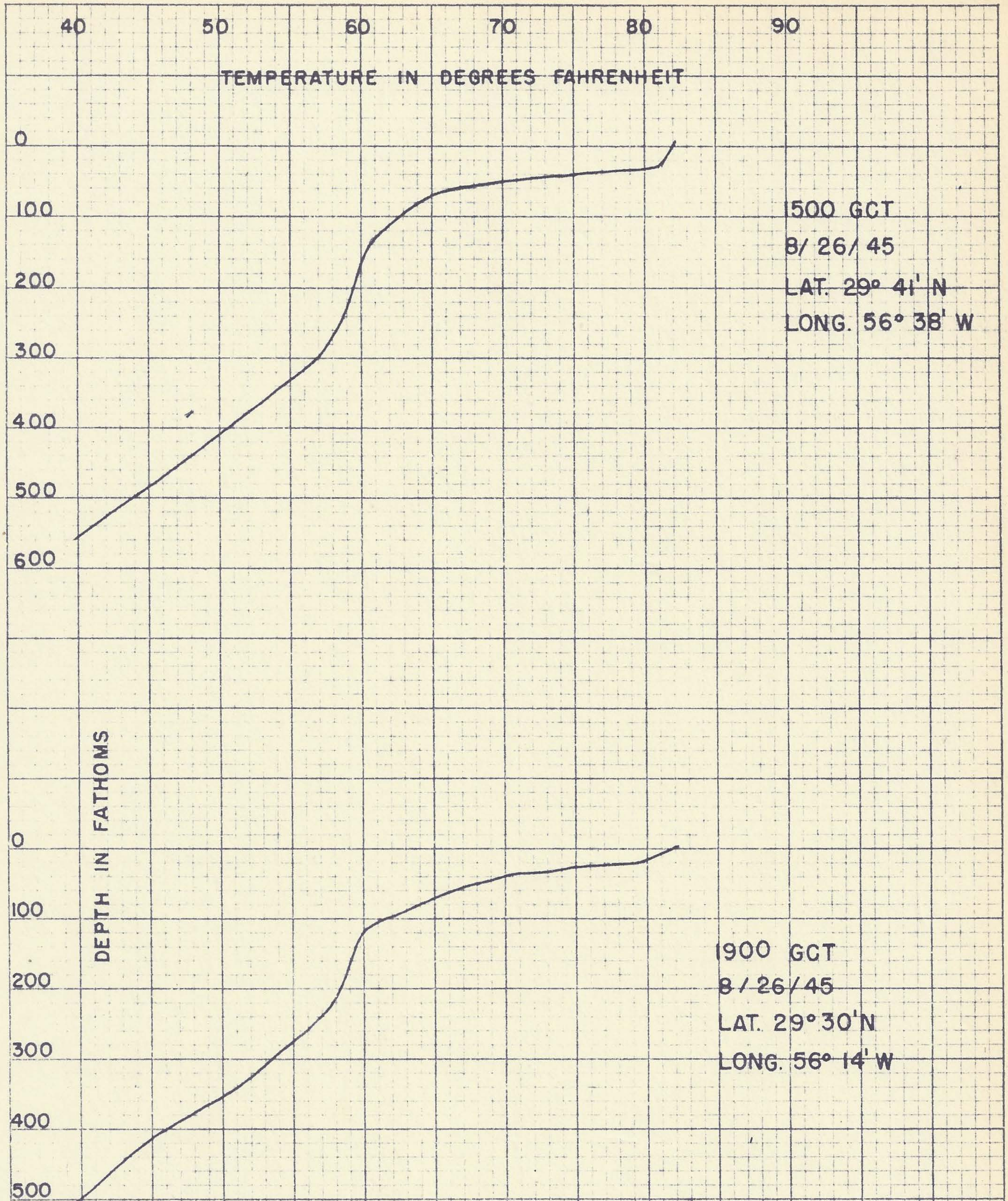
FIG. 5B



TEMPERATURE DEPTH CURVES
TRIANGULATION EXPERIMENT

KEUFFEL & ESSER CO., N. Y. NO. 358H-6
5 x 5 to the ½ inch
MADE IN U. S. A.

FIG. 5c



TEMPERATURE DEPTH CURVES
TRIANGULATION EXPERIMENT

FIG. 5D

Velocity depth curves - Triangulation experiment

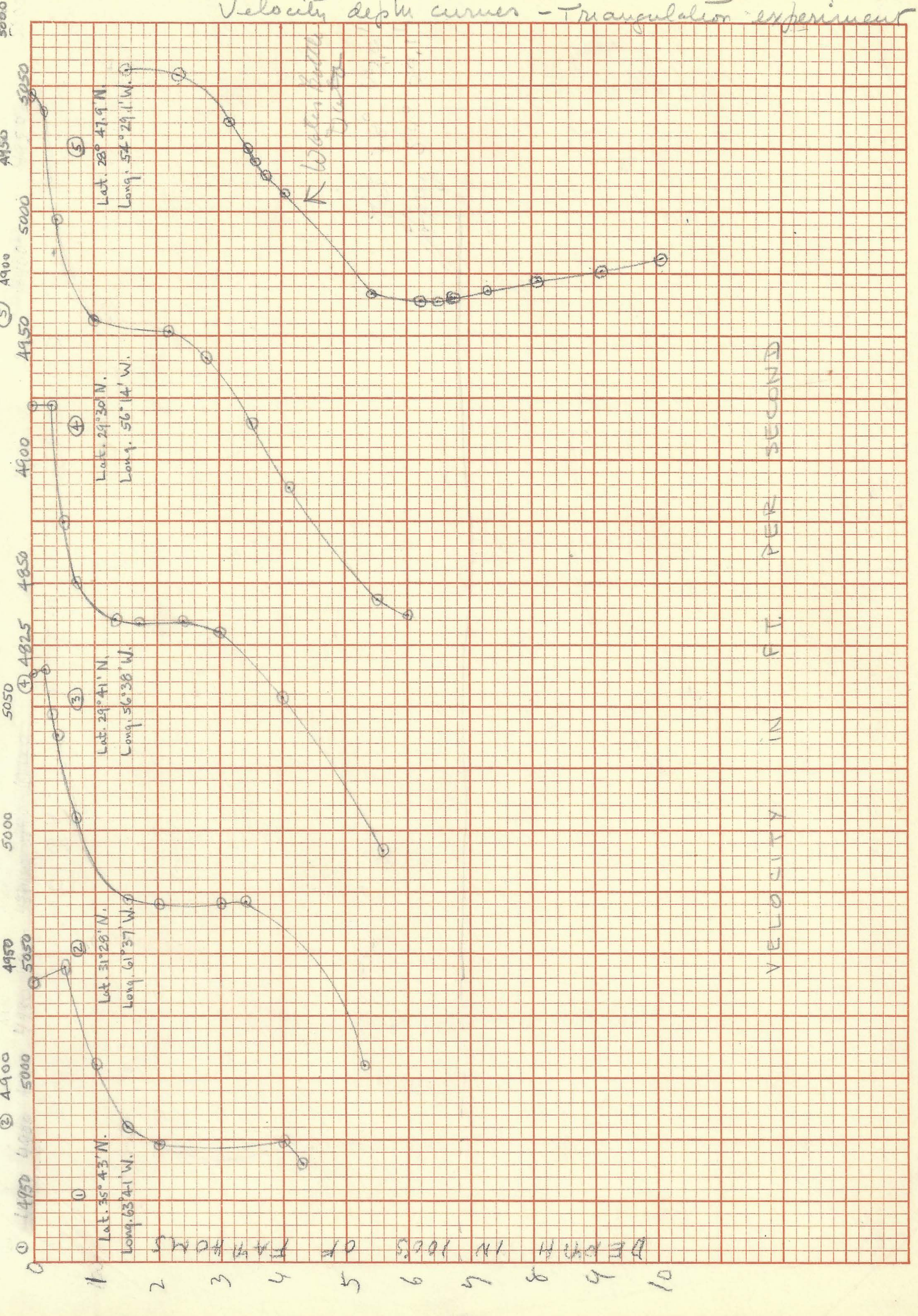


FIGURE 6

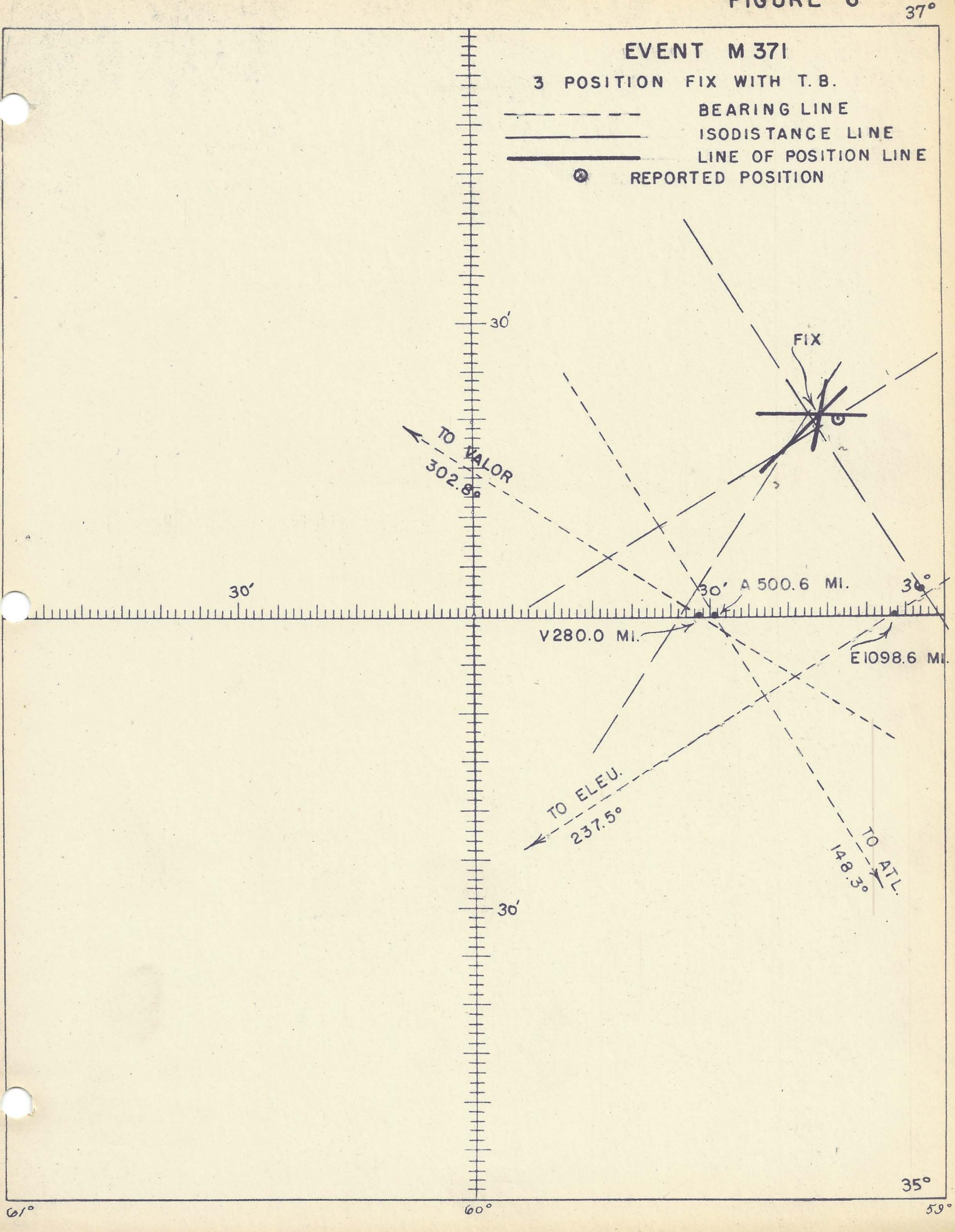
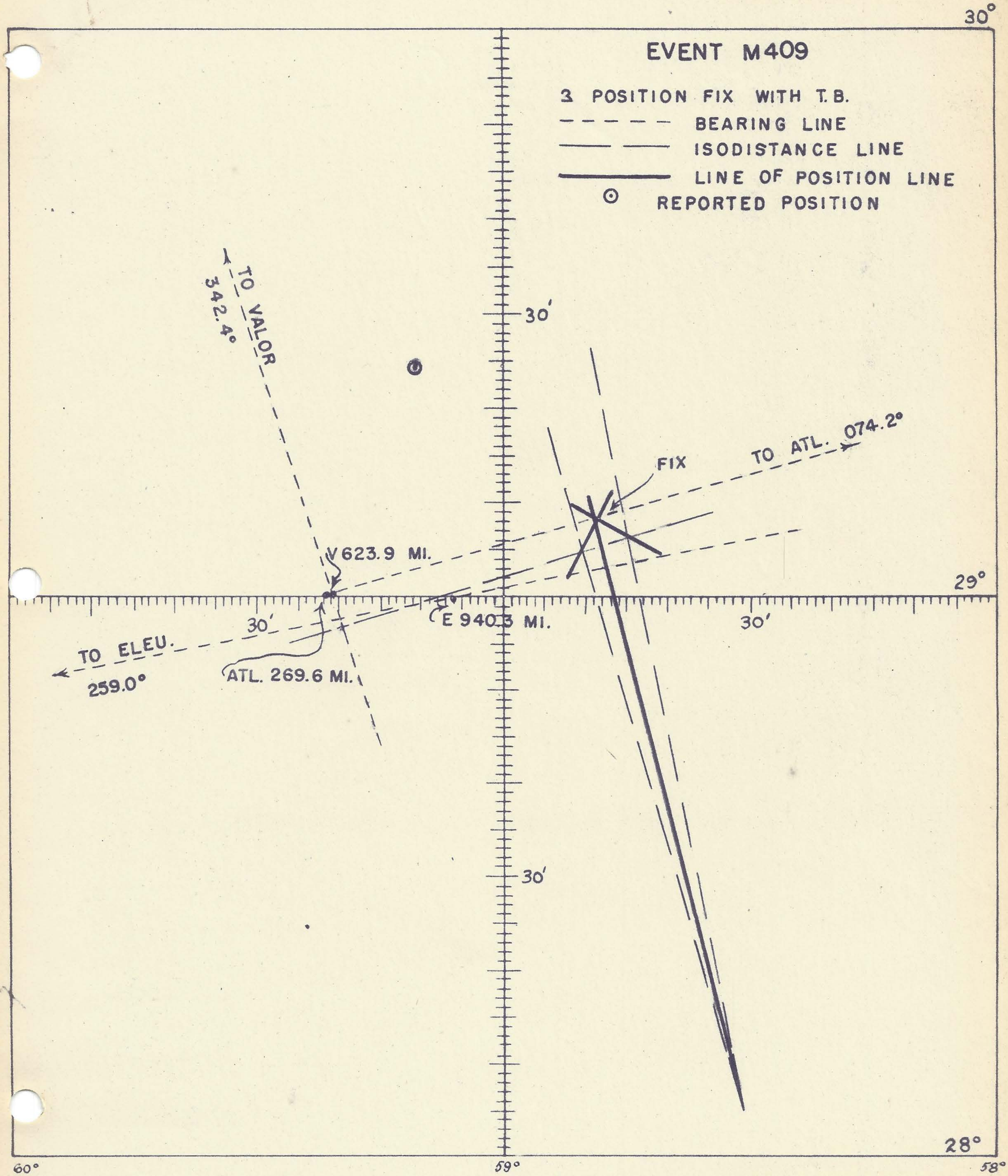

37°
EVENT M 371
3 POSITION FIX WITH T. B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
FIX
30'
TO VALOR
302.8°
30'
30'
A 500.6 MI.
30'
V280.0 MI.
E1098.6 MI.
TO ELEU.
237.5°
TO ATL.
148.3°
30'
35°
61°
60°
59°

FIGURE 7



30°
EVENT M409
3 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
342.4°
30′
FIX
TO ATL. 074.2°
V623.9 MI.
29°
30′
E 940.3 MI.
30′
TO ELEU.
259.0°
ATL. 269.6 MI.
30′
28°
60°
59°
58°

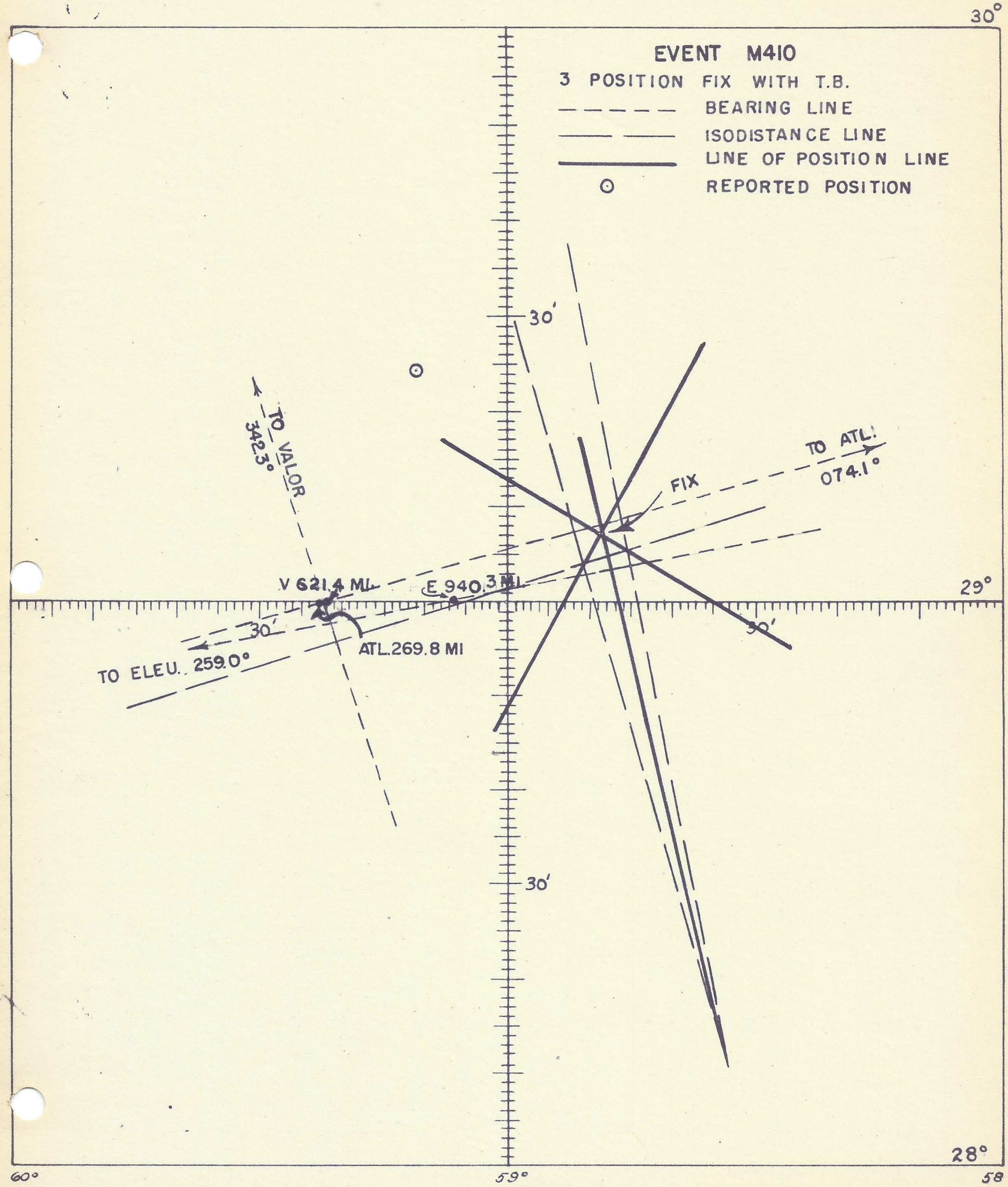
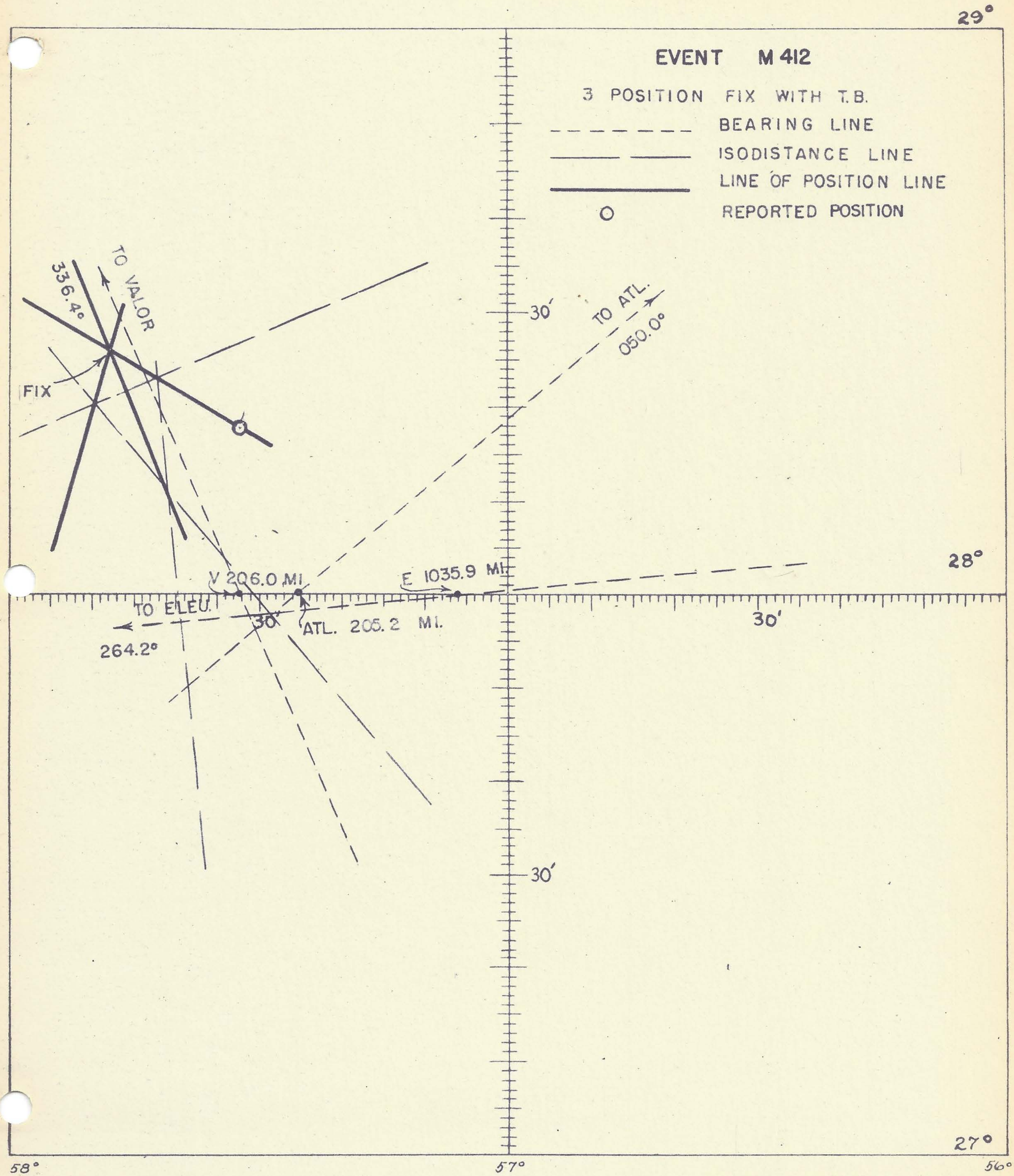

30°
EVENT M410
3 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
TO VALOR
342.3°
TO ATL.
074.1°
FIX
V 621.4 MI.
E 940.3 MI.
29°
30'
30'
ATL. 269.8 MI
TO ELEU. 259.0°
30'
28°
60°
59°
58°

FIGURE 9



29°
EVENT M 412
3 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
336.4°
FIX
30'
TO ATL.
050.0°
28°
V 206.0 MI.
E 1035.9 MI.
TO ELEU.
30'
ATL. 205.2 MI.
264.2°
30'
30'
27°
58°
57°
56°

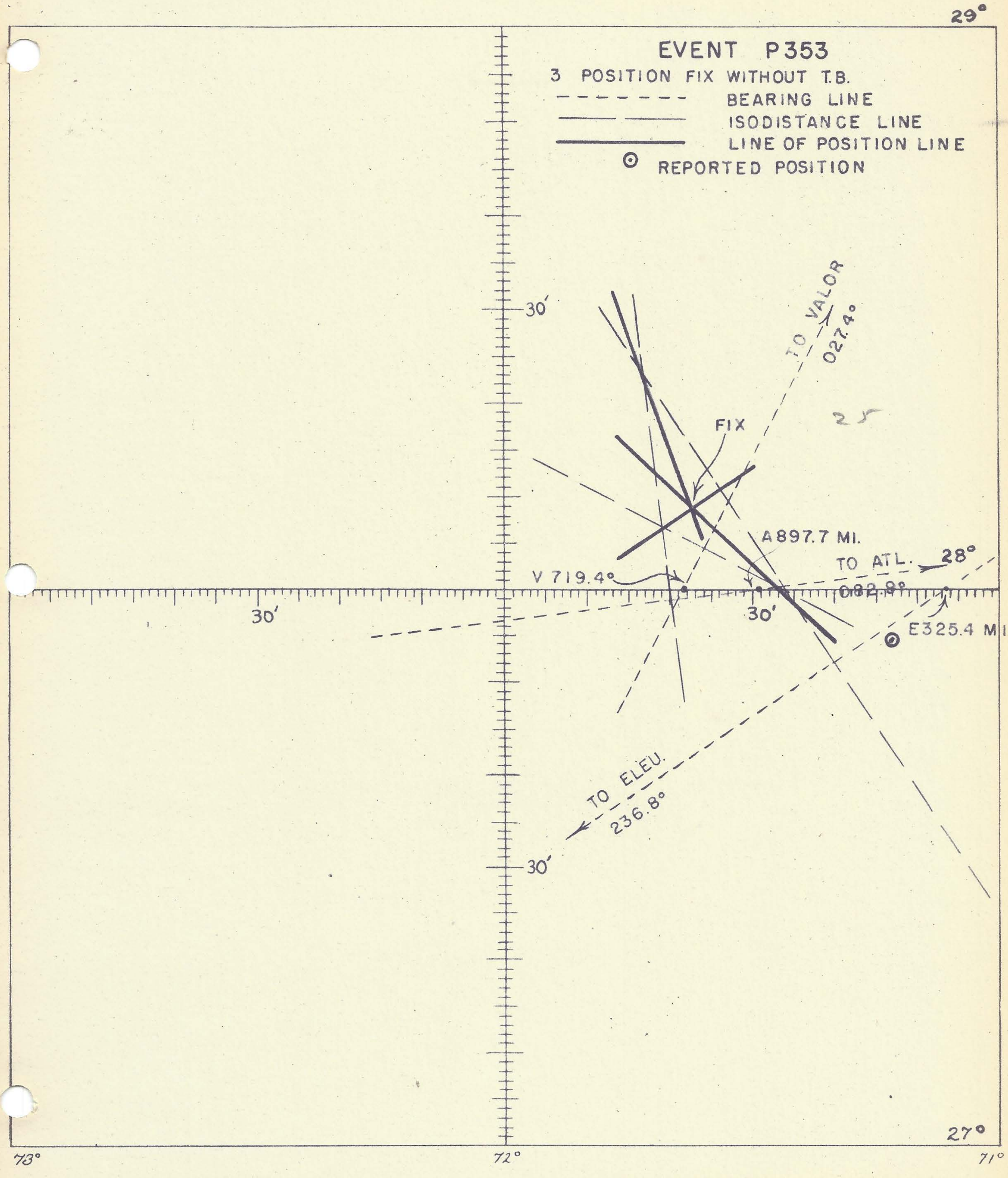
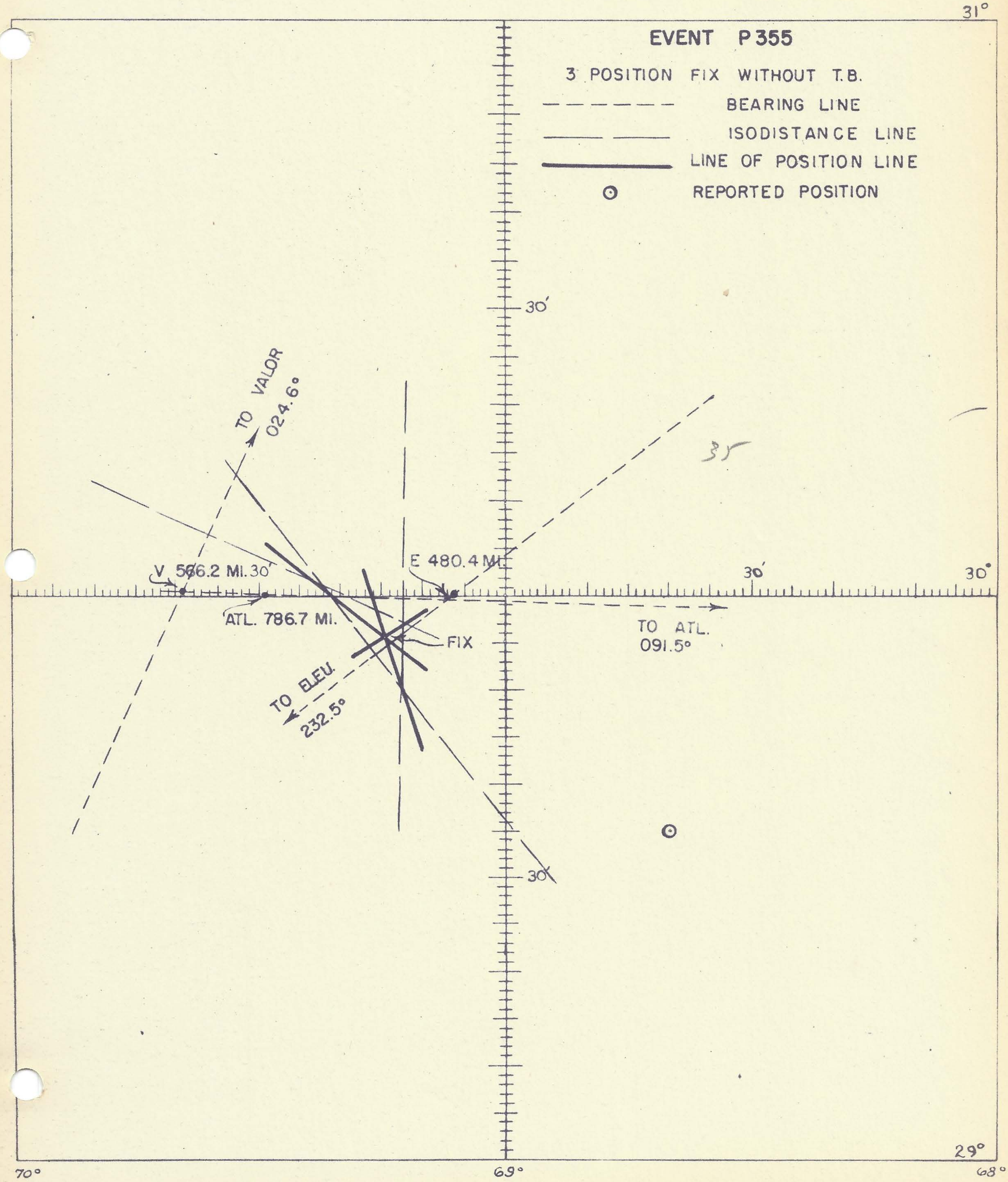
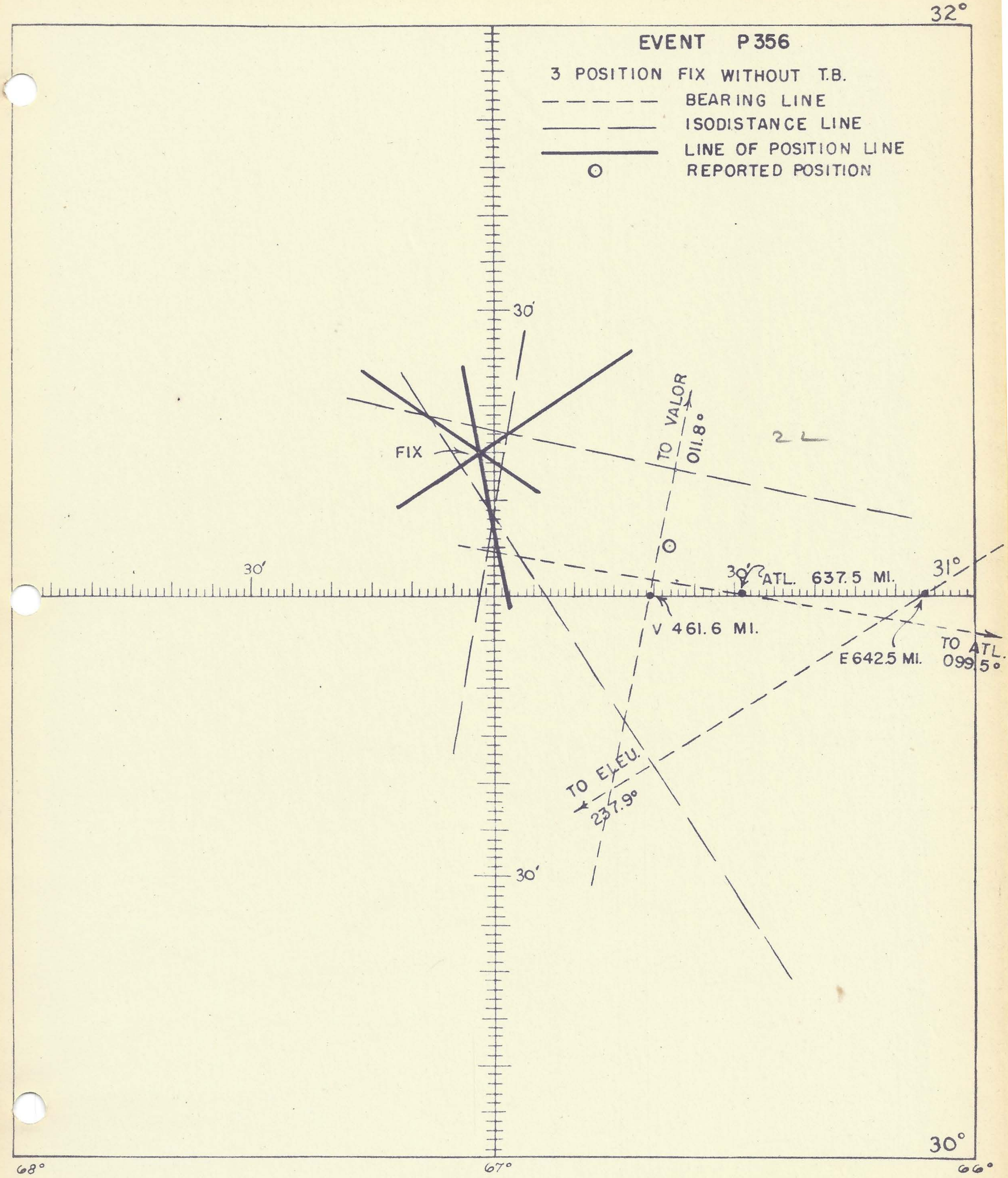

29°
EVENT P353
3 POSITION FIX WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
TO VALOR
027.4°
25
FIX
A 897.7 MI.
TO ATL.
28°
082.8°
V 719.4°
30'
30'
E 325.4 MI.
TO ELEU.
236.8°
30'
27°
73°
72°
71°

FIGURE 11



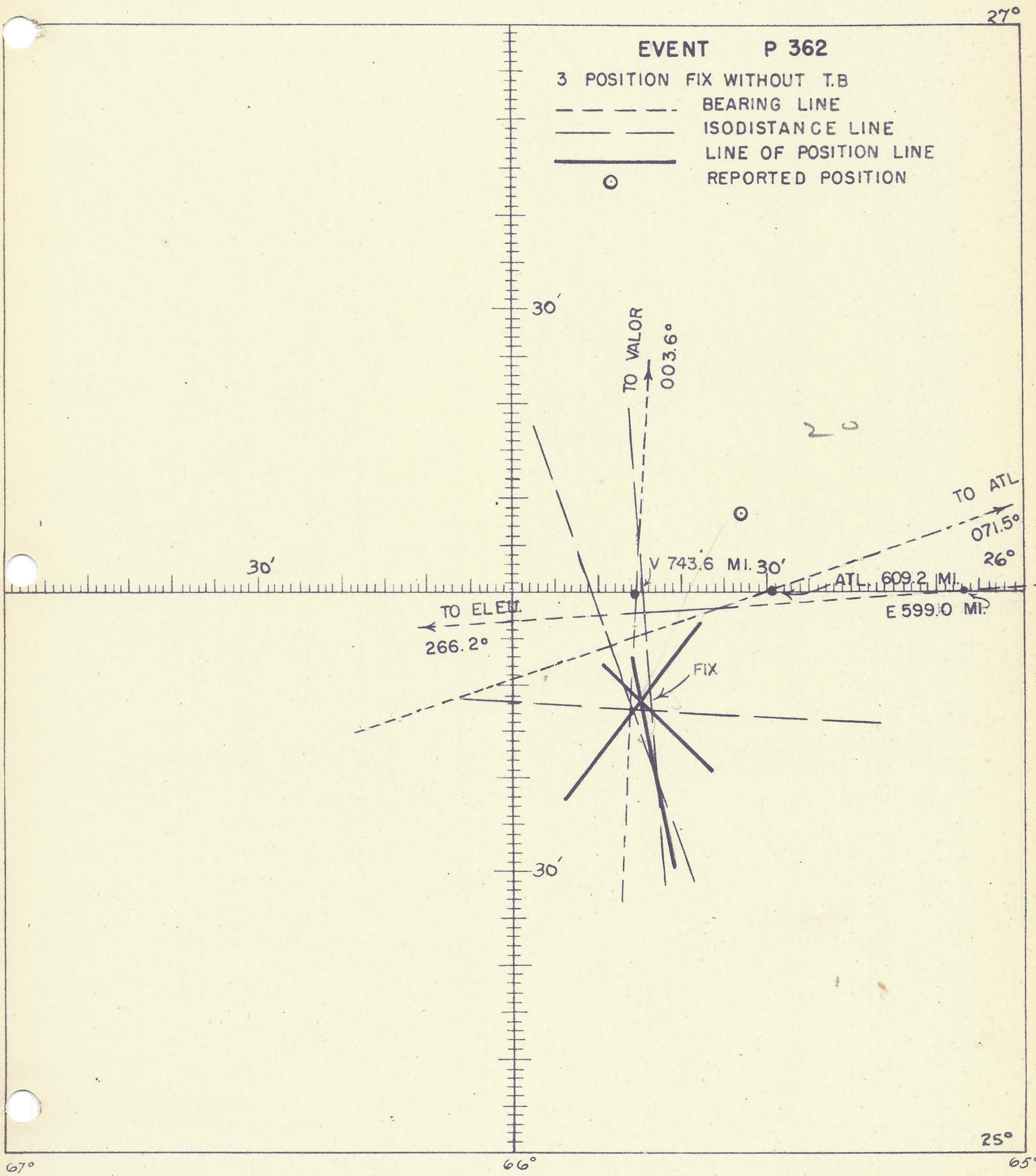
EVENT P355
3 POSITION FIX WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
31°
30'
TO VALOR
024.6°
35
V 566.2 MI.
30'
E 480.4 MI.
30'
30°
ATL. 786.7 MI.
TO ATL.
091.5°
FIX
TO ELEU.
232.5°
30
29°
70°
69°
68°

FIGURE 12



32°
EVENT P356
3 POSITION FIX WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
FIX
TO VALOR
011.8°
30'
30'
ATL. 637.5 MI.
31°
V 461.6 MI.
E 642.5 MI.
TO ATL.
099.5°
TO ELEU.
237.9°
30'
30°
68°
67°
66°

FIGURE 13



EVENT P 362
3 POSITION FIX WITHOUT T.B
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
27°
30′
TO VALOR
003.6°
TO ATL
071.5°
26°
30′
V 743.6 MI.
30′
ATL. 609.2 MI.
E 599.0 MI.
TO ELE
266.2°
FIX
30′
25°
67°
66°
65°

FIGURE 14

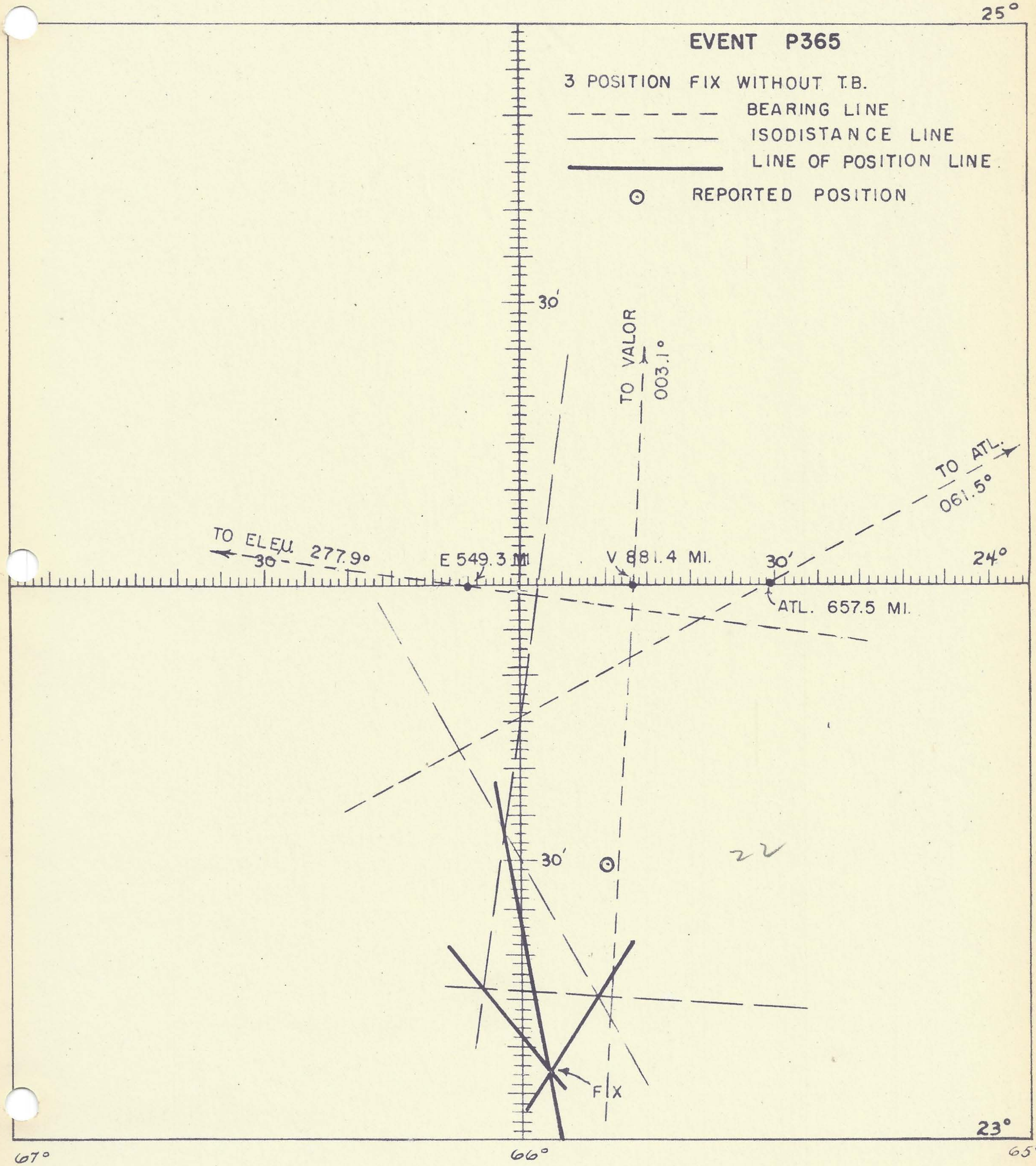
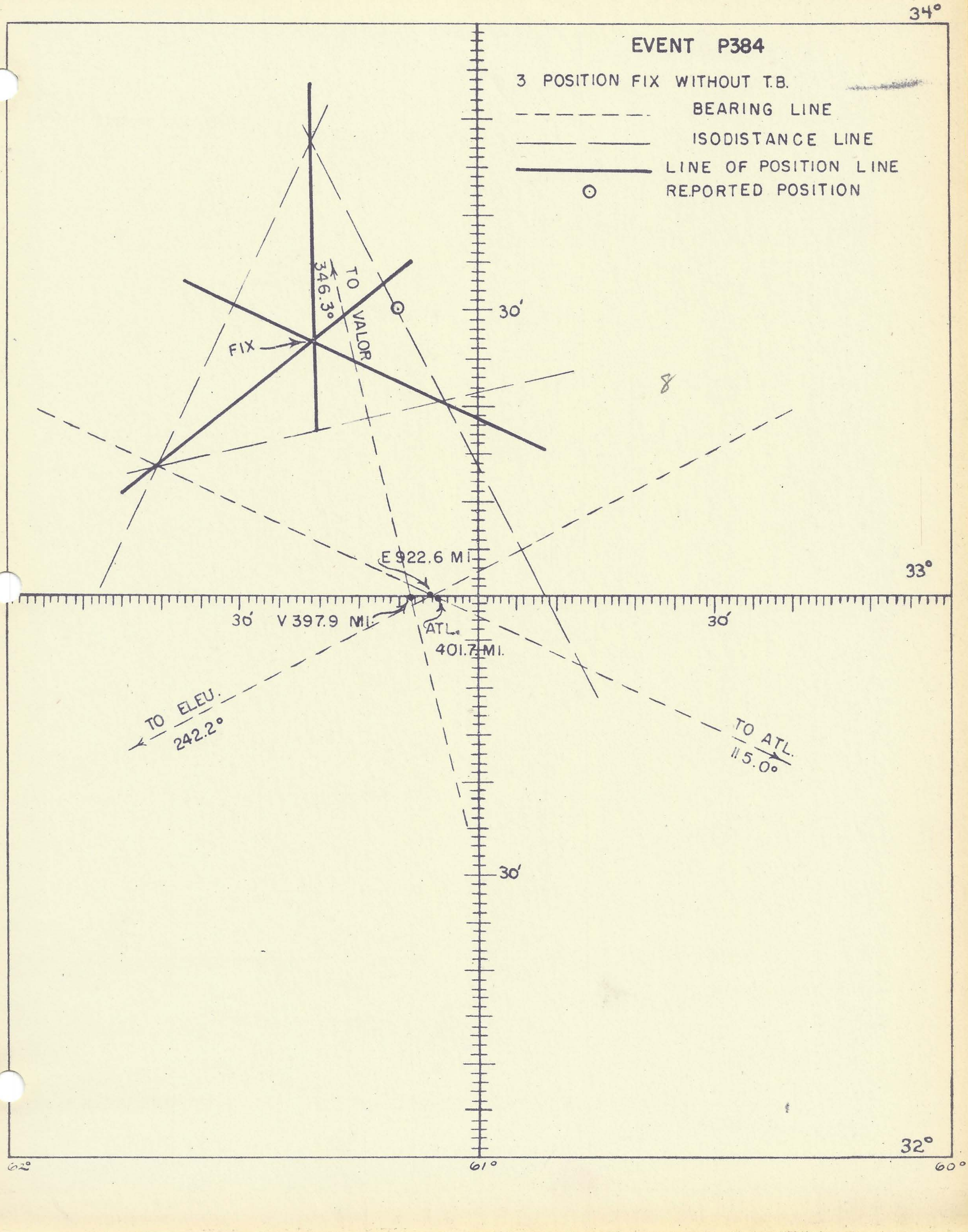
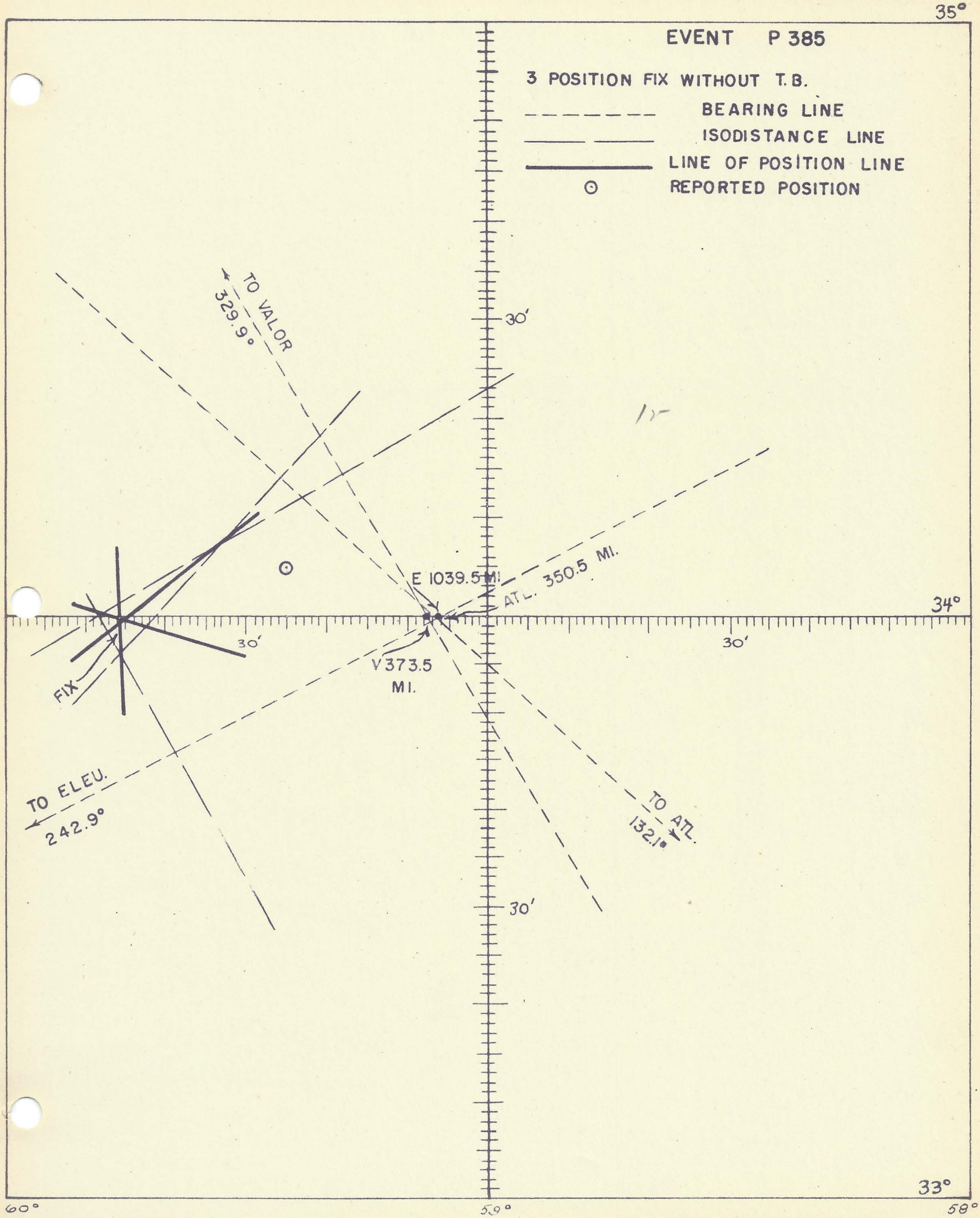

25°
EVENT P365
3 POSITION FIX WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
TO VALOR
003.1°
TO ATL.
061.5°
TO ELEU. 277.9°
30
E 549.3 MI
V 881.4 MI.
30′
24°
ATL. 657.5 MI.
30′
22
FIX
23°
67°
66°
65°

FIGURE 15



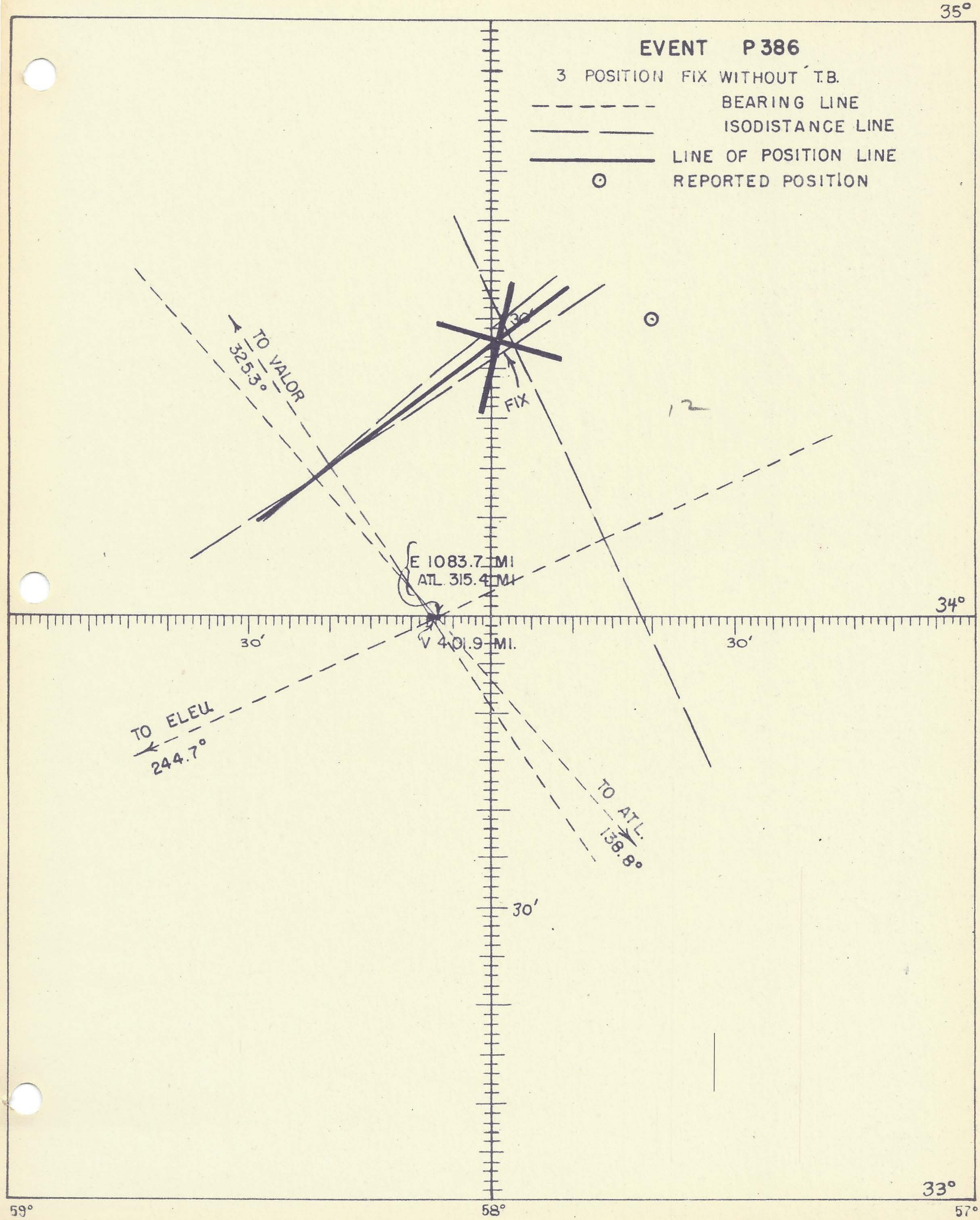
EVENT P384
3 POSITION FIX WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
34°
33°
32°
62°
61°
60°
30'
30'
30'
30'
TO VALOR
346.3°
FIX
E 922.6 MI
V 397.9 MI.
ATL.
401.7 MI.
TO ELEU.
242.2°
TO ATL.
115.0°
8

FIGURE 16



35°
EVENT P 385
3 POSITION FIX WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
329.9°
30'
E 1039.5 MI.
ATL. 350.5 MI.
34°
30'
30'
V 373.5
MI.
FIX
TO ELEU.
242.9°
TO ATL.
132.1°
30'
33°
60°
59°
58°

FIGURE 17



35°
EVENT P386
3 POSITION FIX WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
TO VALOR
325.3°
FIX
E 1083.7 MI
ATL 315.4 MI
34°
30'
V 401.9 MI.
30'
TO ELEU.
244.7°
TO ATL.
138.8°
30'
33°
59°
58°
57°

FIGURE 18

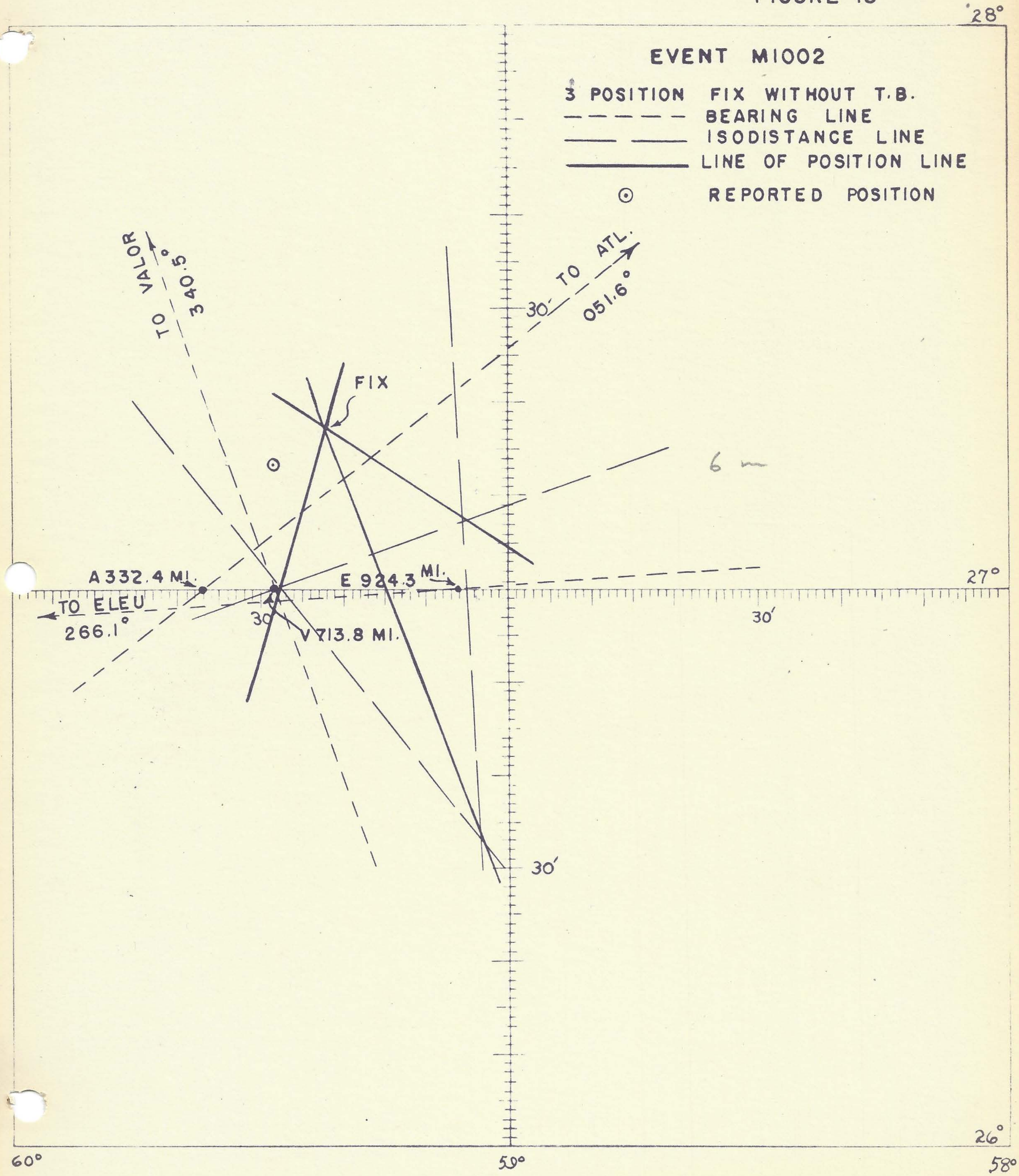
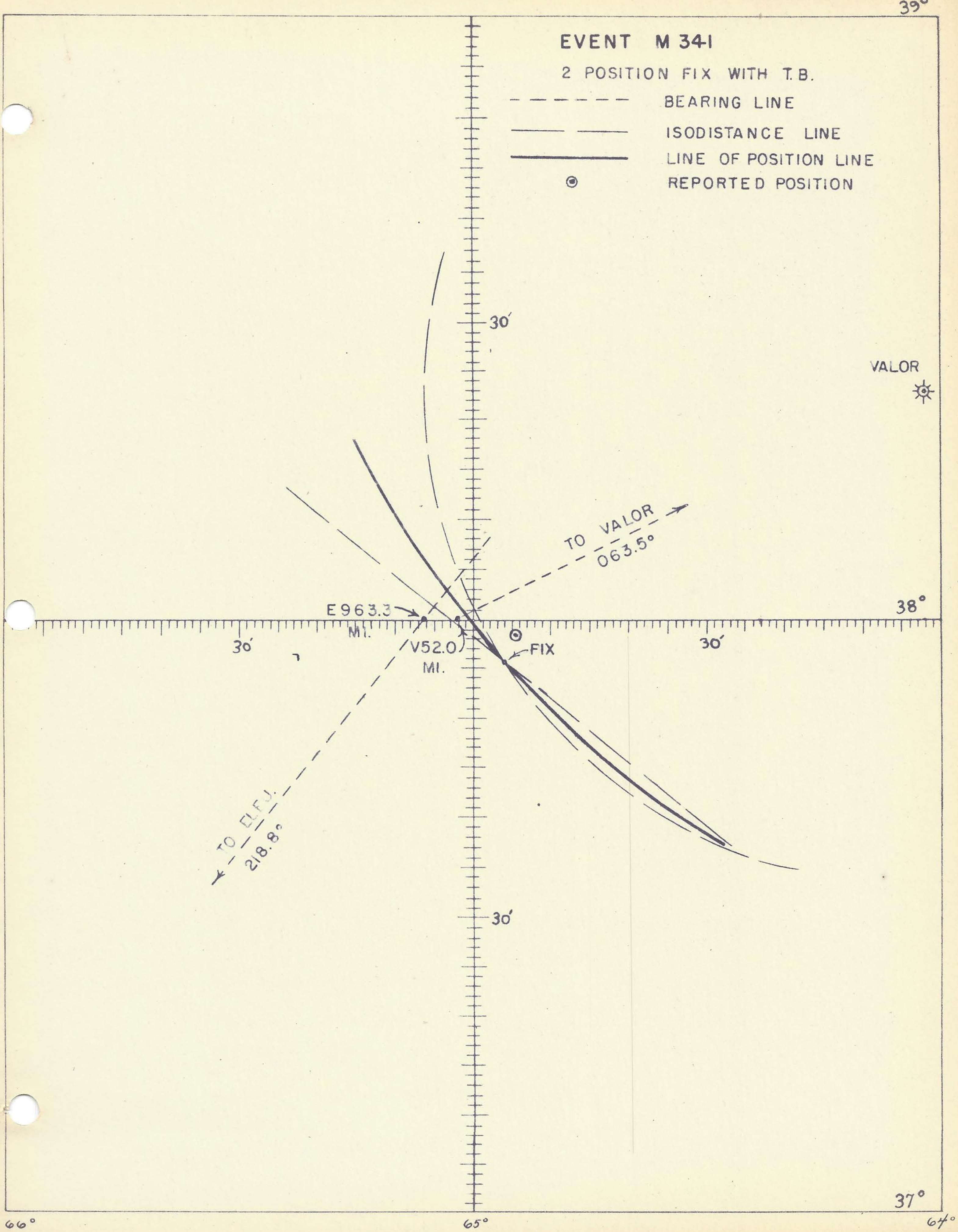

28°
EVENT M1002
3 POSITION FIX WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
340.5°
TO ATL.
051.6°
30'
FIX
6 m
A 332.4 MI.
E 924.3 MI.
27°
TO ELEU
266.1°
30
V 713.8 MI.
30'
30'
26°
60°
59°
58°

FIGURE 19



EVENT M 341
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
39°
38°
37°
66°
65°
64°
30′
30′
30′
30′
VALOR
TO VALOR 063.5°
TO ELFJ. 218.8°
E963.3 MI.
V52.0 MI.
FIX

FIGURE 20

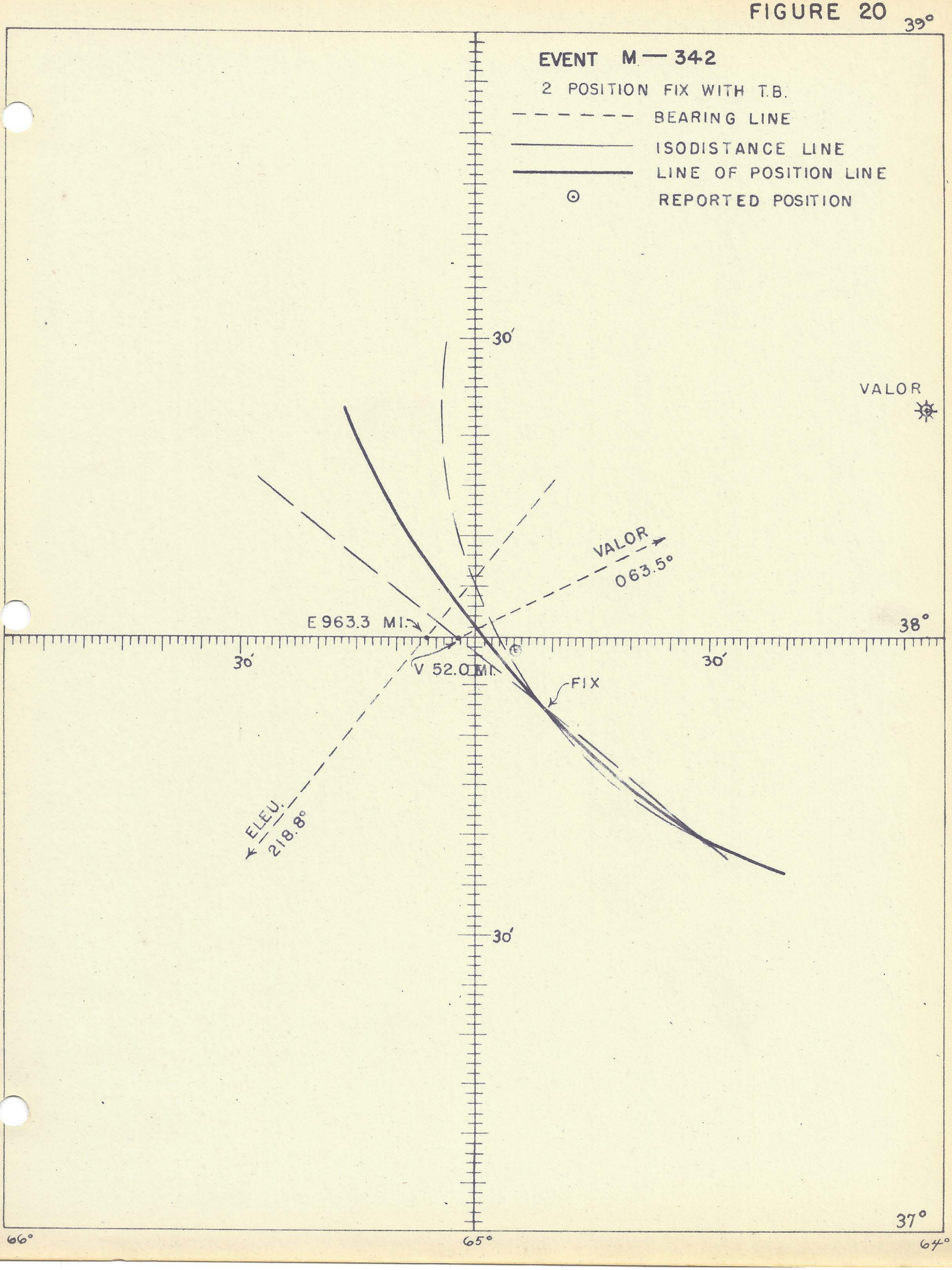
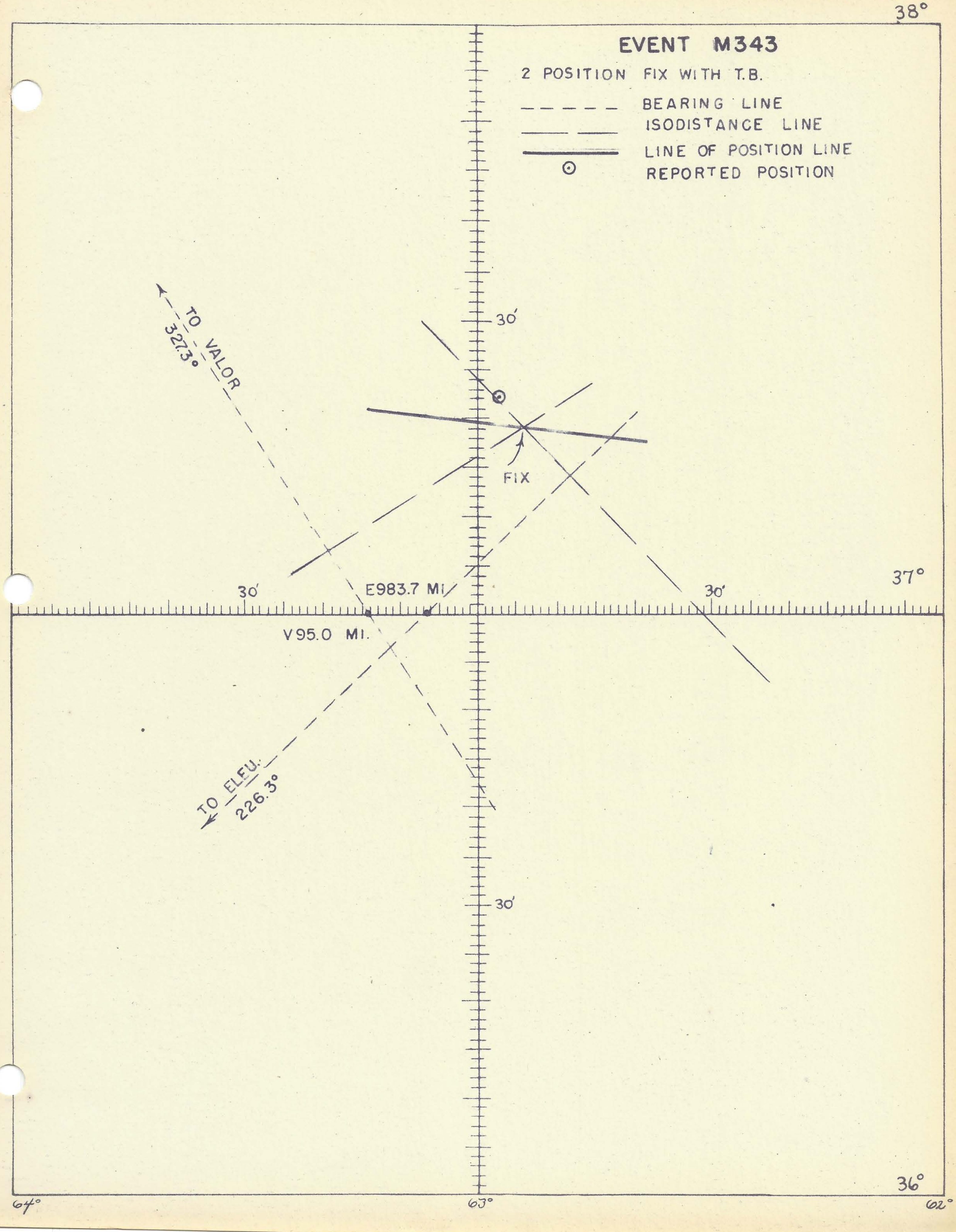
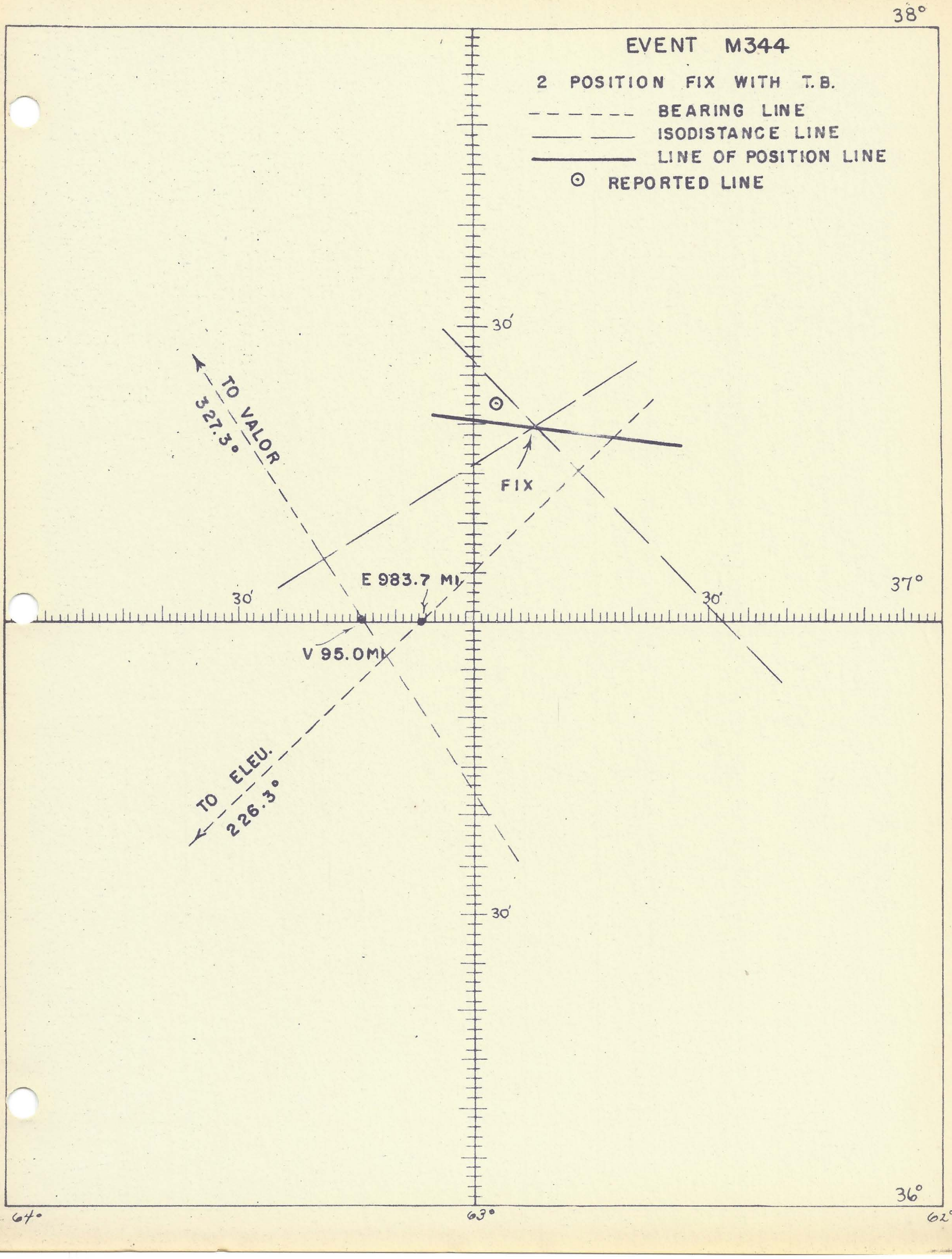

39°
EVENT M — 342
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
VALOR
VALOR
063.5°
E 963.3 MI.
38°
30′
V 52.0 MI.
30′
FIX
ELEV.
218.8°
30′
37°
66°
65°
64°

FIGURE 21



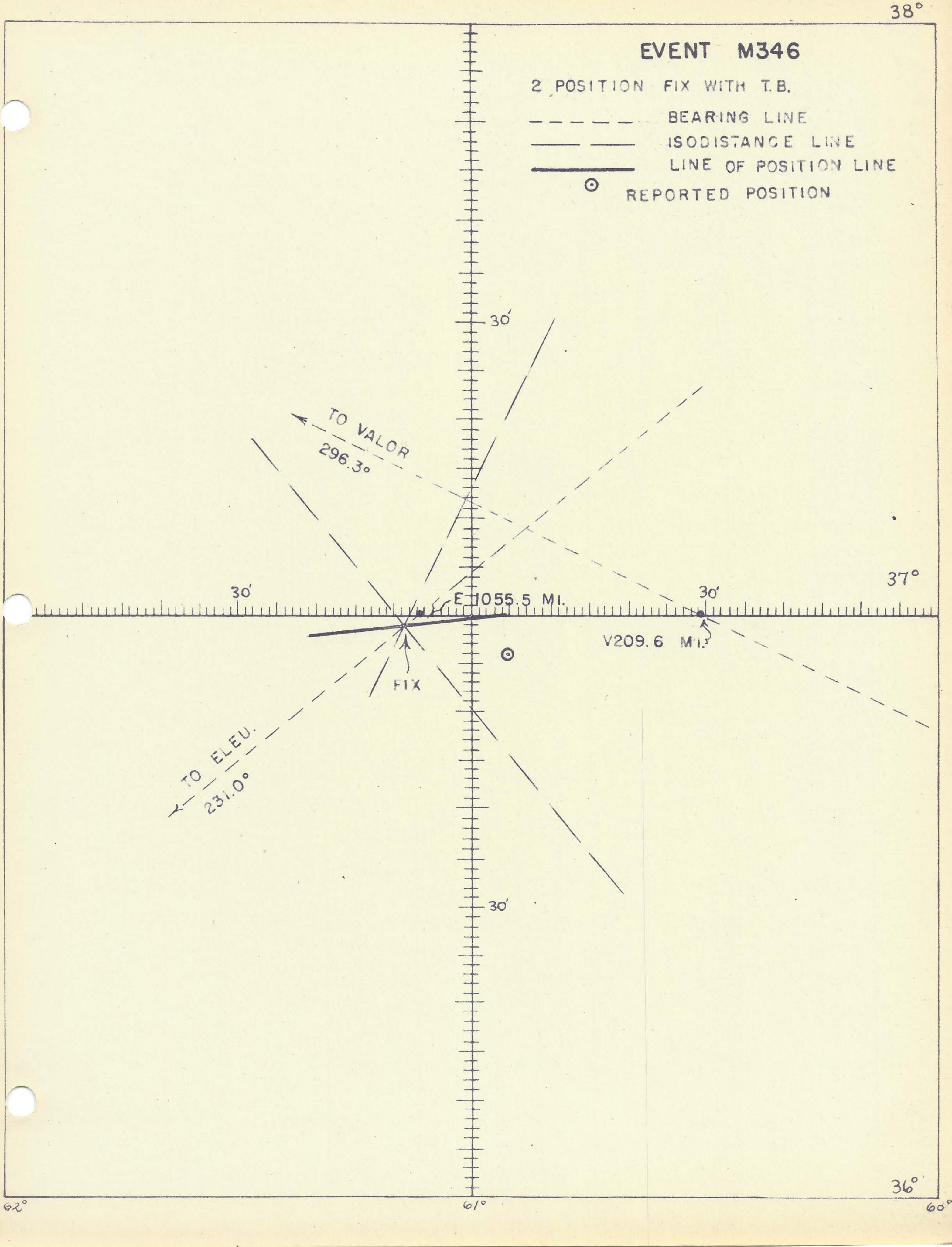
38°
EVENT M343
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
TO VALOR
327.3°
FIX
30'
E983.7 MI.
30'
37°
V95.0 MI.
TO ELEU.
226.3°
30'
36°
64°
63°
62°

FIGURE 22



EVENT M344
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED LINE
TO VALOR 327.3°
TO ELEU. 226.3°
E 983.7 MI
V 95.0MI
FIX
30′
38°
37°
36°
64°
63°
62°

FIGURE 23



38°
EVENT M346
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
TO VALOR
296.3°
30'
E 1055.5 MI.
30'
37°
V209.6 MI.
FIX
TO ELEU.
231.0°
30'
36°
62°
61°
60°

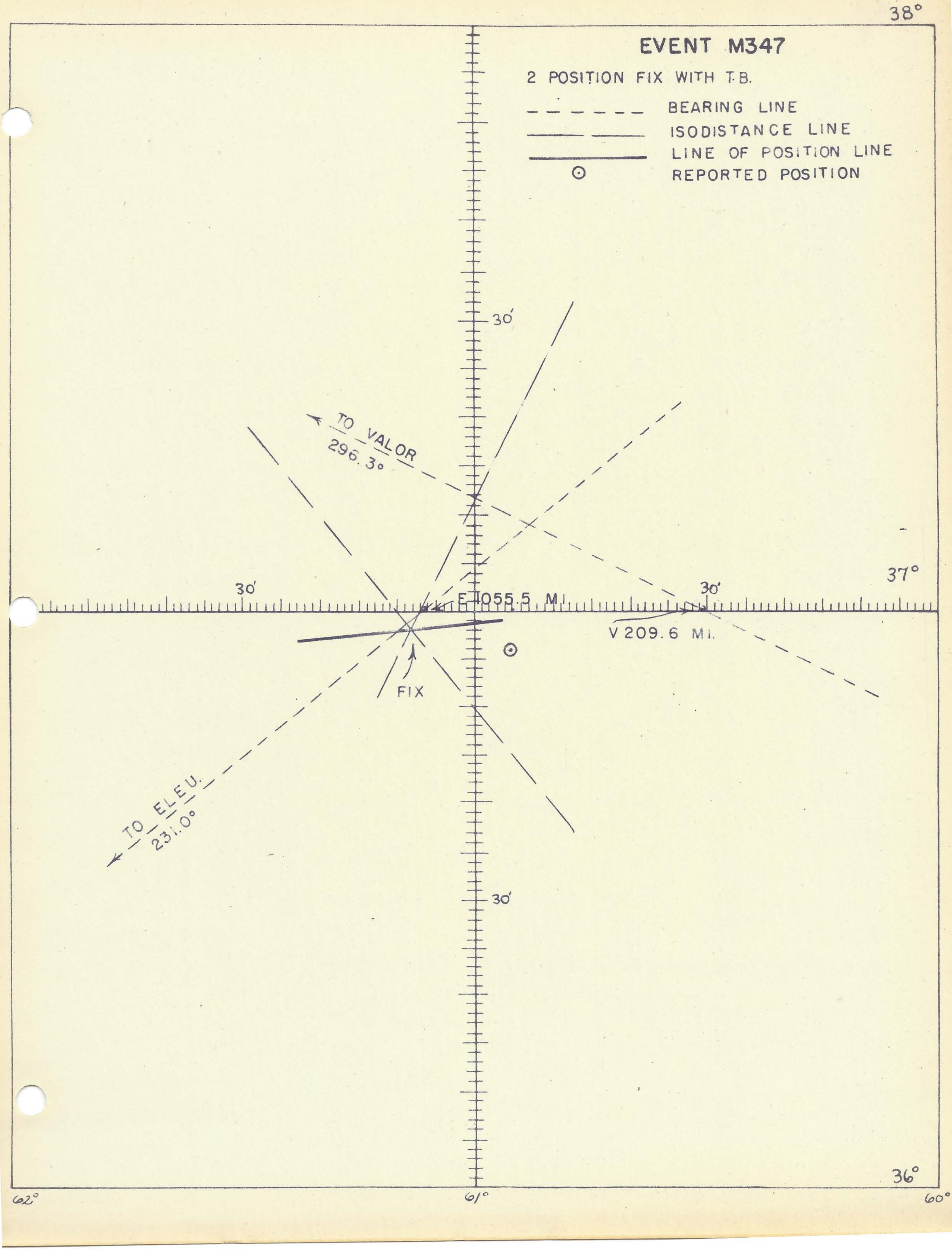
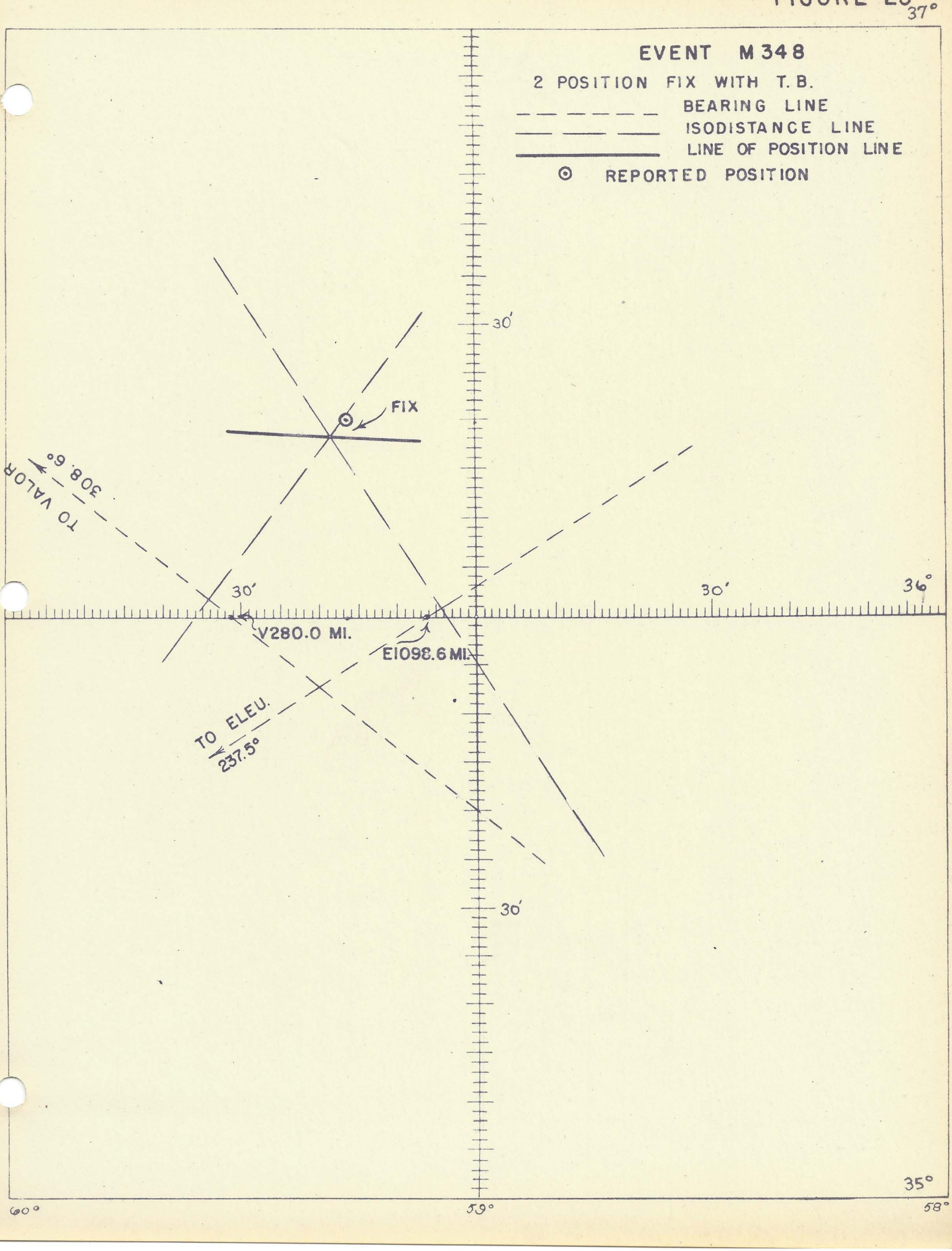
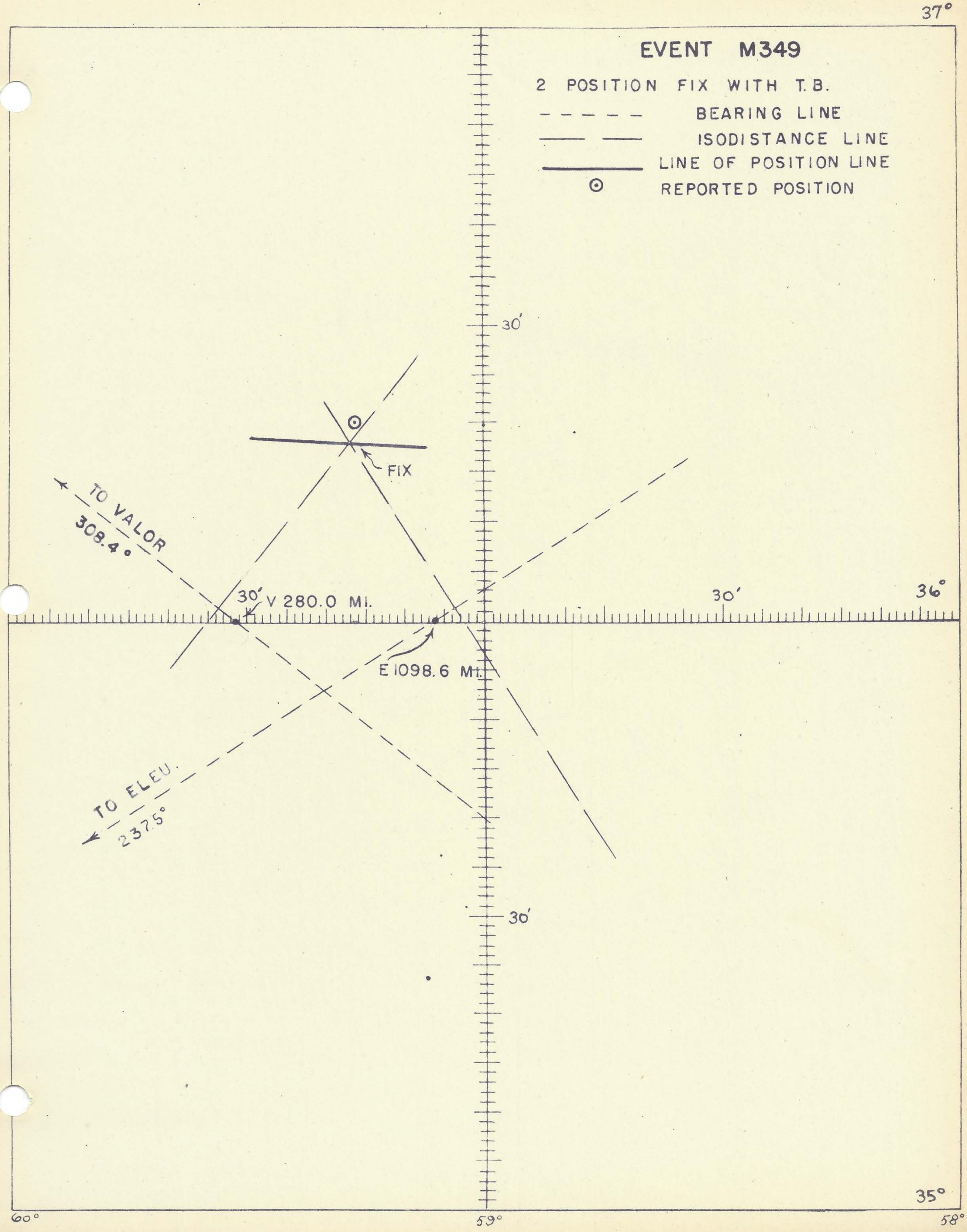

EVENT M347
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
38°
37°
36°
62°
61°
60°
30'
30'
30'
30'
TO VALOR
296.3°
TO ELEU.
231.0°
E 1055.5 MI.
V 209.6 MI.
FIX

FIGURE 25



37°
EVENT M348
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
FIX
TO VALOR
308.6°
30′
30′
36°
V280.0 MI.
E1098.6 MI.
TO ELEU.
237.5°
30′
35°
60°
59°
58°

FIGURE 26


37°
EVENT M349
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
FIX
TO VALOR
308.4°
30′
V 280.0 MI.
30′
36°
E 1098.6 MI.
TO ELEU.
237.5°
30′
35°
60°
59°
58°

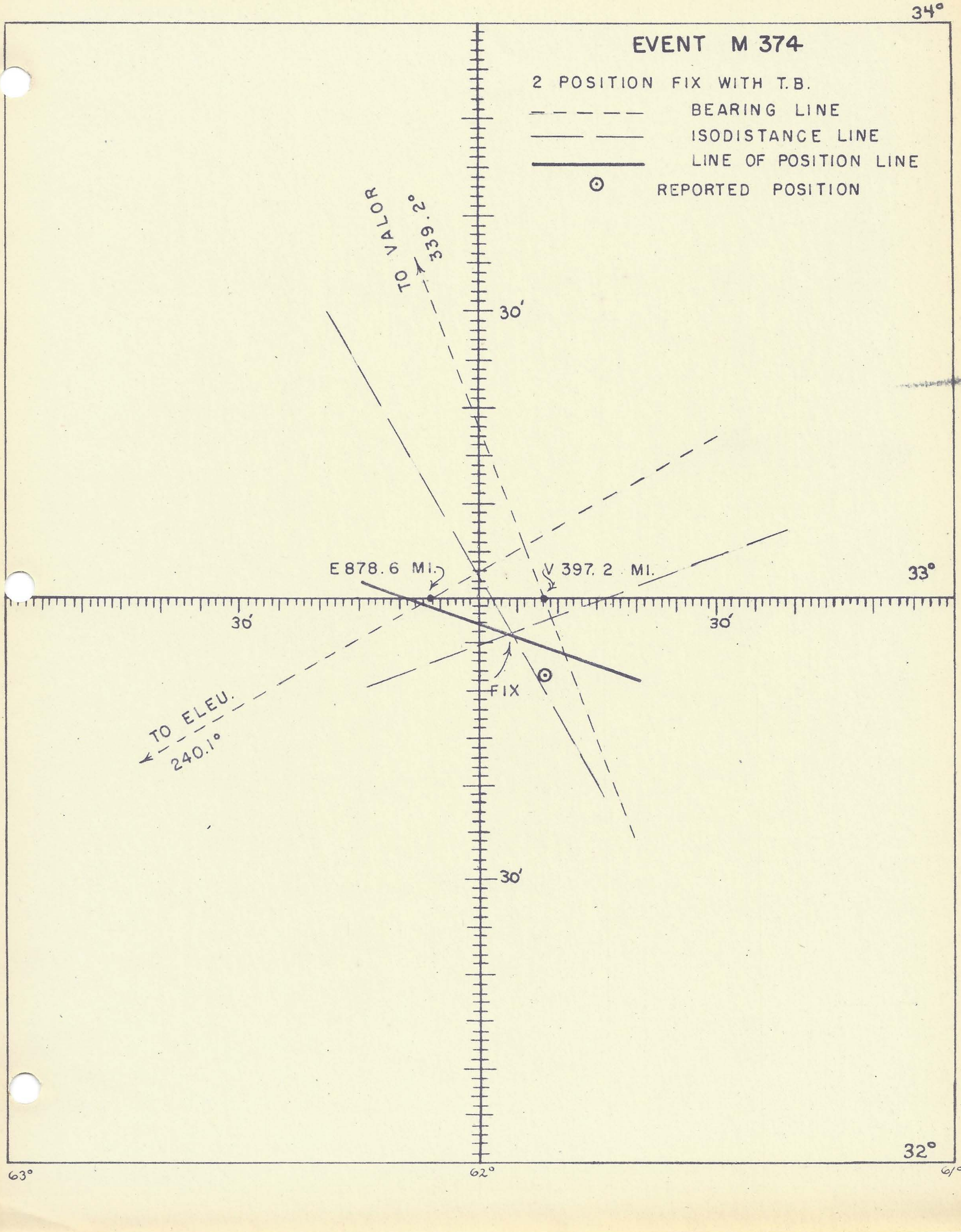
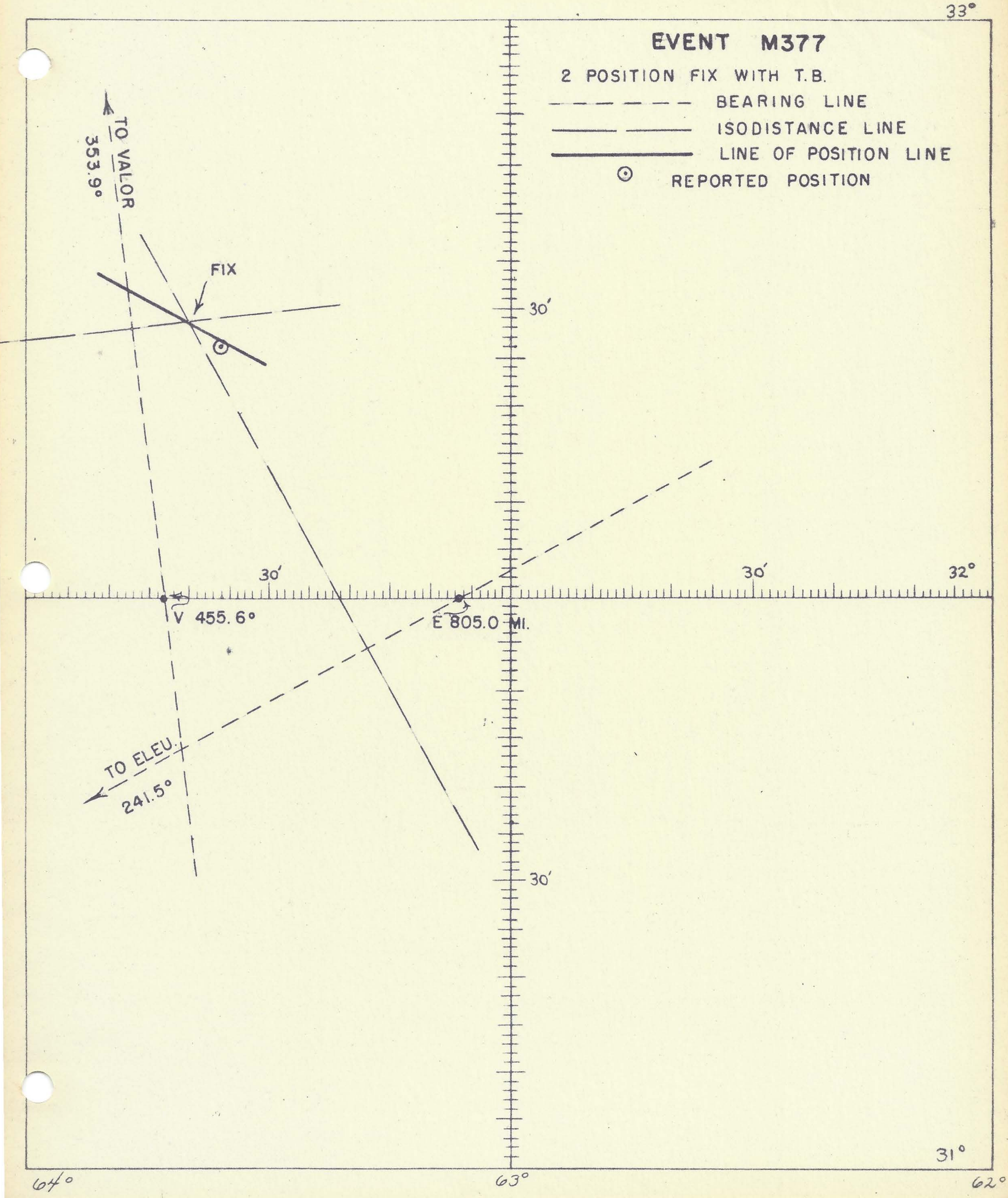
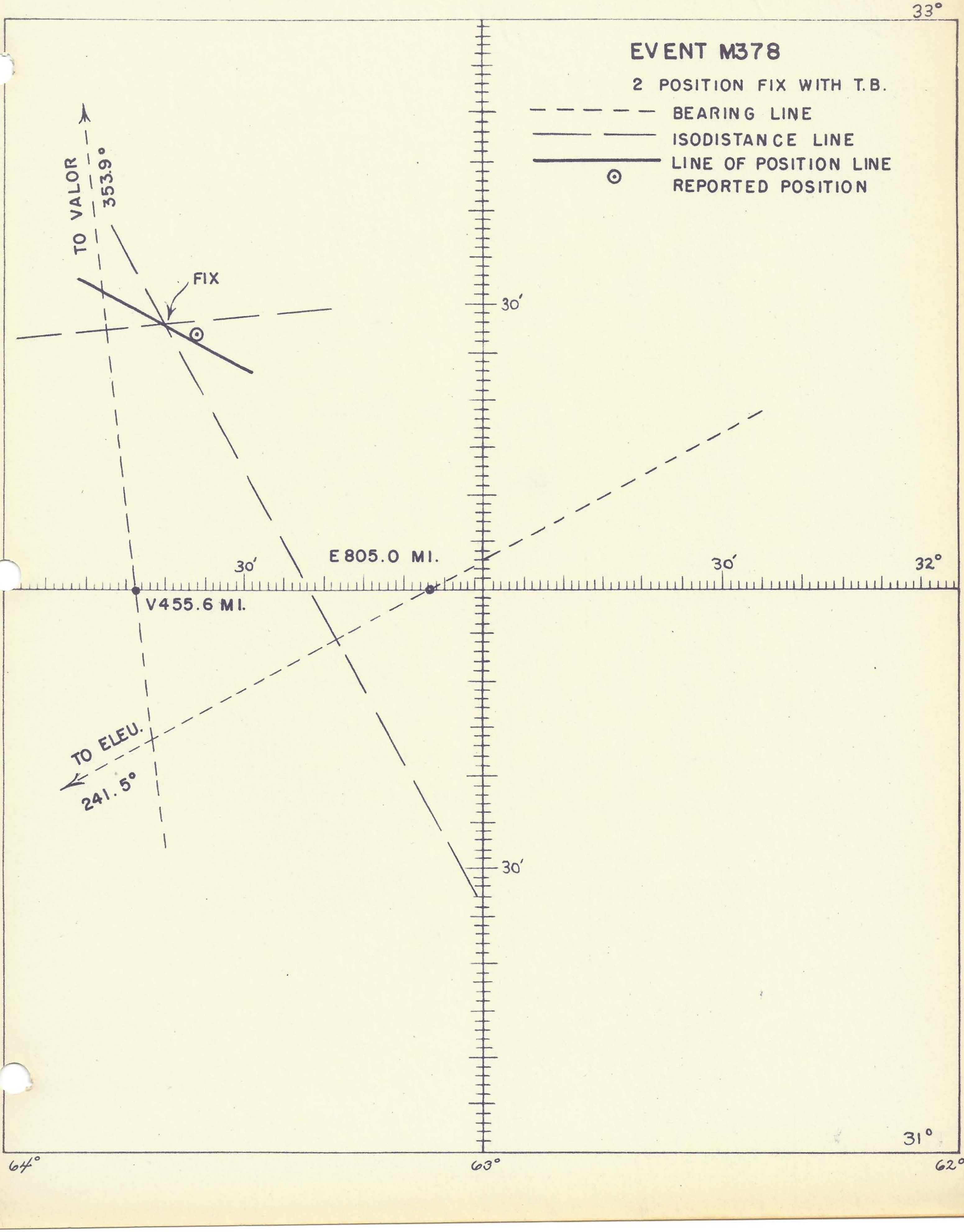

34°
EVENT M 374
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
339.2°
30′
E 878.6 MI.
V 397.2 MI.
33°
30′
30′
FIX
TO ELEU.
240.1°
30′
32°
63°
62°
61°

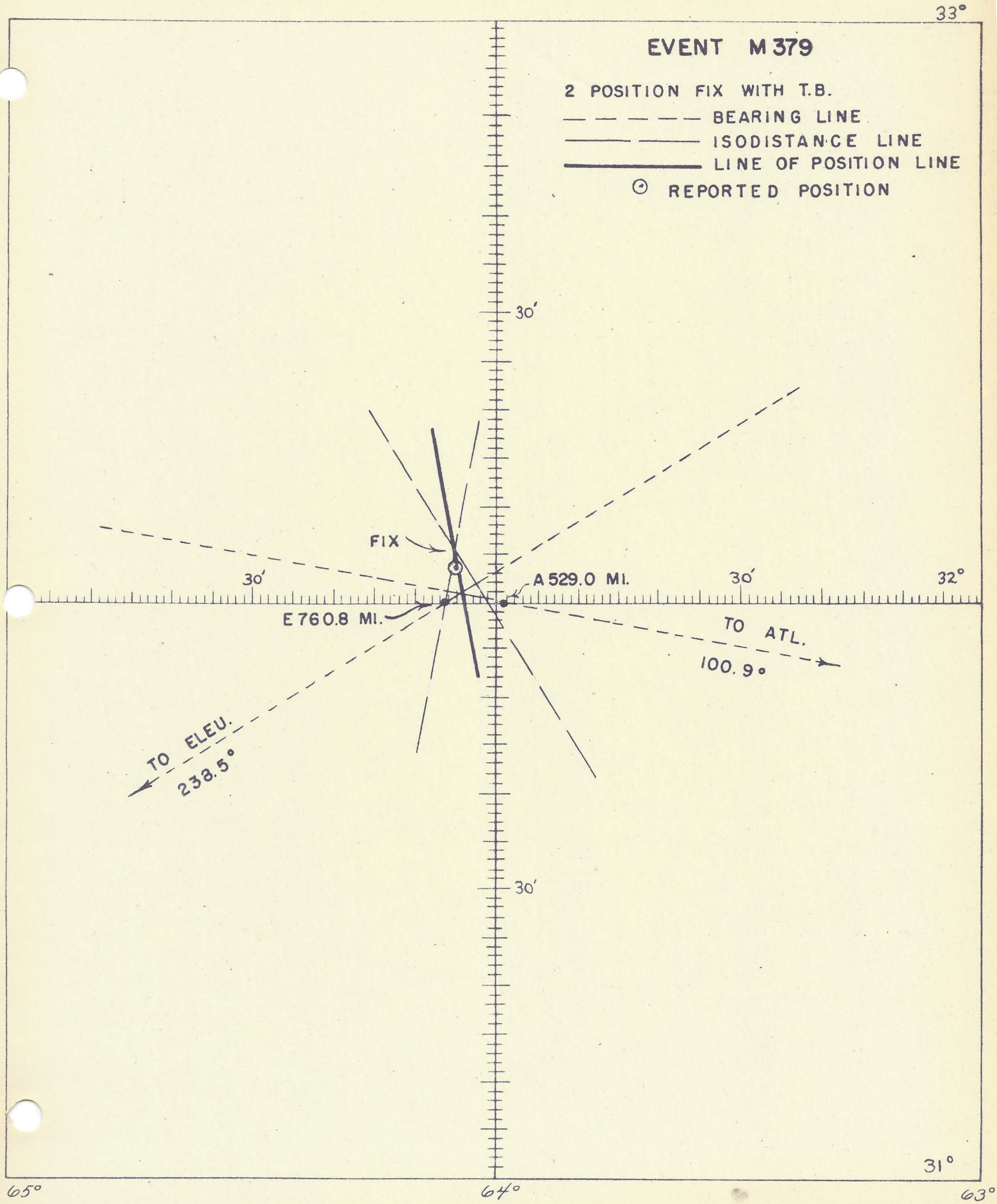
FIGURE 28



33°
EVENT M377
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
353.9°
FIX
30′
30′
30′
32°
V 455.6°
E 805.0 MI.
TO ELEU.
241.5°
30′
31°
64°
63°
62°

FIGURE 29


33°
EVENT M378
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
353.9°
FIX
30′
E 805.0 MI.
30′
30′
32°
V455.6 MI.
TO ELEU.
241.5°
30′
31°
64°
63°
62°



33°
EVENT M 379
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
FIX
30′
A 529.0 MI.
30′
32°
E 760.8 MI.
TO ATL.
100.9°
TO ELEU.
238.5°
30′
31°
65°
64°
63°

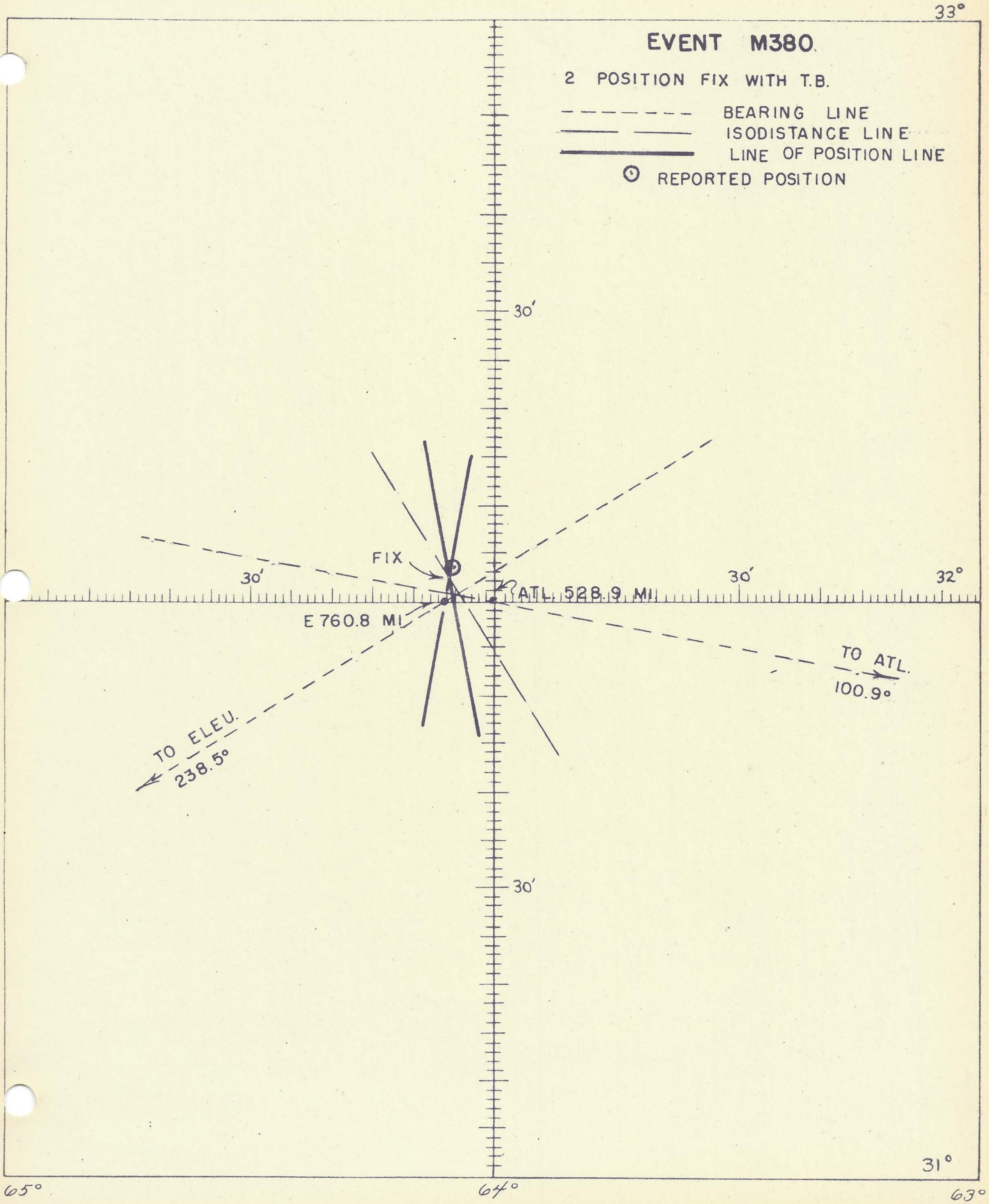
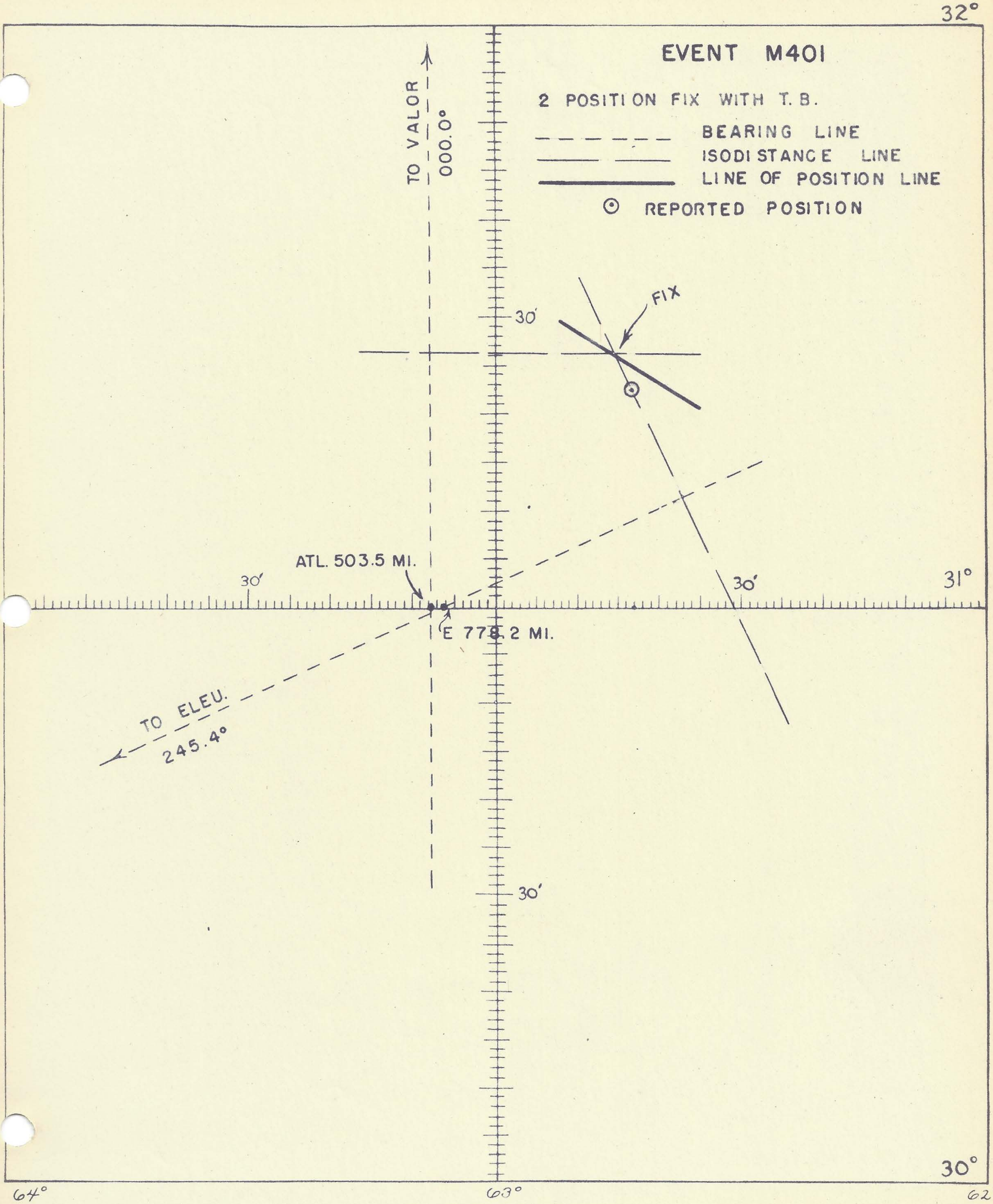
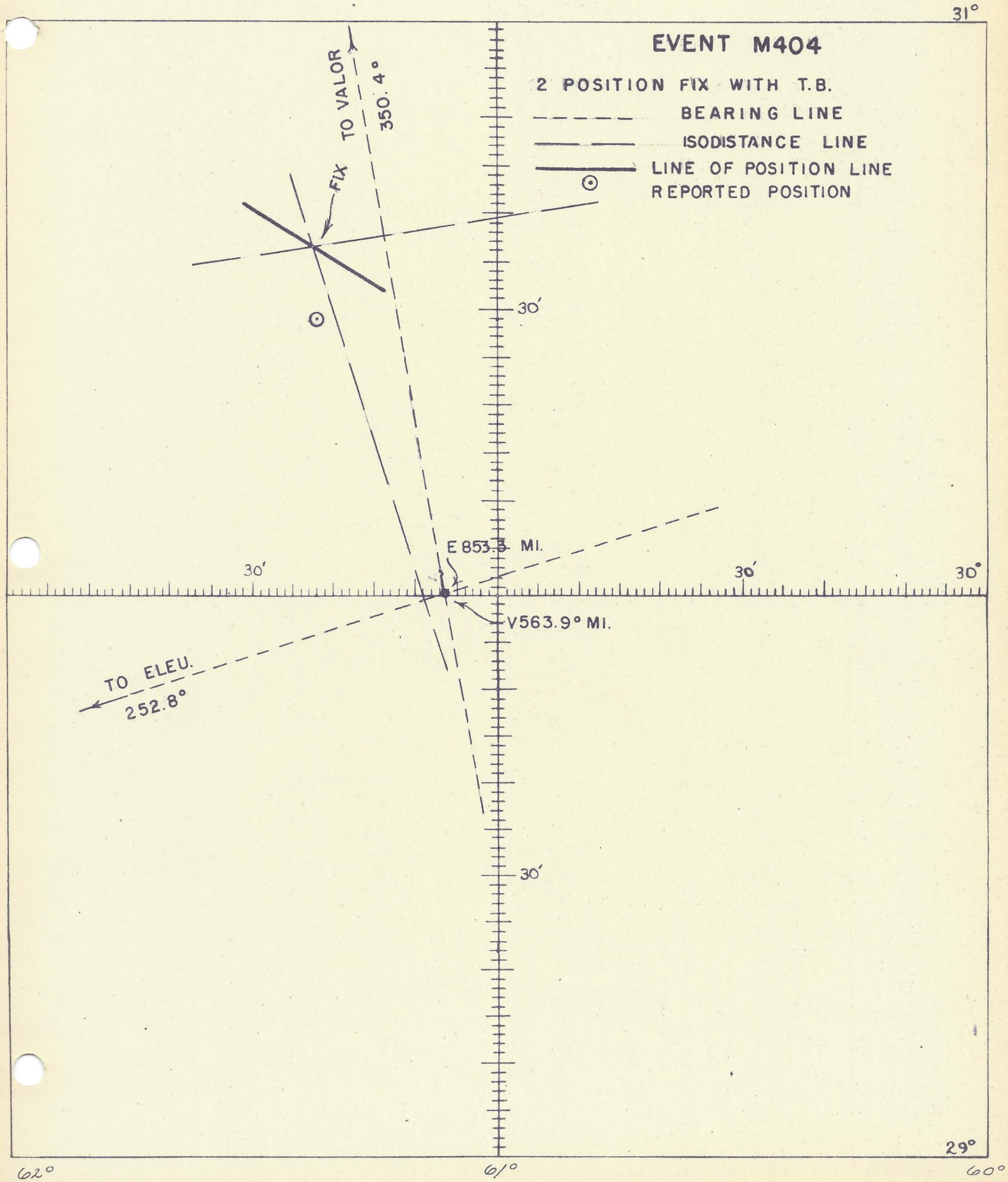

33°
EVENT M380
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
FIX
30′
30′
32°
ATL 528.9 MI
E 760.8 MI
TO ATL.
100.9°
TO ELEU.
238.5°
30′
31°
65°
64°
63°

FIGURE 32



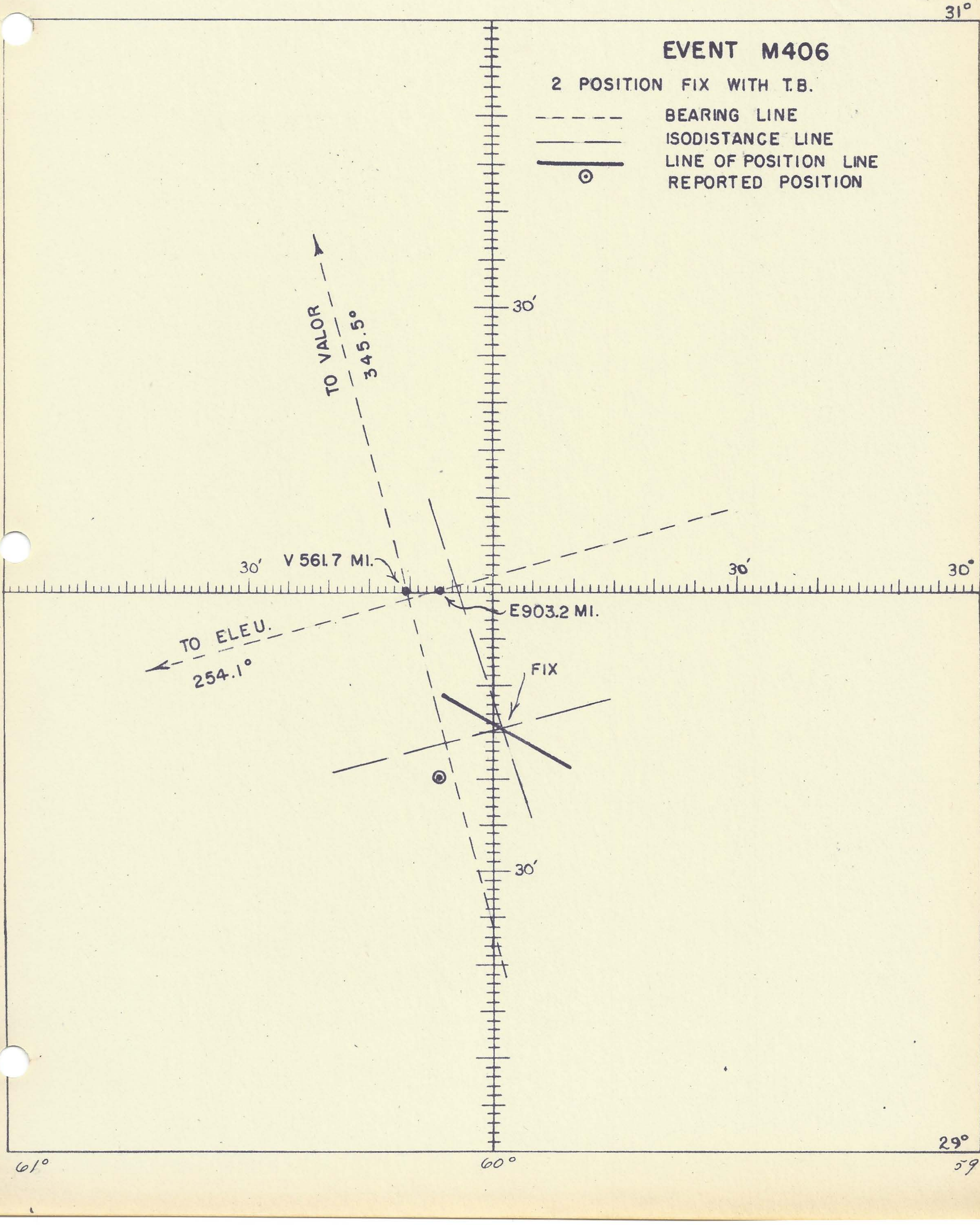
32°
EVENT M401
2 POSITION FIX WITH T. B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
000.0°
FIX
30'
ATL. 503.5 MI.
30'
30'
31°
E 778.2 MI.
TO ELEU.
245.4°
30'
30°
64°
63°
62°

FIGURE 33



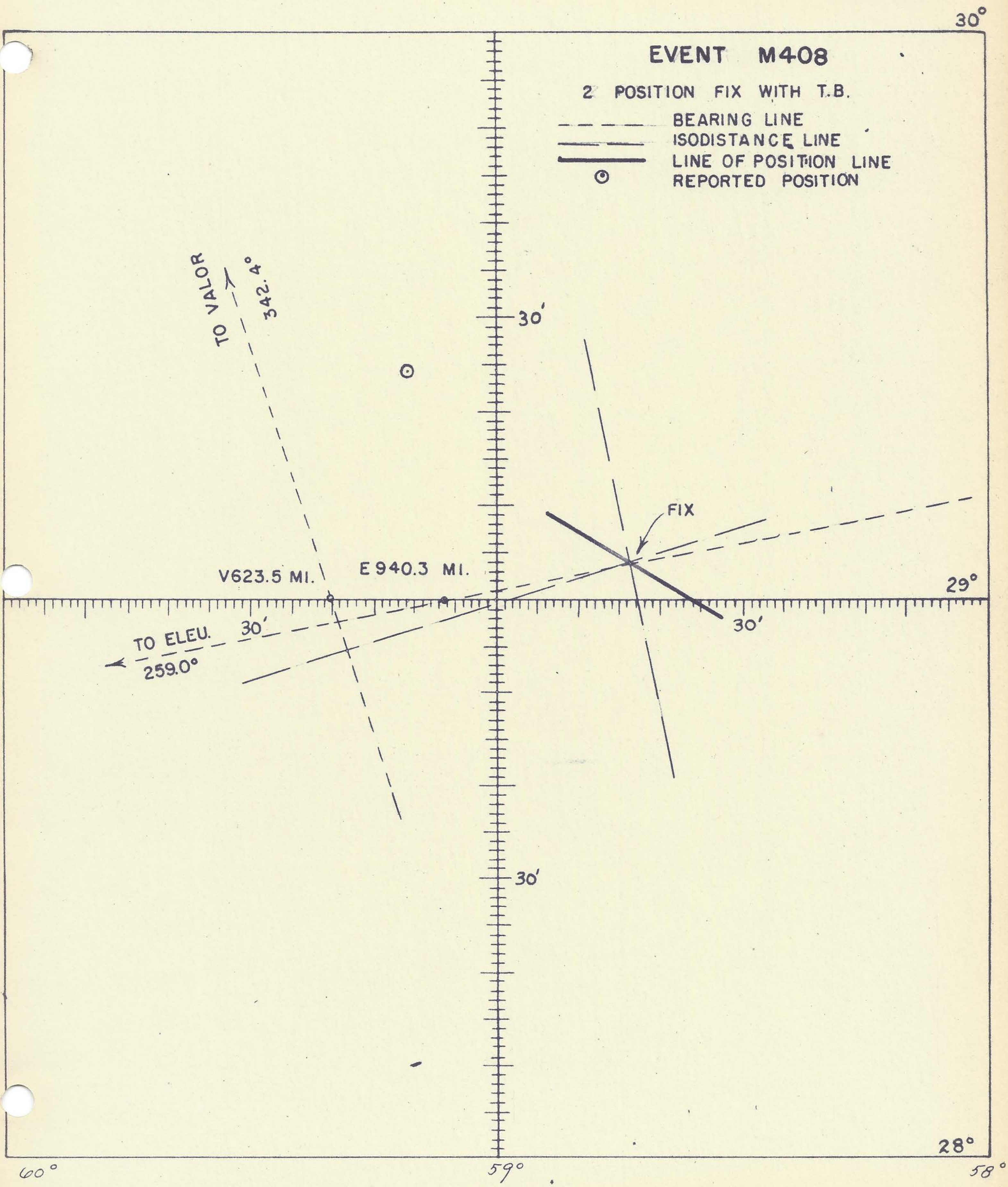
31°
EVENT M404
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
350.4°
FIX
30'
E 853.3 MI.
30'
30'
30°
V563.9° MI.
TO ELEU.
252.8°
30'
29°
62°
61°
60°

FIGURE 34



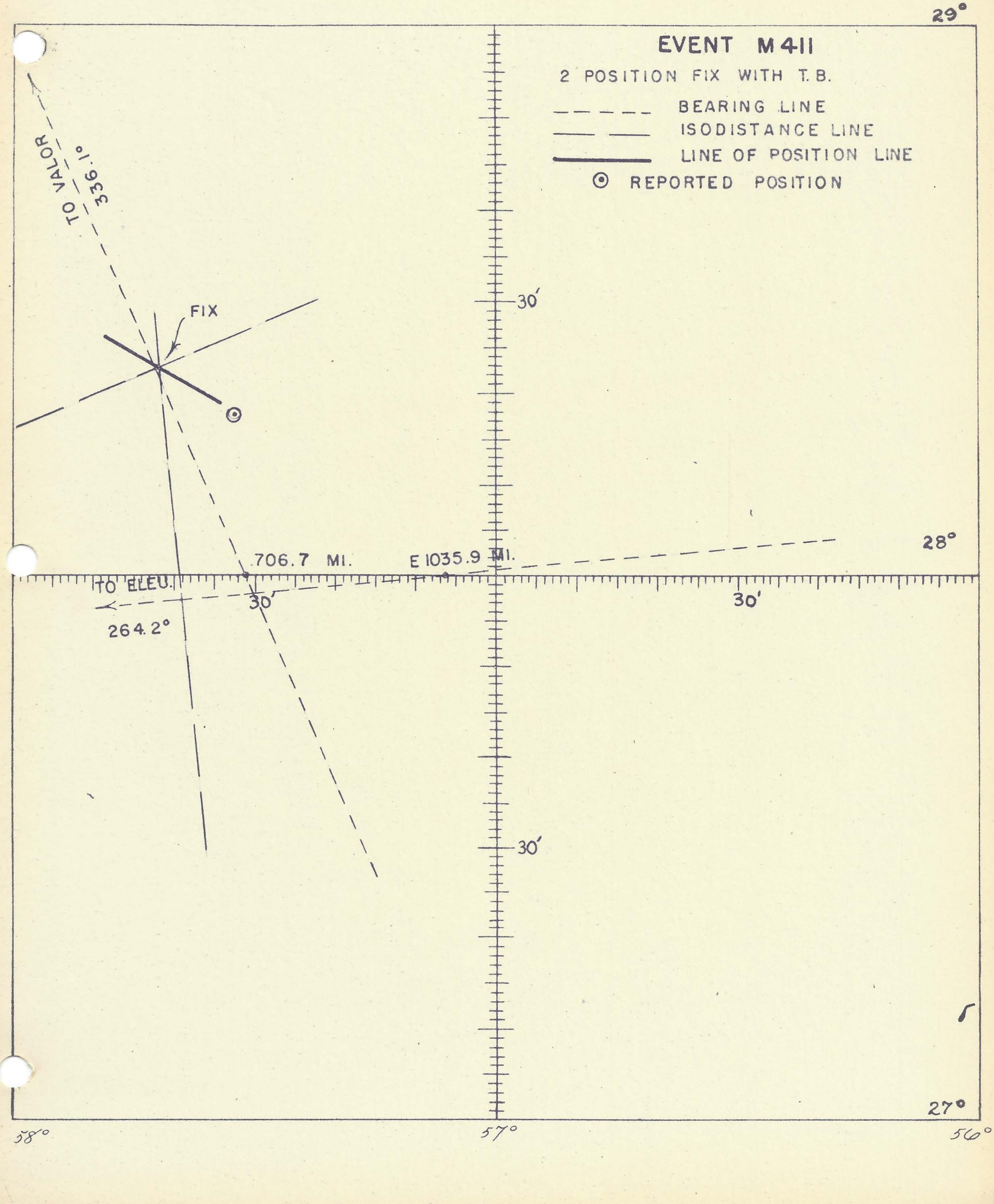
EVENT M406
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
31°
30′
TO VALOR
345.5°
30′
V 561.7 MI.
30′
30°
E903.2 MI.
TO ELEU.
254.1°
FIX
30′
29°
61°
60°
59°

FIGURE 35



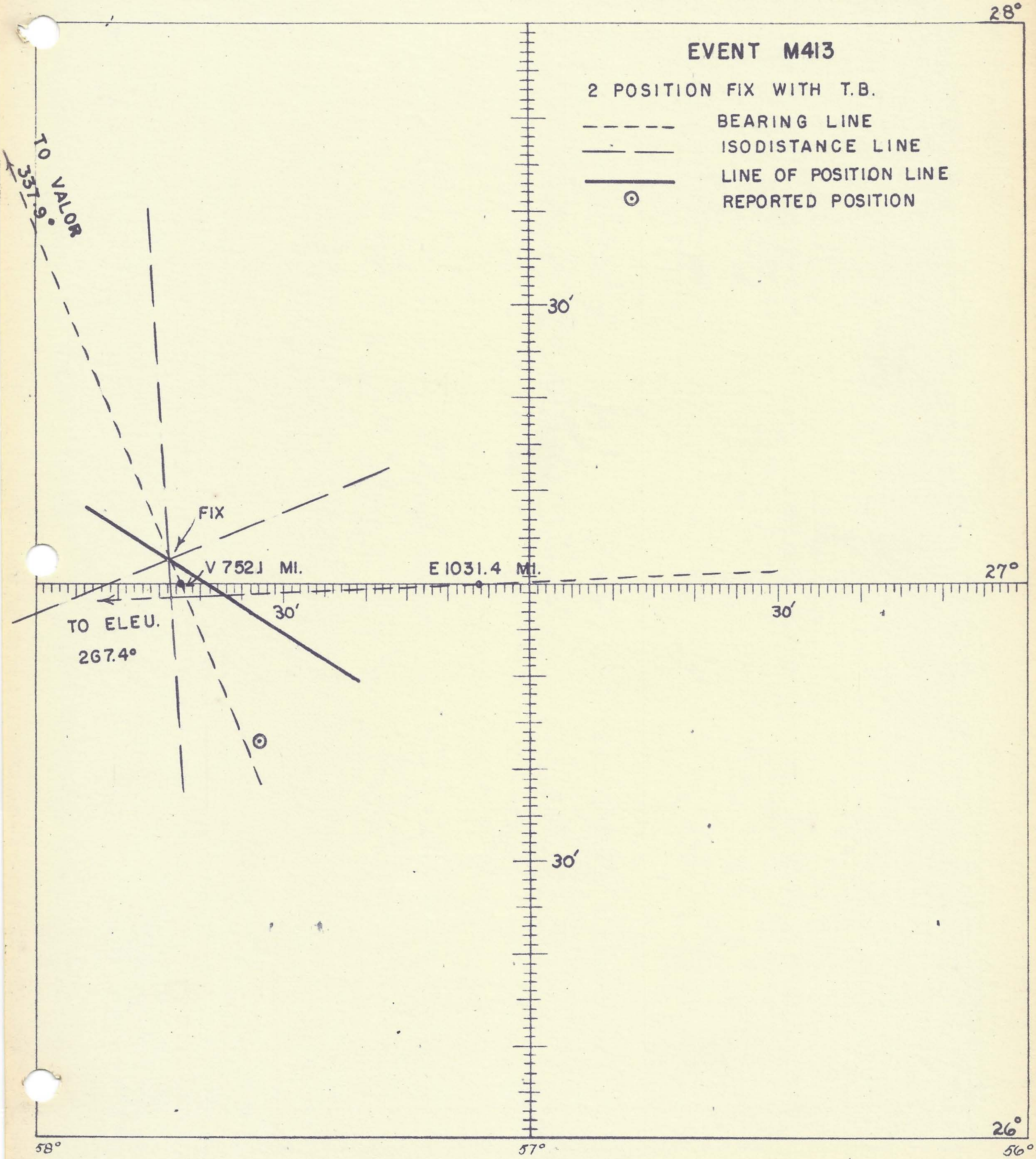
30°
EVENT M408
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
342.4°
30′
FIX
V623.5 MI.
E940.3 MI.
29°
TO ELEU.
259.0°
30′
30′
30′
28°
60°
59°
58°

FIGURE 36



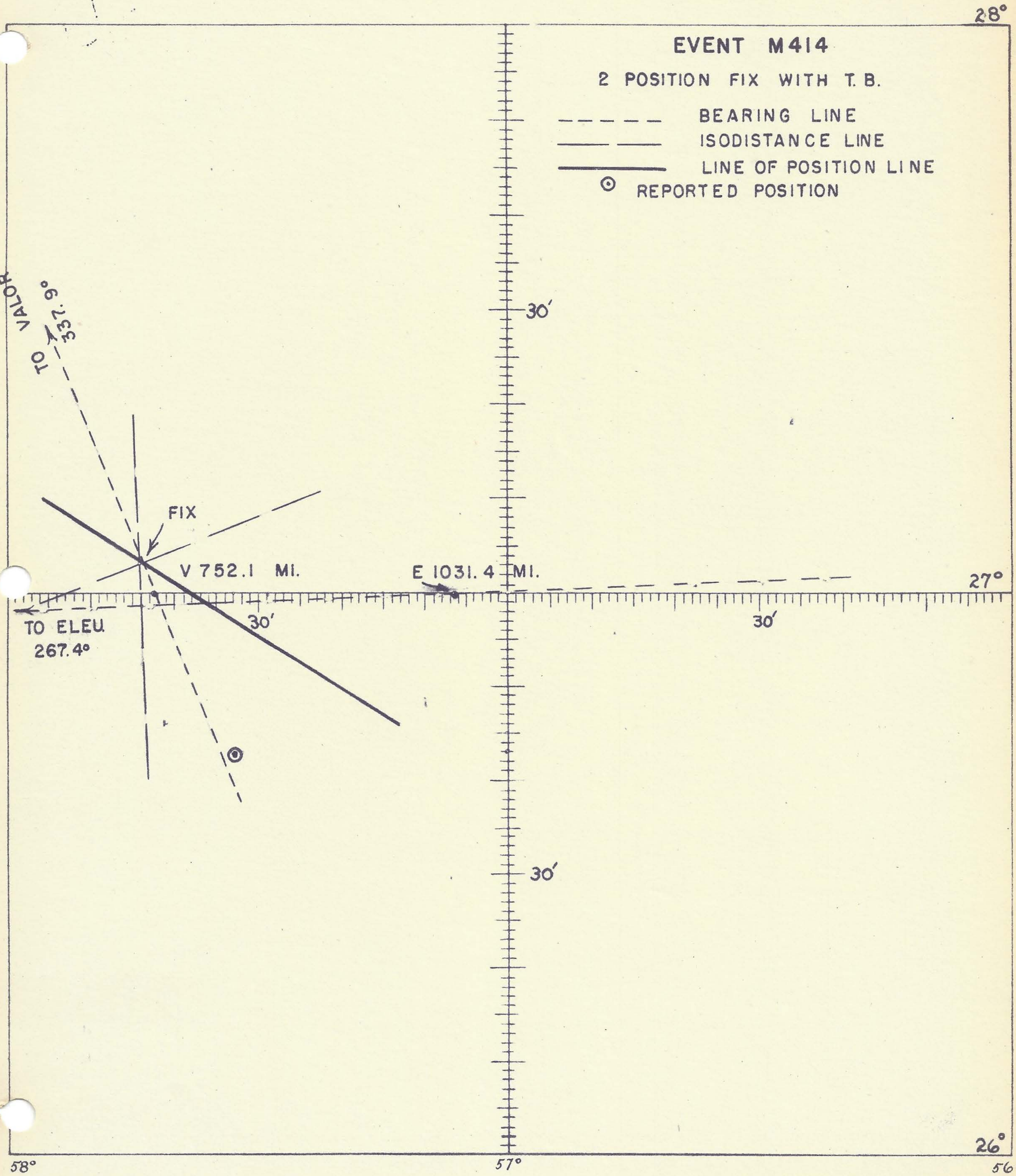
EVENT M411
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
29°
TO VALOR 336.1°
FIX
30'
706.7 MI.
E 1035.9 MI.
28°
TO ELEU.
264.2°
30'
30'
30'
27°
58°
57°
56°

FIGURE 37



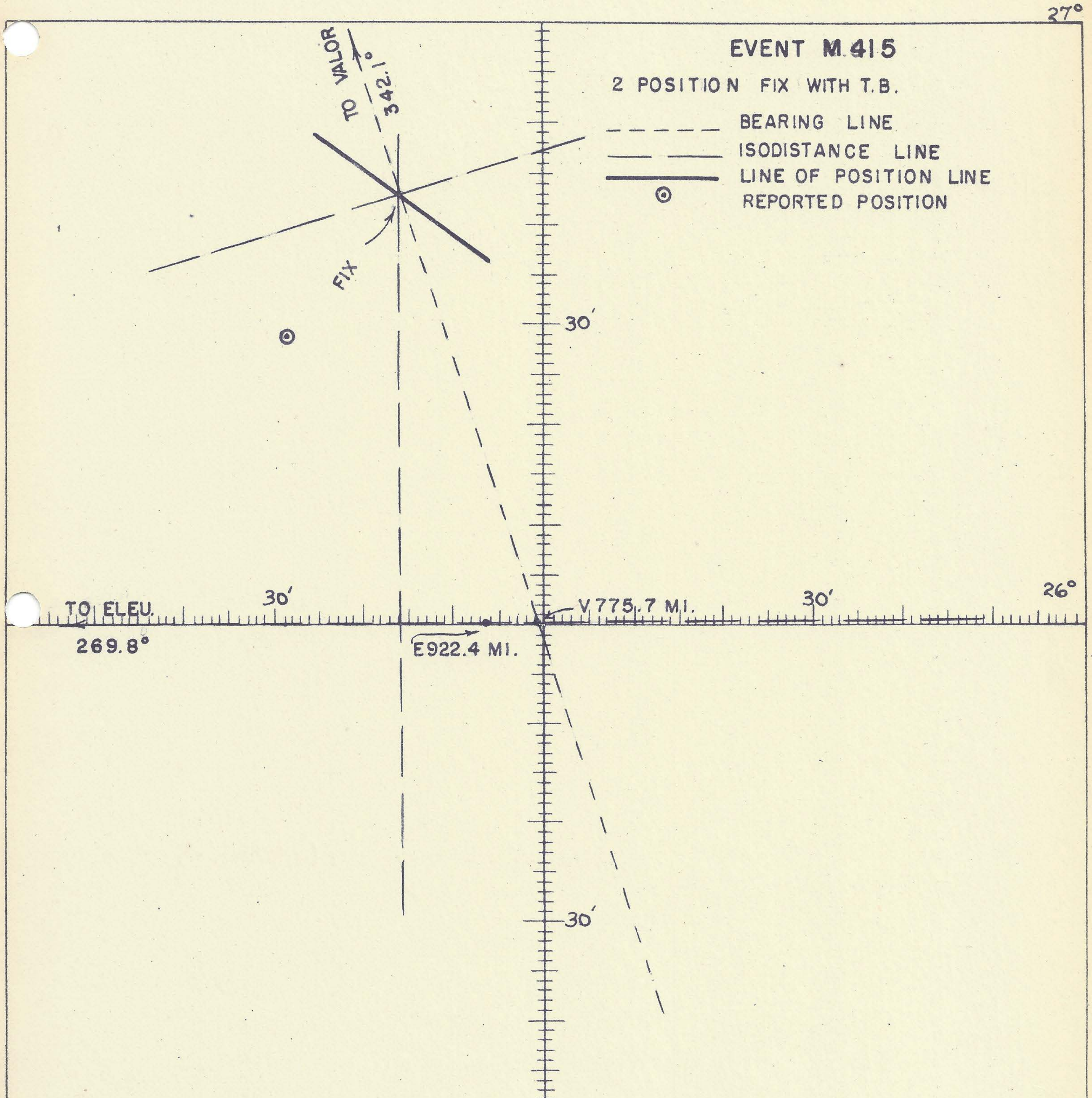
28°
EVENT M413
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
337.9°
30′
FIX
V 752.1 MI.
E 1031.4 MI.
27°
TO ELEU.
267.4°
30′
30′
30′
26°
58°
57°
56°

FIGURE 38



28°
EVENT M414
2 POSITION FIX WITH T. B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
337.9°
30′
FIX
V 752.1 MI.
E 1031.4 MI.
27°
TO ELEU.
267.4°
30′
30′
30′
26°
58°
57°
56°

FIGURE 39



EVENT M 415
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
342.1°
FIX
30′
TO ELEU.
269.8°
30′
V 775.7 MI.
E922.4 MI.
30′
26°
30′
27°
25°
60°
59°
58°

FIGURE 40

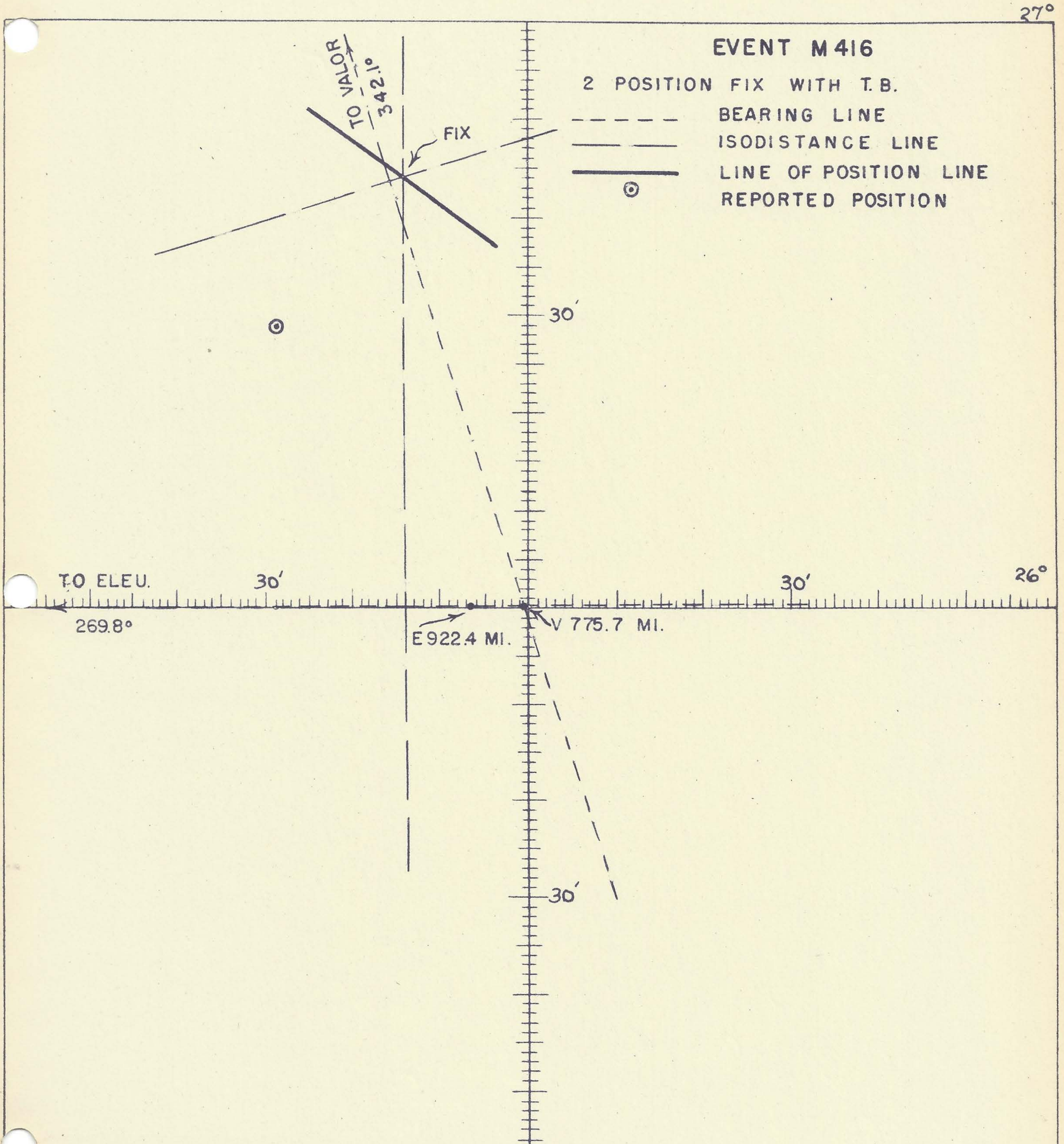
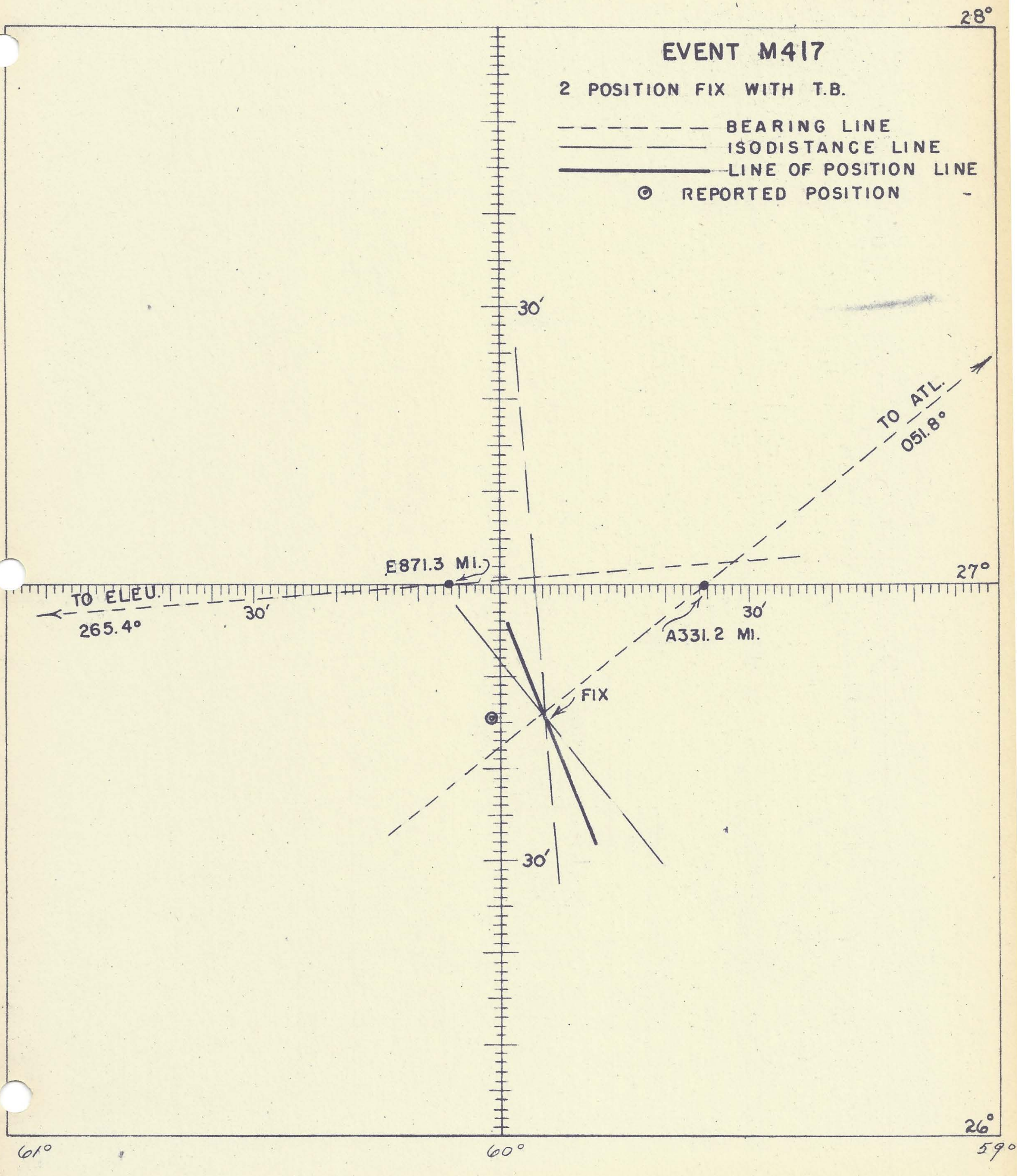

EVENT M416
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
342.1°
FIX
TO ELEU.
269.8°
30′
30′
30′
-30′
E 922.4 MI.
V 775.7 MI.
27°
26°
25°
60°
59°
58°

FIGURE 41



EVENT M417
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
28°
27°
26°
30′
30′
30′
30′
E871.3 MI.
A331.2 MI.
TO ELEU.
265.4°
TO ATL.
051.8°
FIX
61°
60°
59°

FIGURE 42

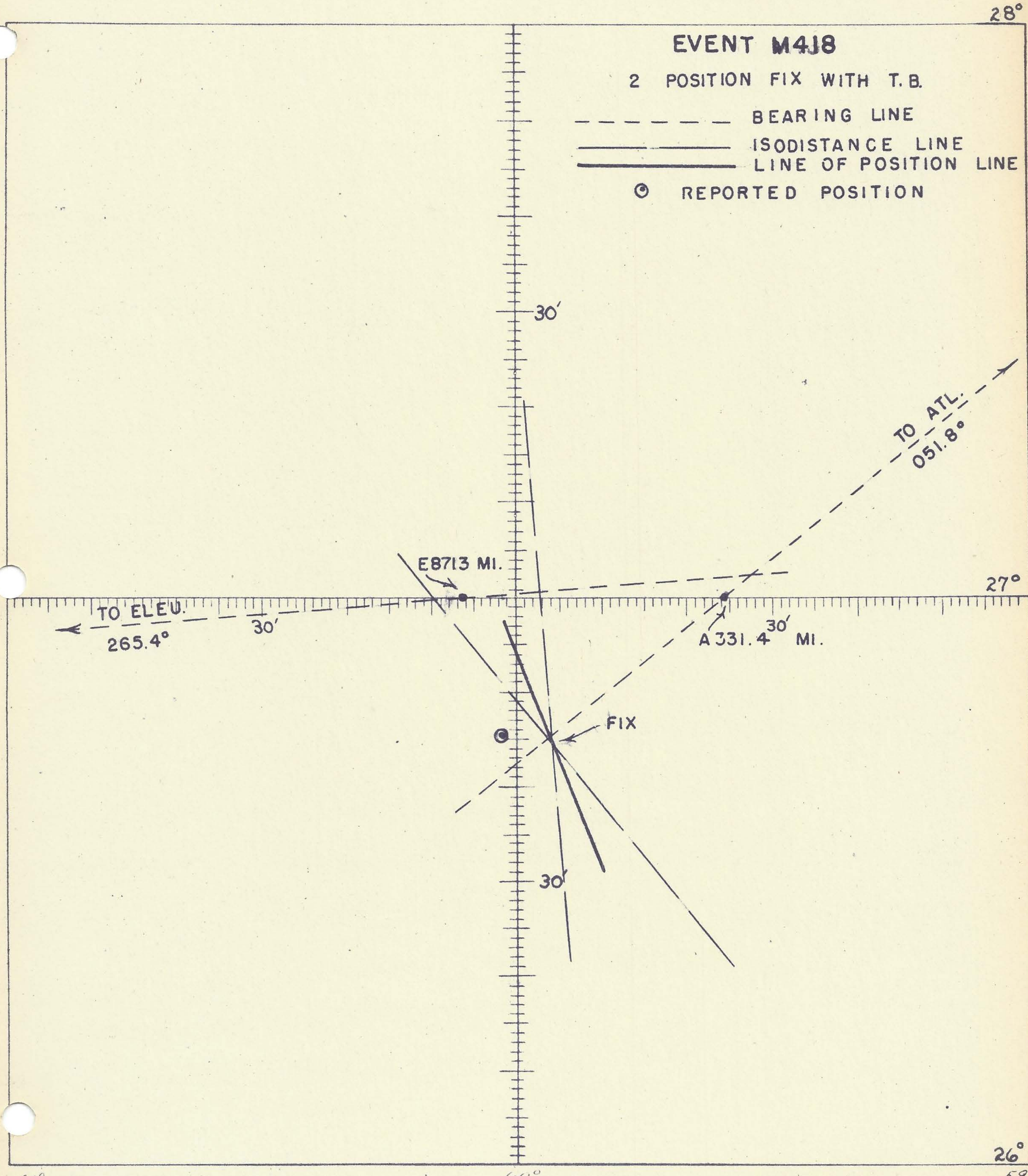
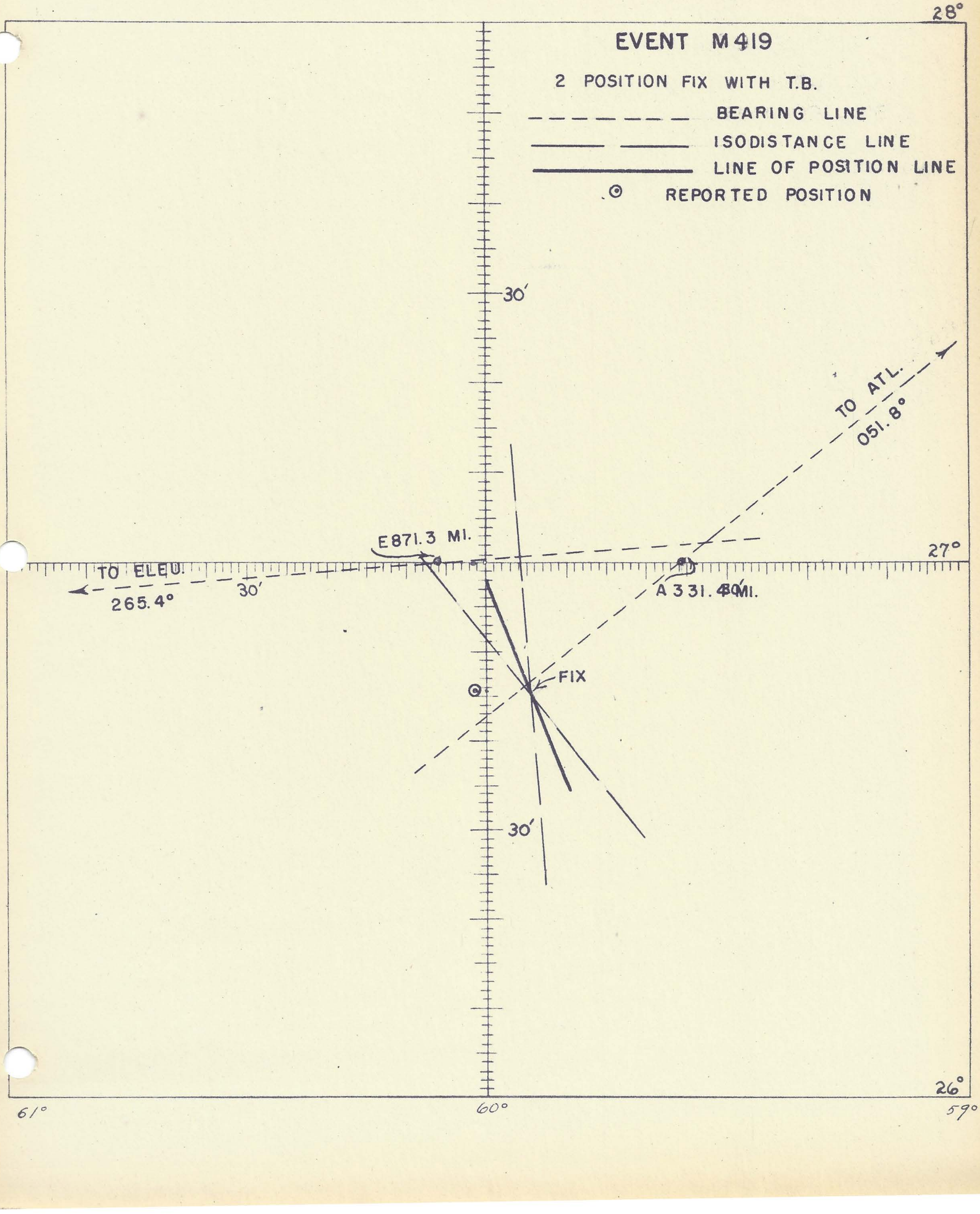

EVENT M418
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
28°
27°
26°
61°
60°
59°
30'
30'
30'
30'
TO ATL.
051.8°
TO ELEU.
265.4°
E8713 MI.
A 331.4 MI.
FIX

FIGURE 43



EVENT M 419
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
28°
30'
TO ATL.
051.8°
E871.3 MI.
TO ELEU.
265.4°
30'
A 331.4 MI.
30'
27°
FIX
30'
26°
61°
60°
59°

FIGURE 44

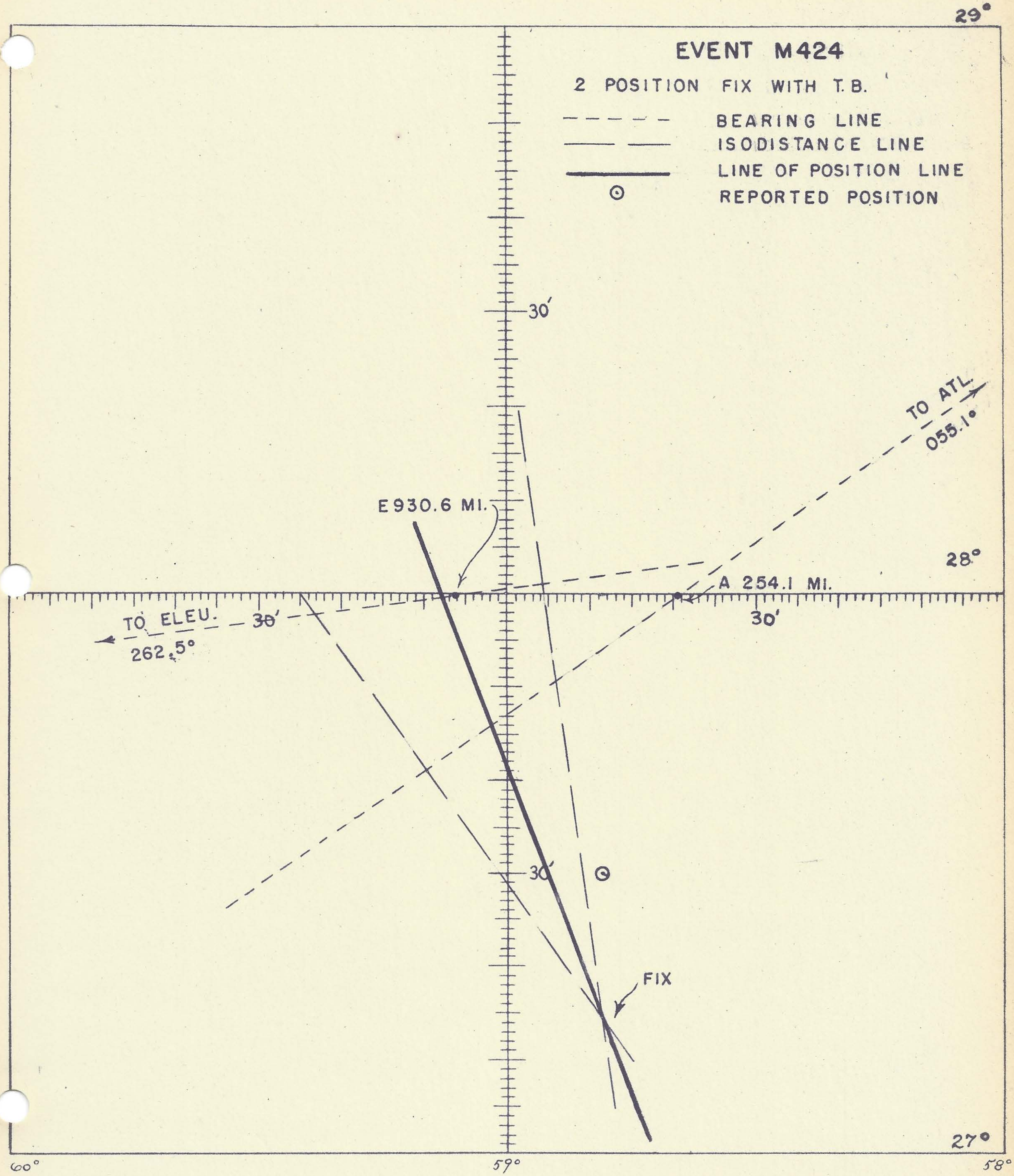

29°
EVENT M424
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
TO ATL.
055.1°
E930.6 MI.
28°
A 254.1 MI.
TO ELEU.
262.5°
30'
30'
30'
FIX
27°
60°
59°
58°

FIGURE 45

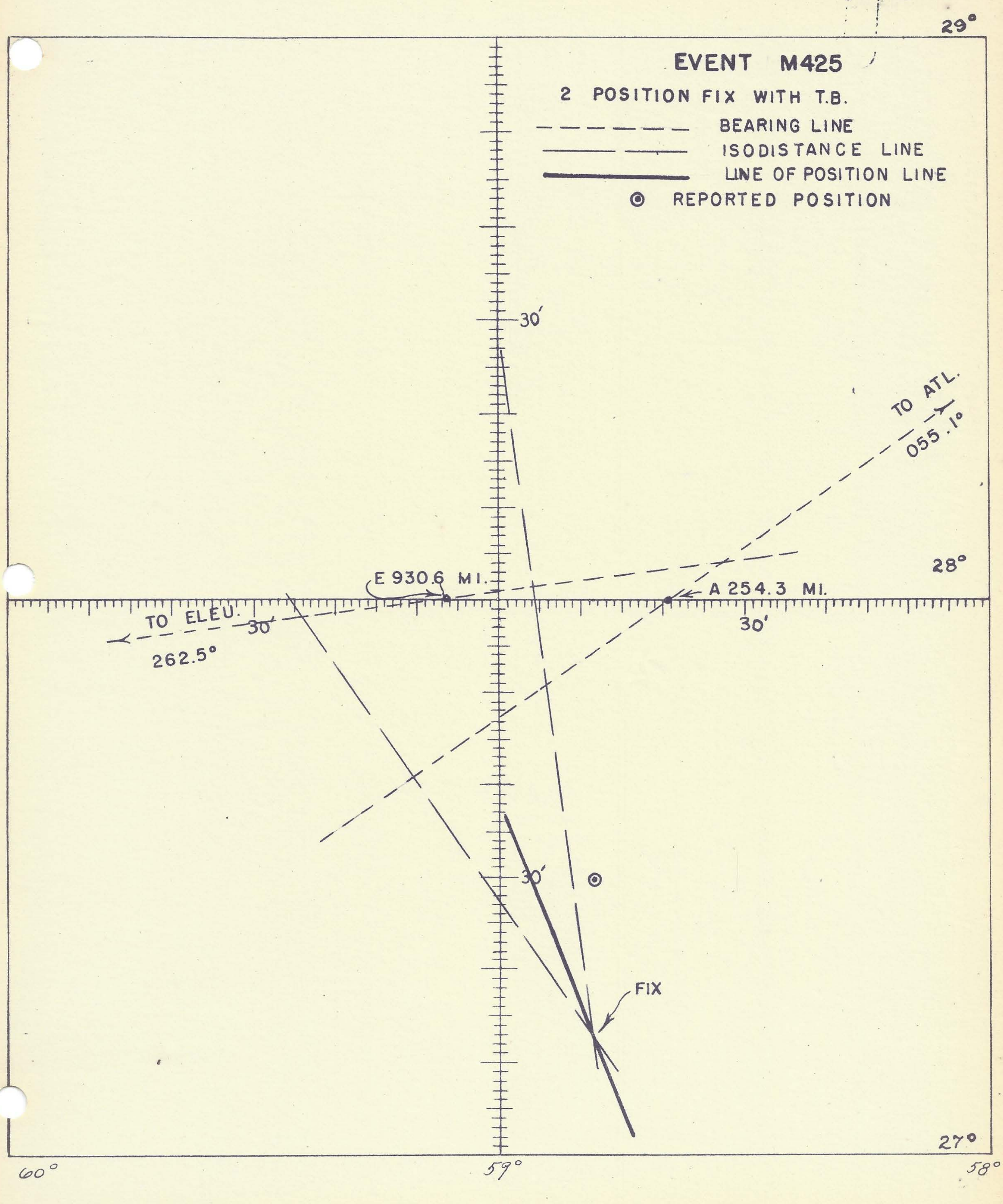
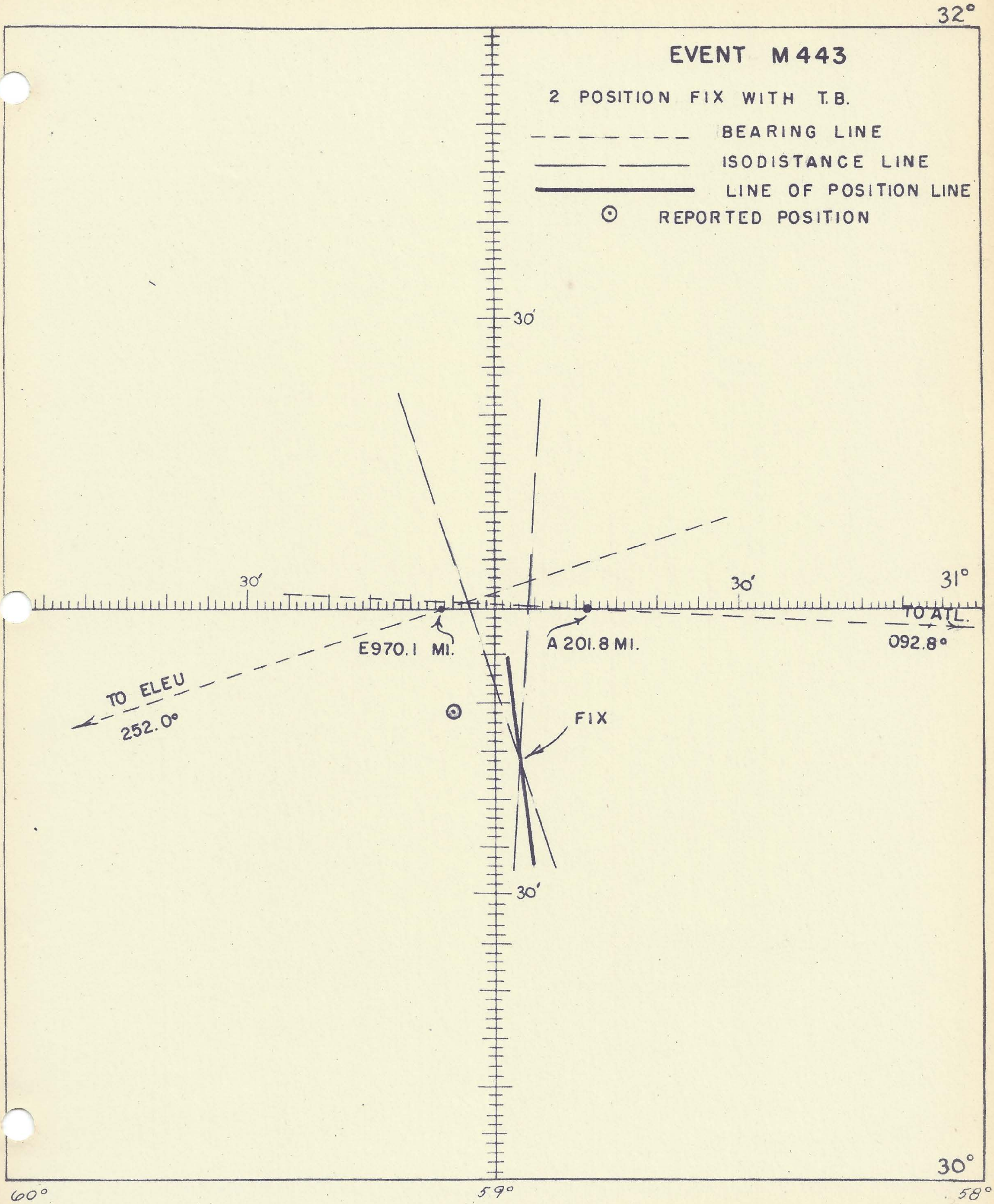

29°
EVENT M425
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
TO ATL.
055.1°
E 930.6 MI.
A 254.3 MI.
28°
TO ELEU.
30′
262.5°
30′
30′
FIX
27°
60°
59°
58°

FIGURE 46



EVENT M 443
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
32°
31°
30°
30′
30′
30′
30′
60°
59°
58°
E970.1 MI.
A 201.8 MI.
TO ATL.
092.8°
TO ELEU
252.0°
FIX

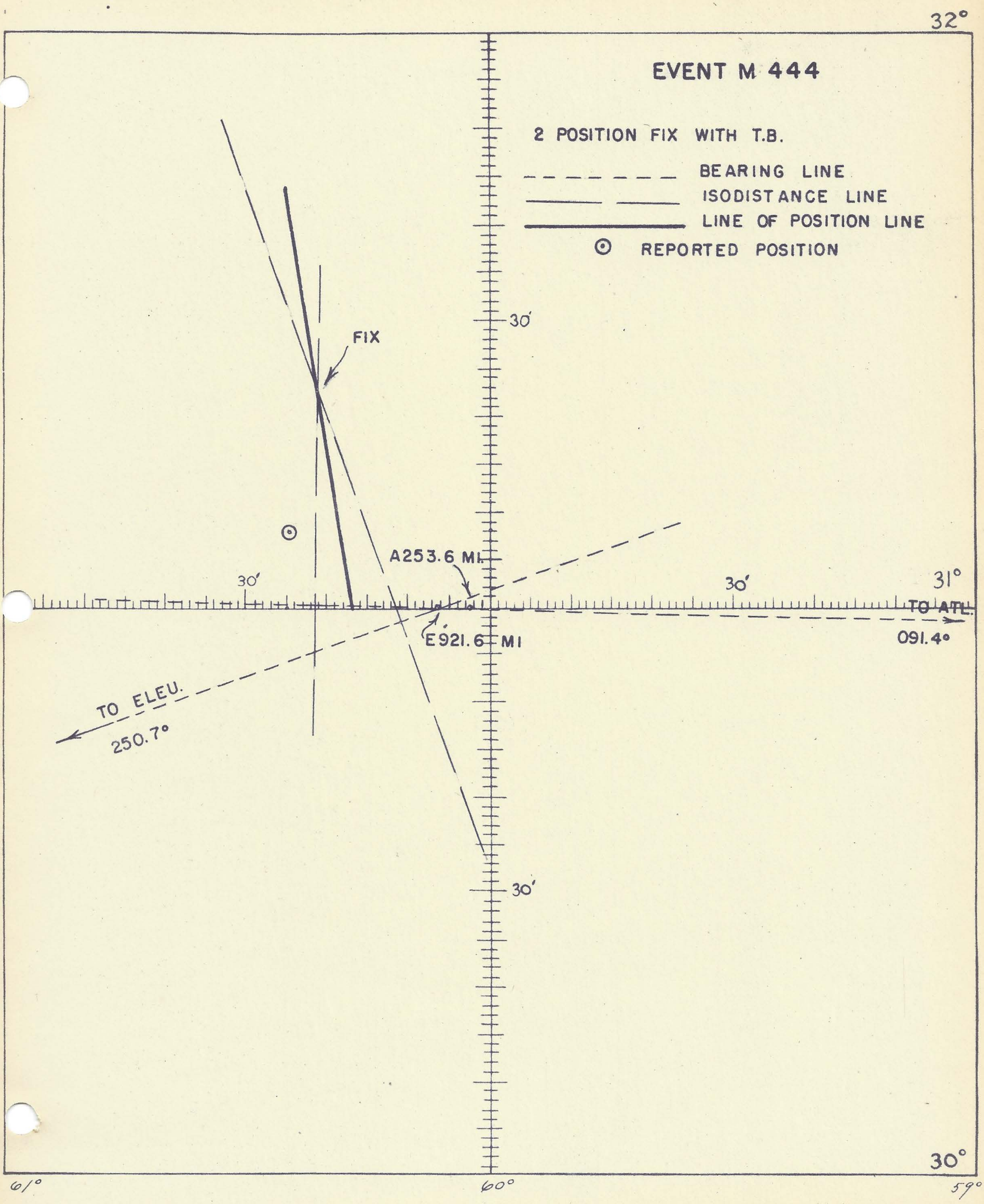
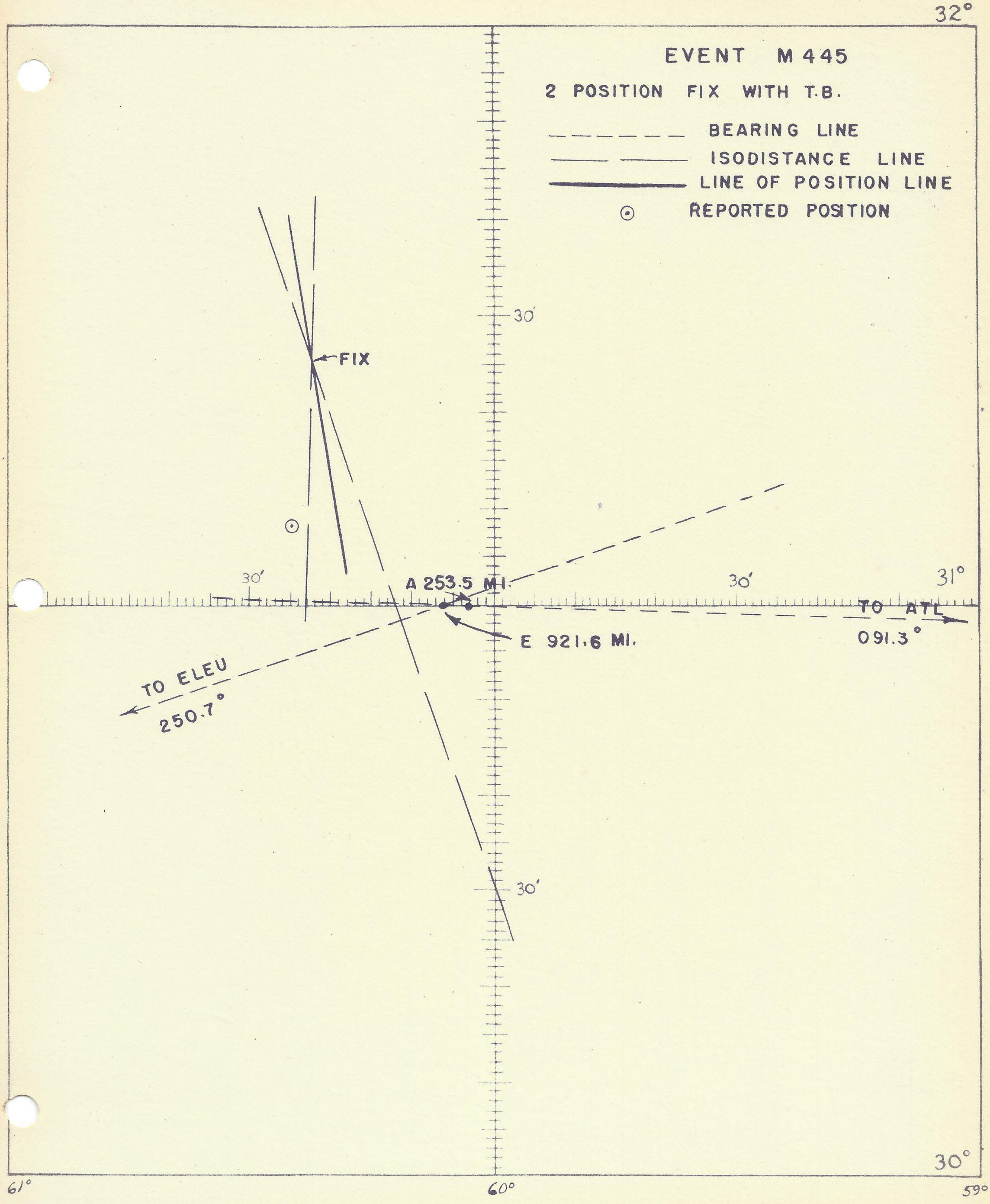
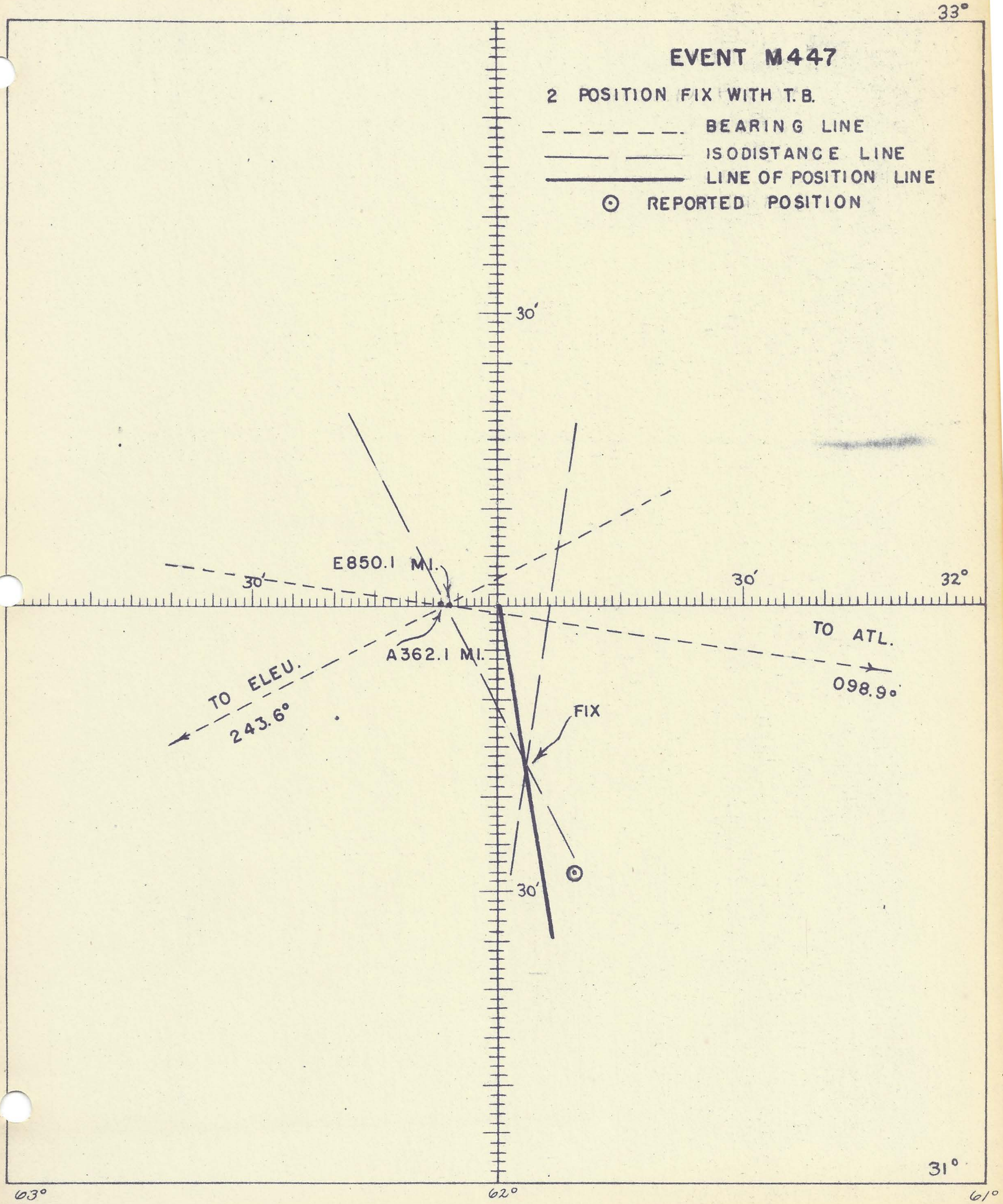

32°
EVENT M 444
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
FIX
A253.6 MI.
30′
30′
31°
TO ATL.
091.4°
E921.6 MI
TO ELEU.
250.7°
30′
30°
61°
60°
59°

FIGURE 48



32°
EVENT M 445
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
FIX
30'
A 253.5 MI.
30'
31°
TO ATL
091.3°
E 921.6 MI.
TO ELEU
250.7°
30'
30°
61°
60°
59°

FIGURE 49



EVENT M447
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
33°
30'
30'
30'
32°
E850.1 MI.
A362.1 MI.
TO ATL.
098.9°
TO ELEU.
243.6°
FIX
30'
31°
63°
62°
61°

FIGURE 50

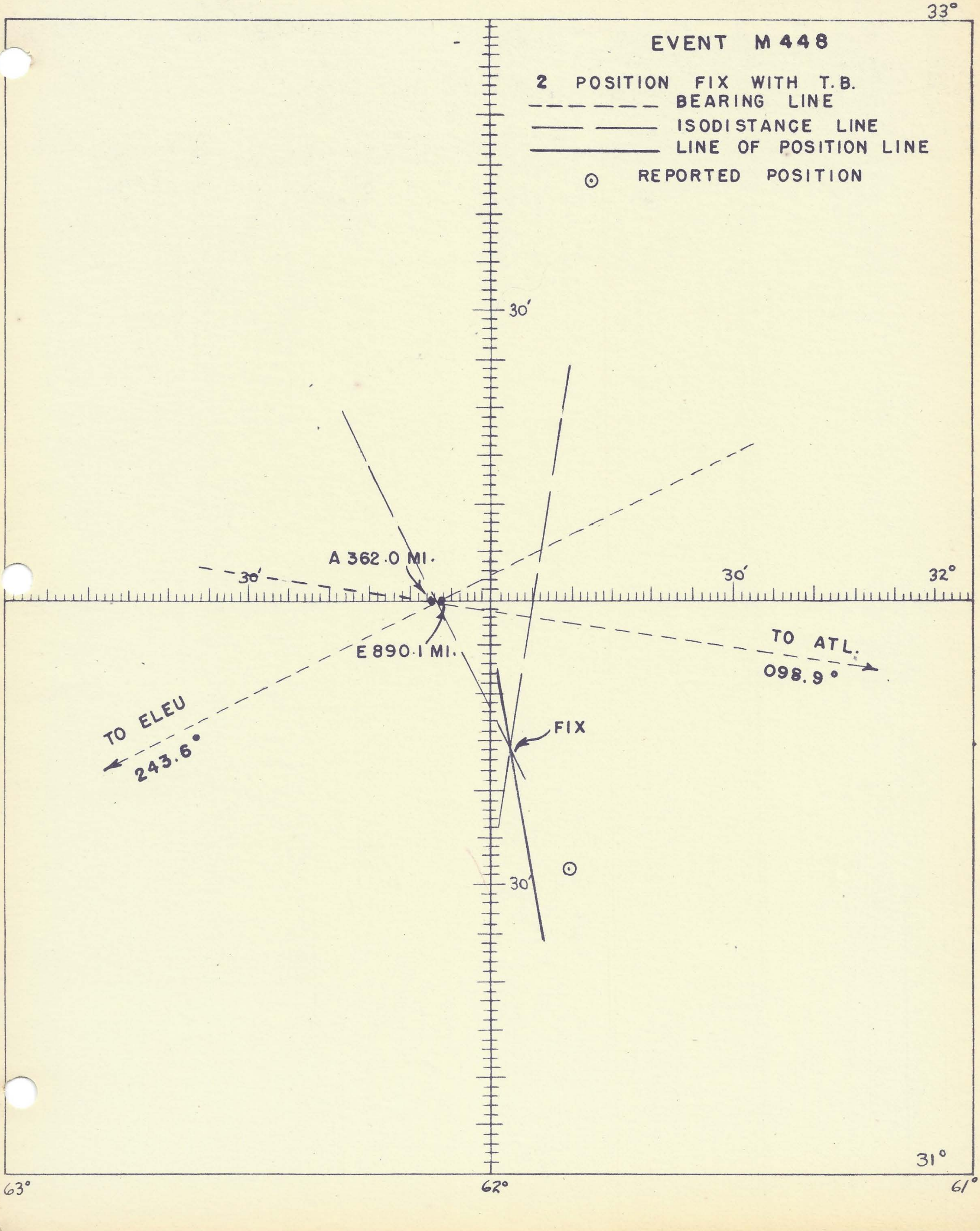
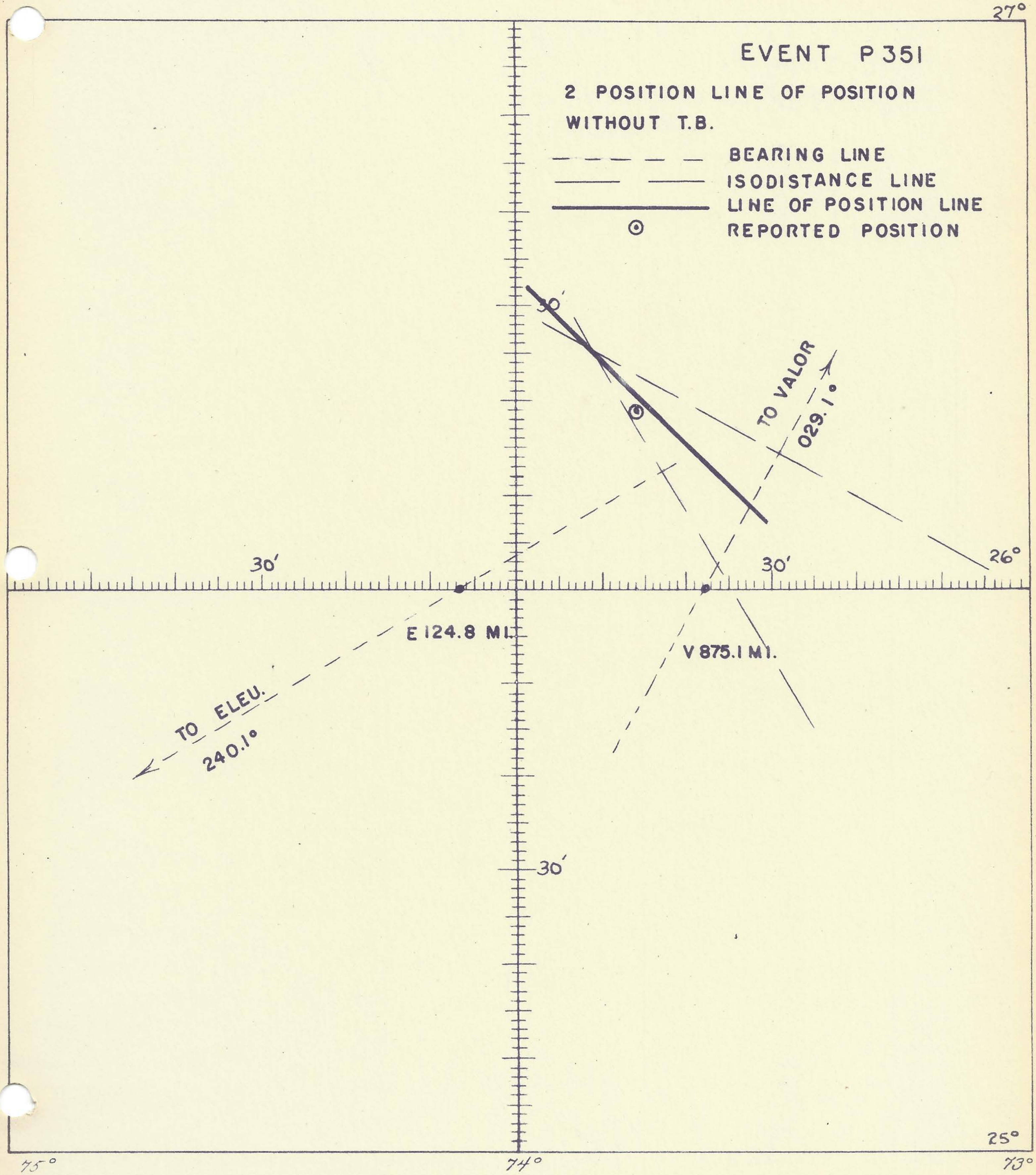
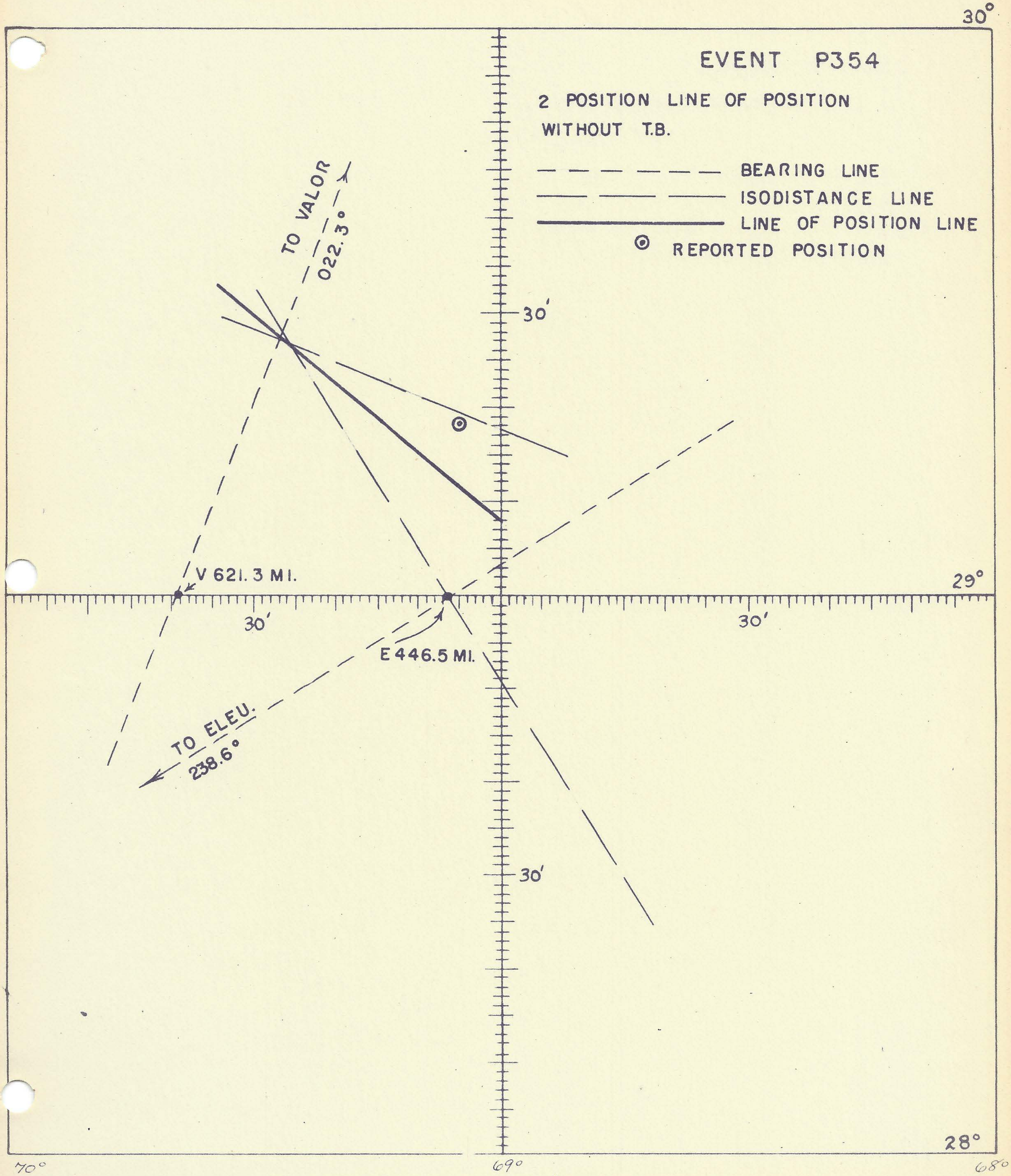
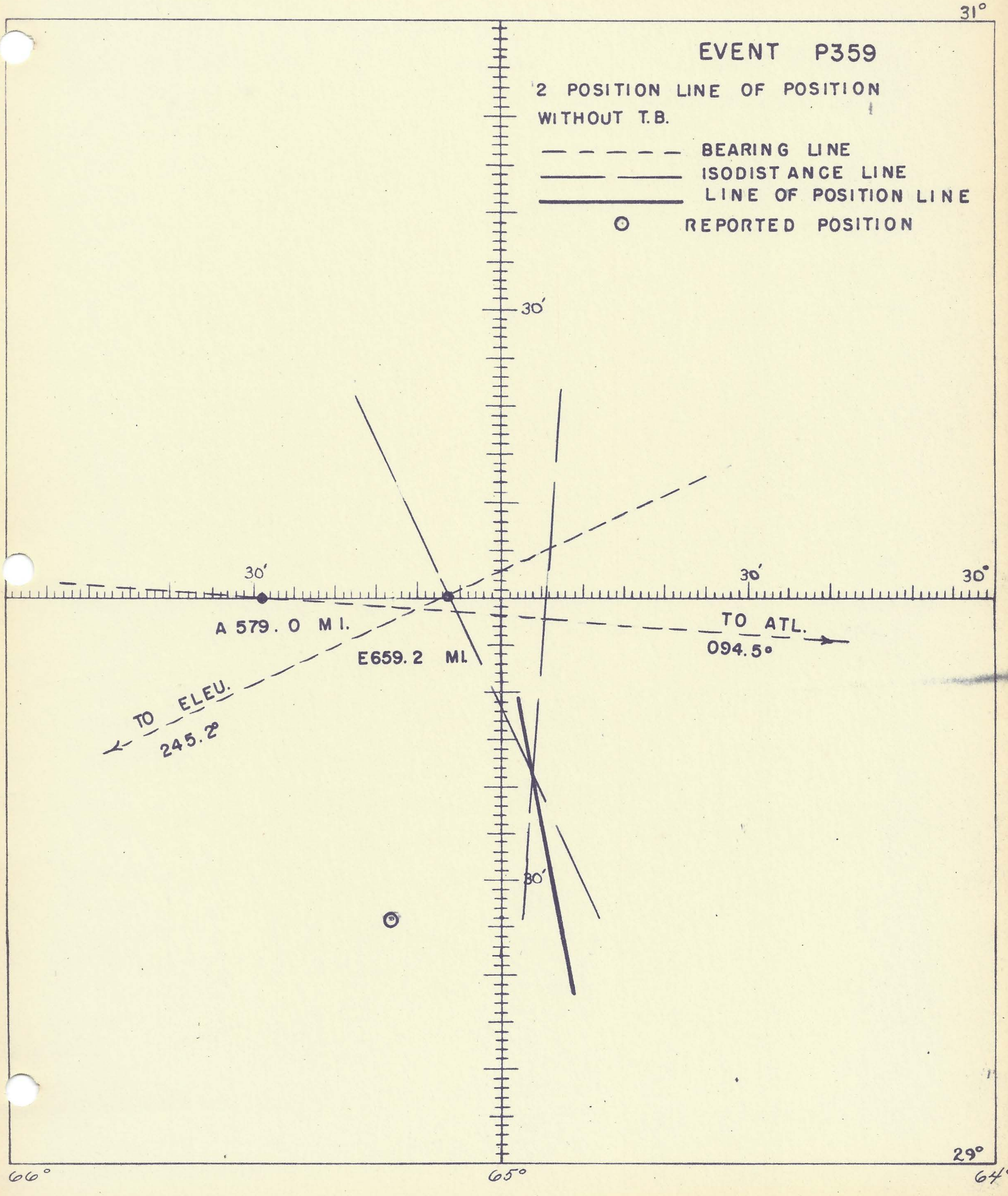

33°
EVENT M 448
2 POSITION FIX WITH T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
A 362.0 MI.
30′
30′
32°
E 890.1 MI.
TO ATL.
098.9°
TO ELEU
243.6°
FIX
30′
31°
63°
62°
61°

FIGURE 51


EVENT P351
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
27°
30′
TO VALOR
029.1°
30′
30′
26°
E 124.8 MI.
V 875.1 MI.
TO ELEV.
240.1°
30′
25°
75°
74°
73°



30°
EVENT P354
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
TO VALOR
022.3°
30'
V 621.3 MI.
29°
30'
30'
E 446.5 MI.
TO ELEU.
238.6°
30'
28°
70°
69°
68°



31°
EVENT P359
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
30'
30'
30°
A 579.0 MI.
E659.2 MI.
TO ATL.
094.5°
TO ELEU.
245.2°
30'
29°
66°
65°
64°

FIGURE 54

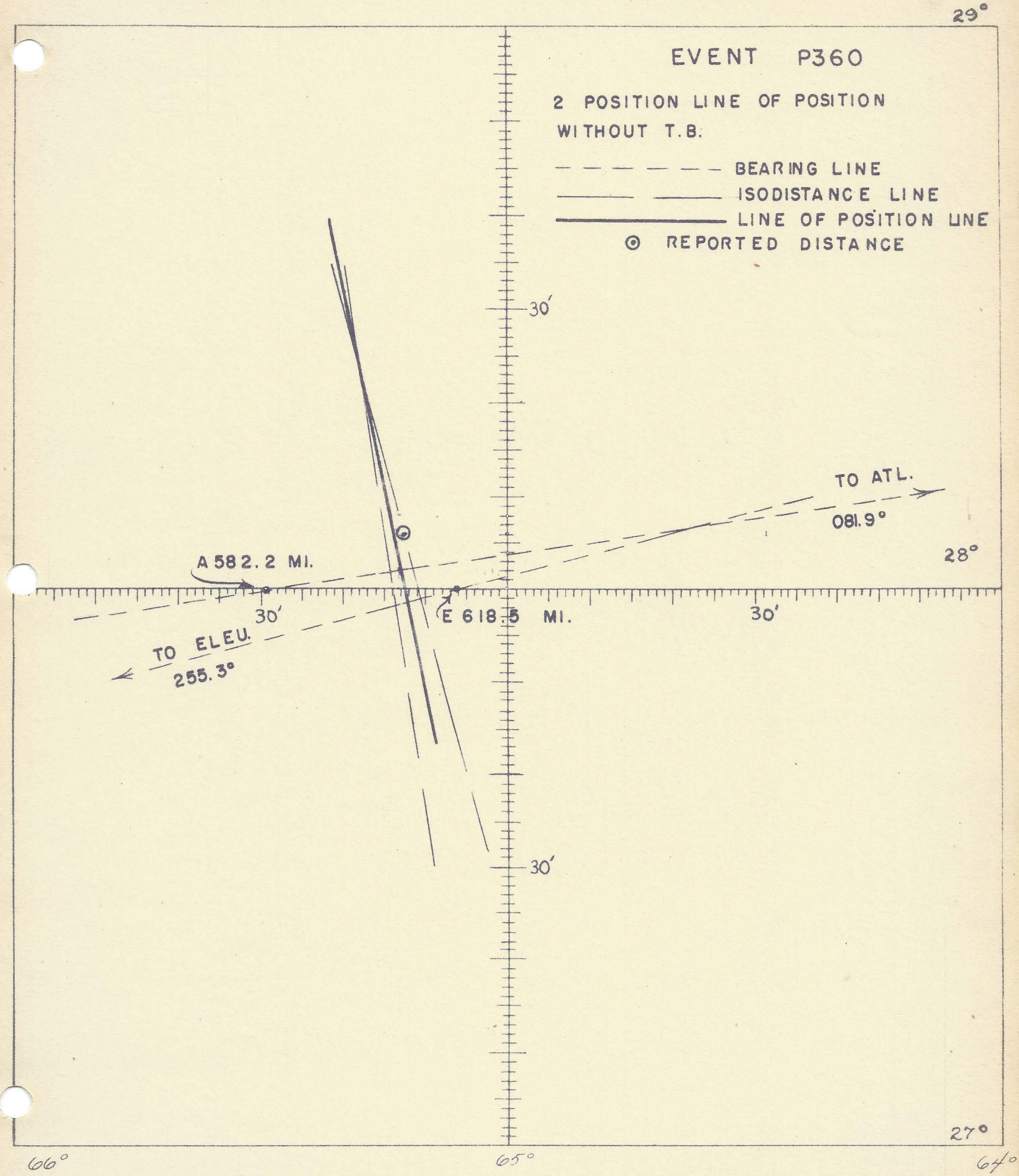
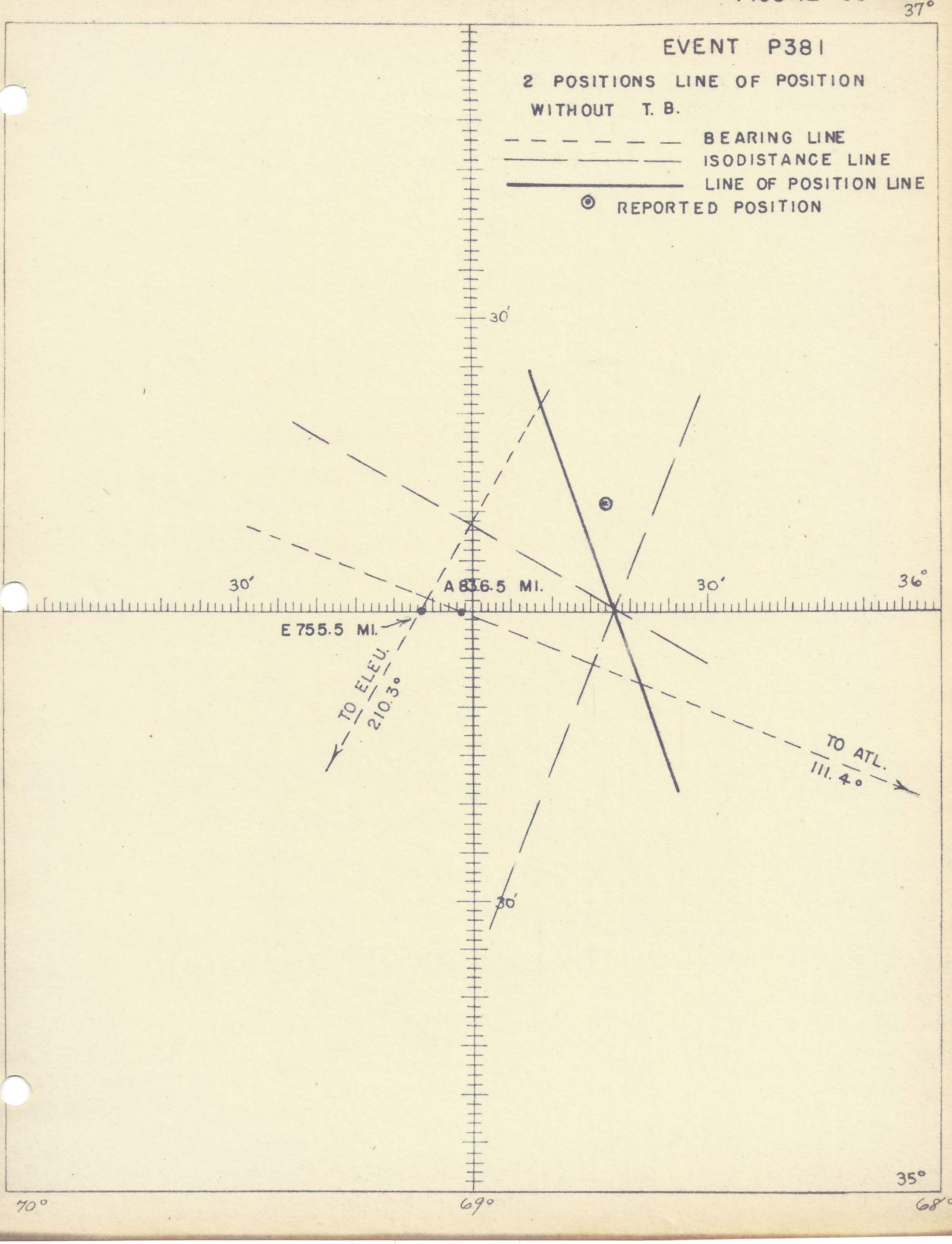

29°
EVENT P360
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED DISTANCE
30′
TO ATL.
081.9°
28°
A 582.2 MI.
30′
E 618.5 MI.
30′
TO ELEU.
255.3°
30′
27°
66°
65°
64°

FIGURE 55


37°
EVENT P381
2 POSITIONS LINE OF POSITION
WITHOUT T. B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
30′
A 836.5 MI.
30′
36°
E 755.5 MI.
TO ELEU.
210.3°
TO ATL.
111.4°
30′
35°
70°
69°
68°

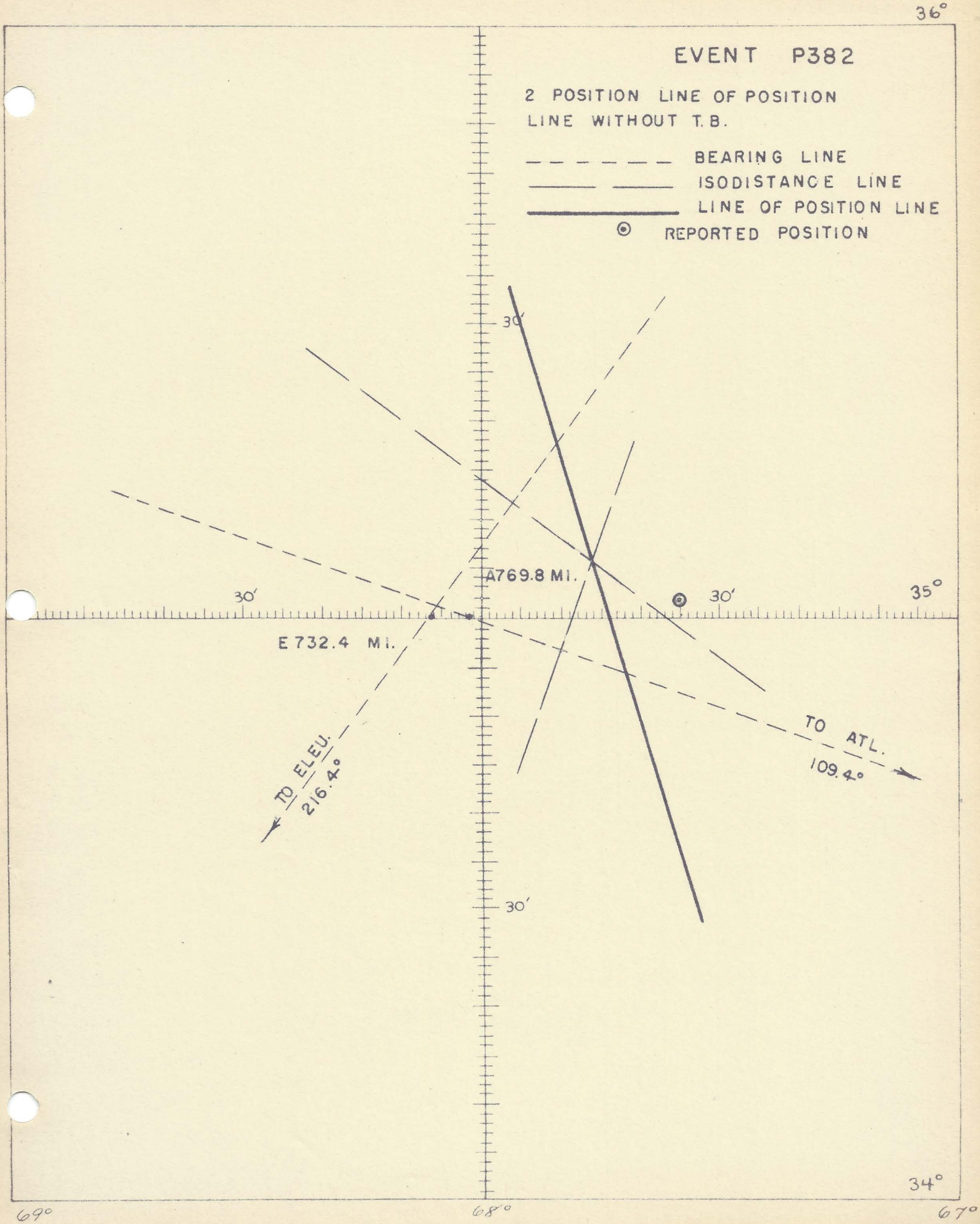
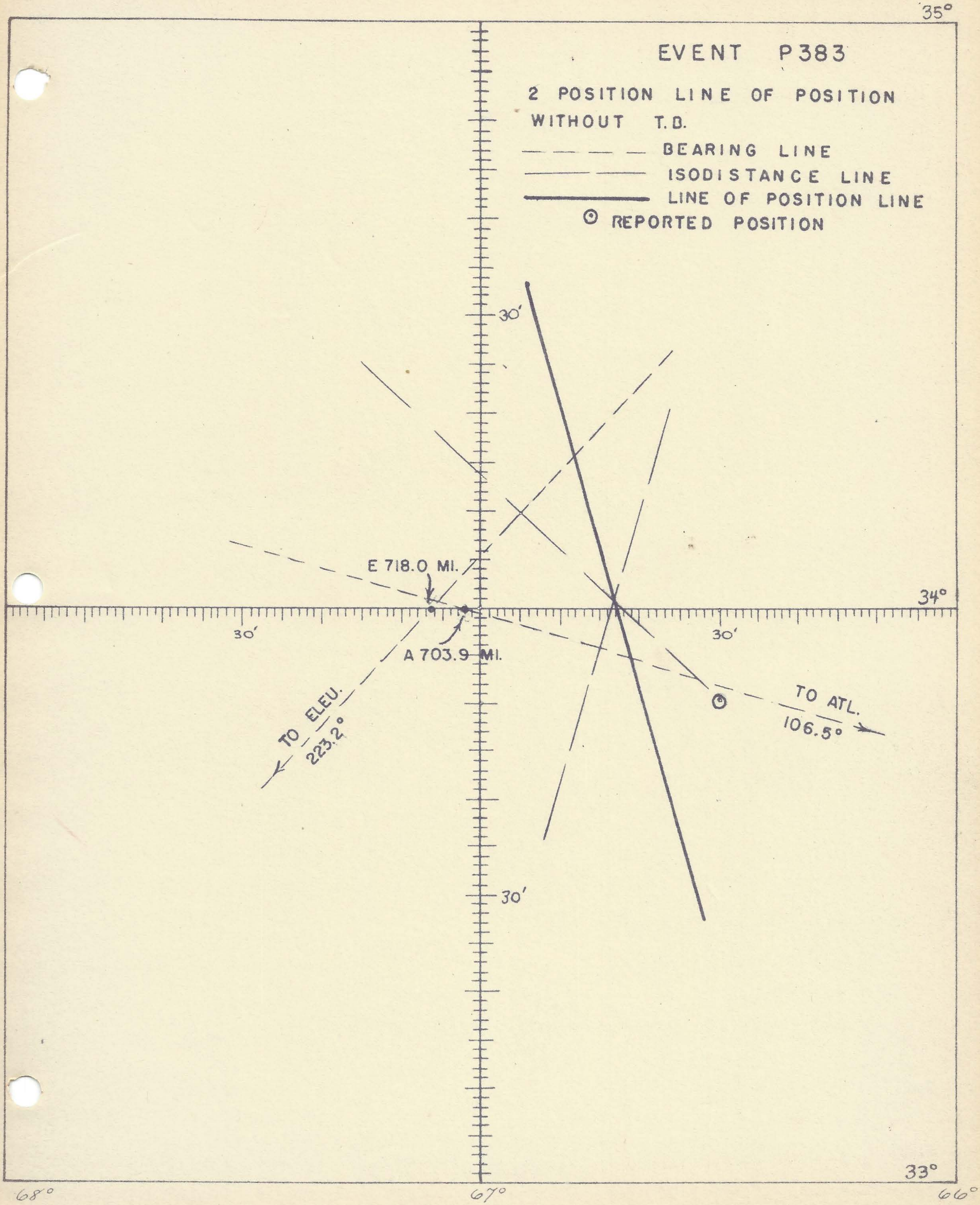
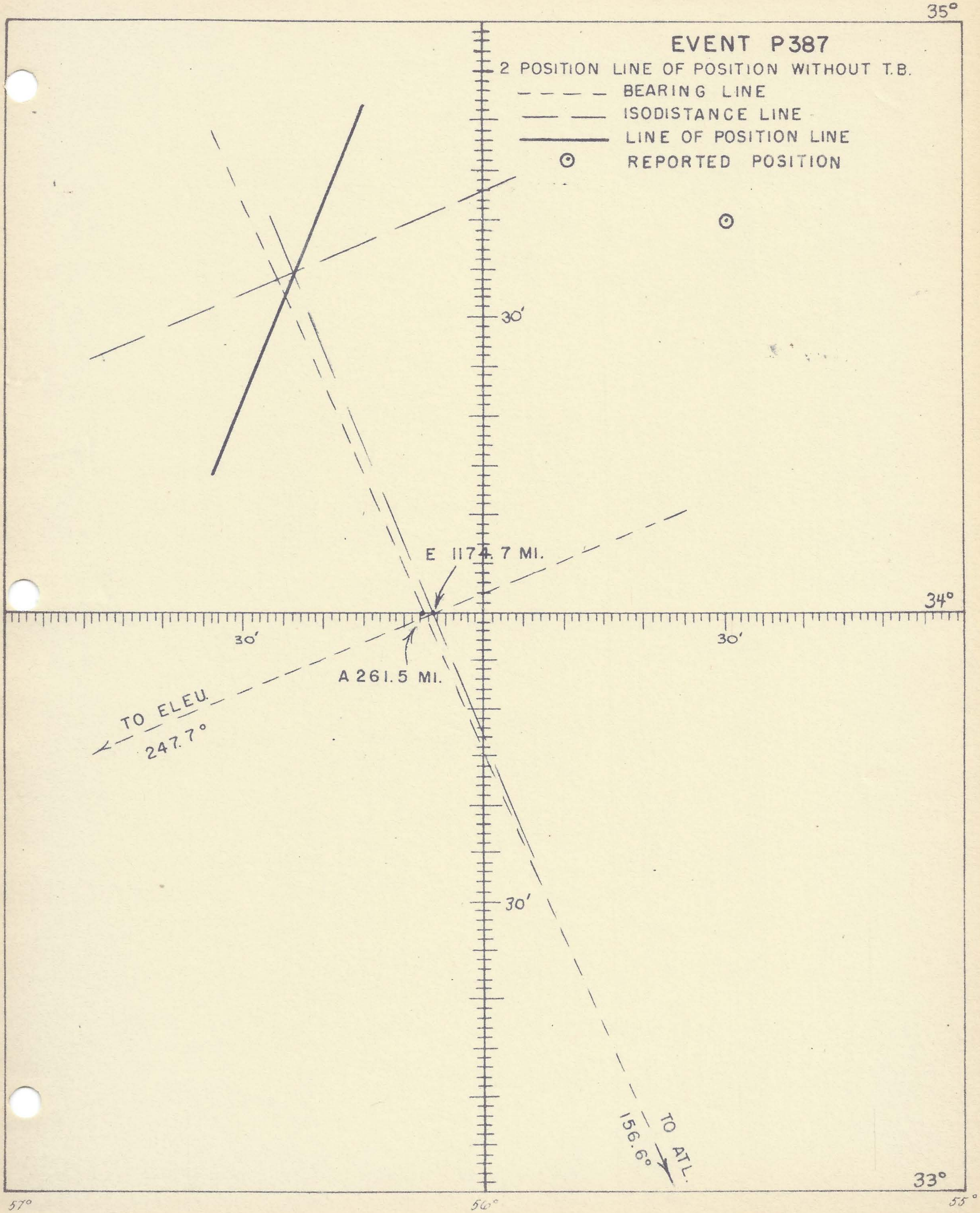

36°
EVENT P382
2 POSITION LINE OF POSITION
LINE WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
A769.8 MI.
30′
30′
35°
E732.4 MI.
TO ELEU.
216.4°
TO ATL.
109.4°
30′
34°
69°
68°
67°

FIGURE 57



35°
EVENT P383
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
E 718.0 MI.
34°
30'
30'
A 703.9 MI.
TO ELEU.
223.2°
TO ATL.
106.5°
30'
33°
68°
67°
66°

FIGURE 58



35°
EVENT P387
2 POSITION LINE OF POSITION WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
E 1174.7 MI.
34°
30'
30'
A 261.5 MI.
TO ELEU.
247.7°
30'
TO ATL.
156.6°
33°
57°
56°
55°

FIGURE 59

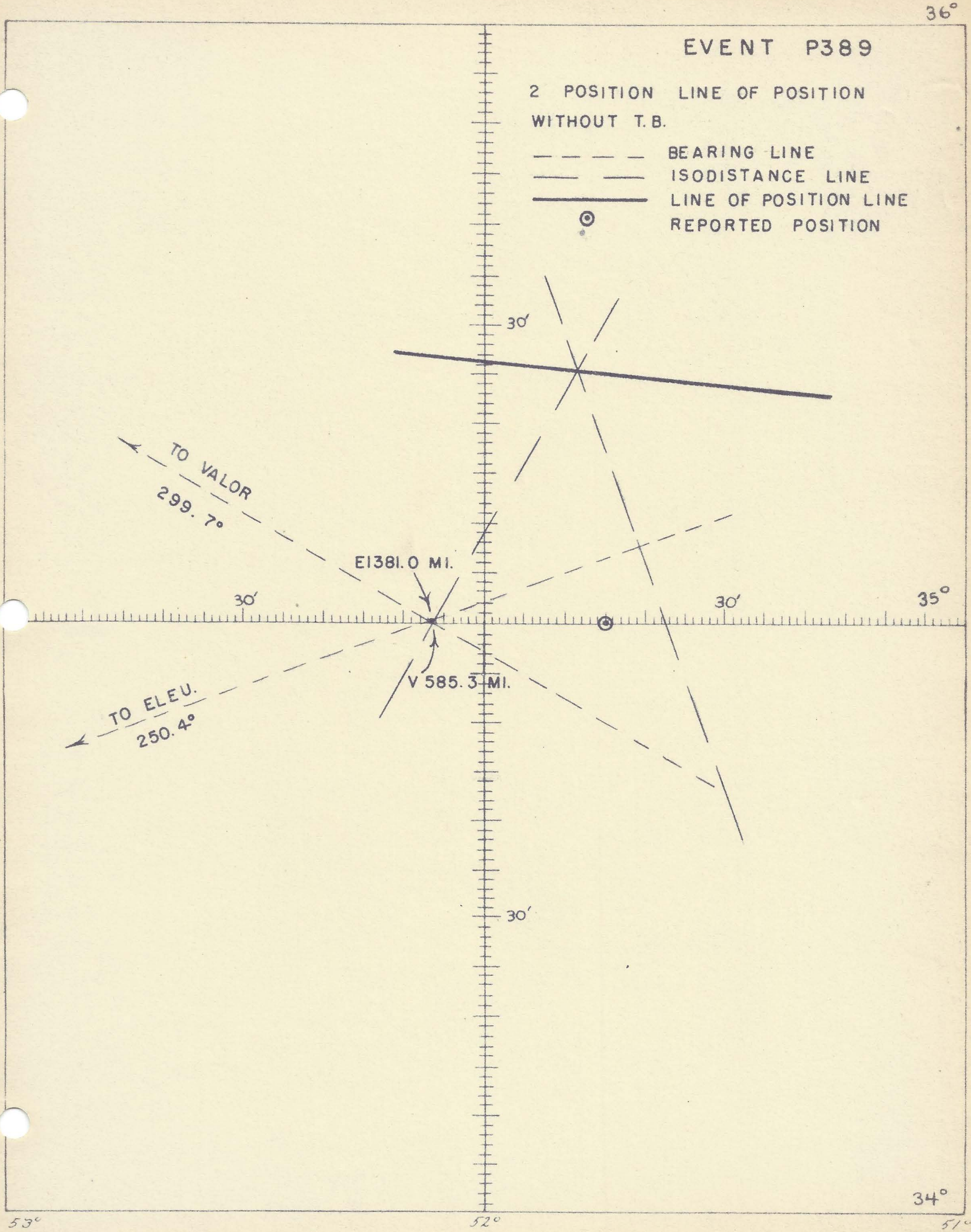
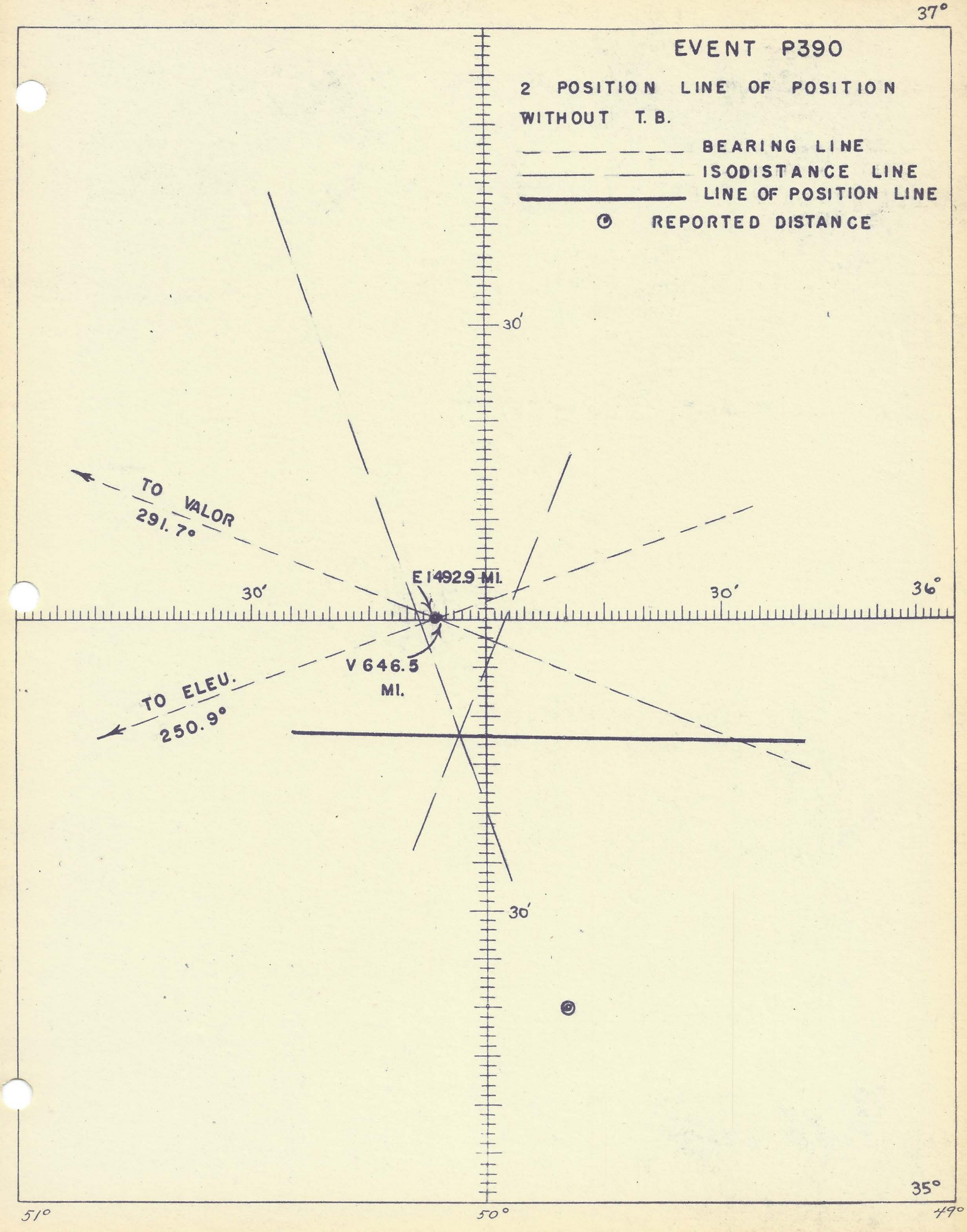

36°
EVENT P389
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
TO VALOR
299. 7°
E1381.0 MI.
30′
30′
35°
V 585. 3 MI.
TO ELEU.
250. 4°
30′
34°
53°
52°
51°

FIGURE 60


37°
EVENT P390
2 POSITION LINE OF POSITION
WITHOUT T. B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED DISTANCE
30'
TO VALOR
291.7°
E 1492.9 MI.
30'
30'
36°
V 646.5
MI.
TO ELEU.
250.9°
30'
35°
51°
50°
49°

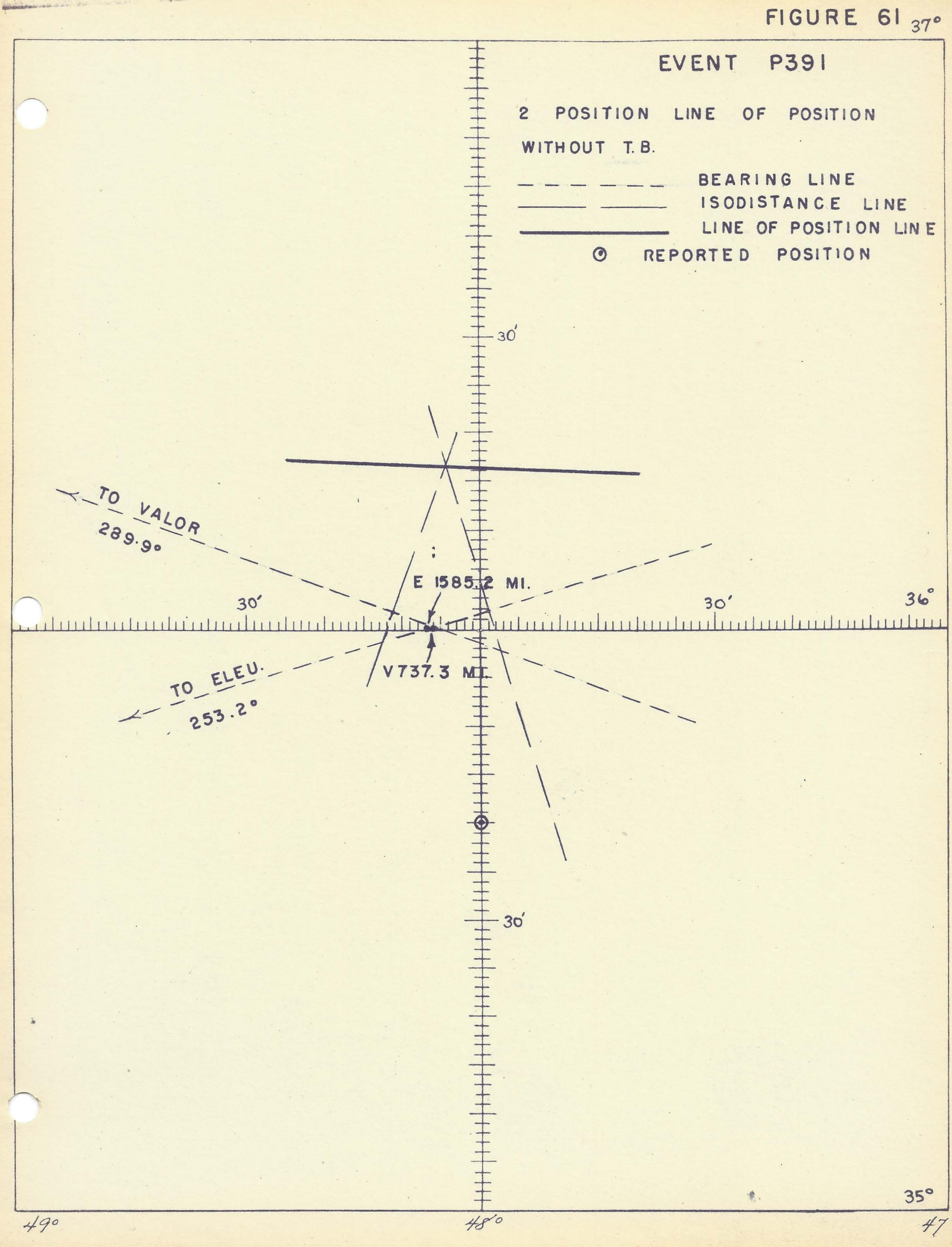

37°
EVENT P391
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
TO VALOR
289.9°
E 1585.2 MI.
30'
30'
36°
TO ELEU.
253.2°
V737.3 MI.
30'
35°
49°
48°
47°

FIGURE 62

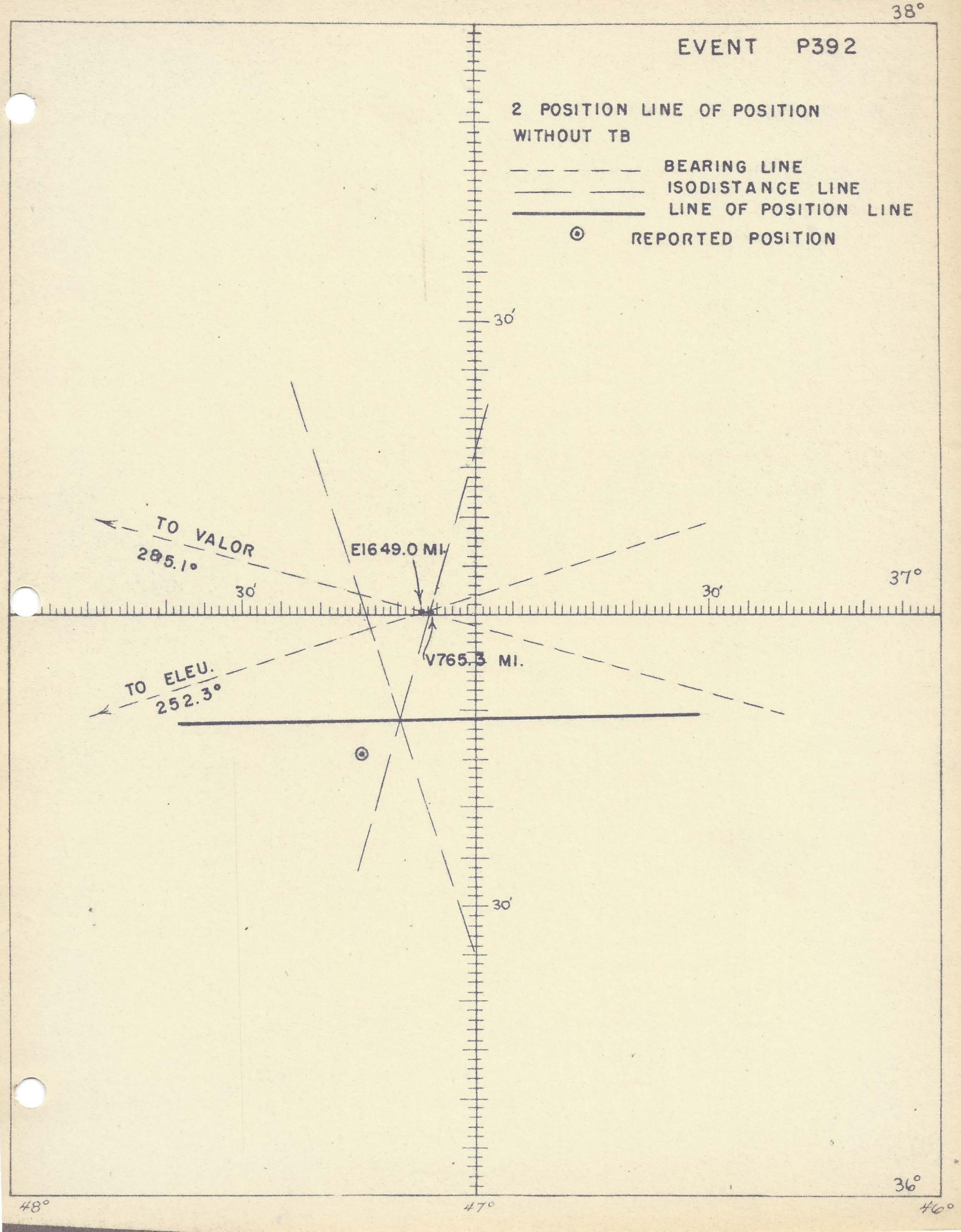
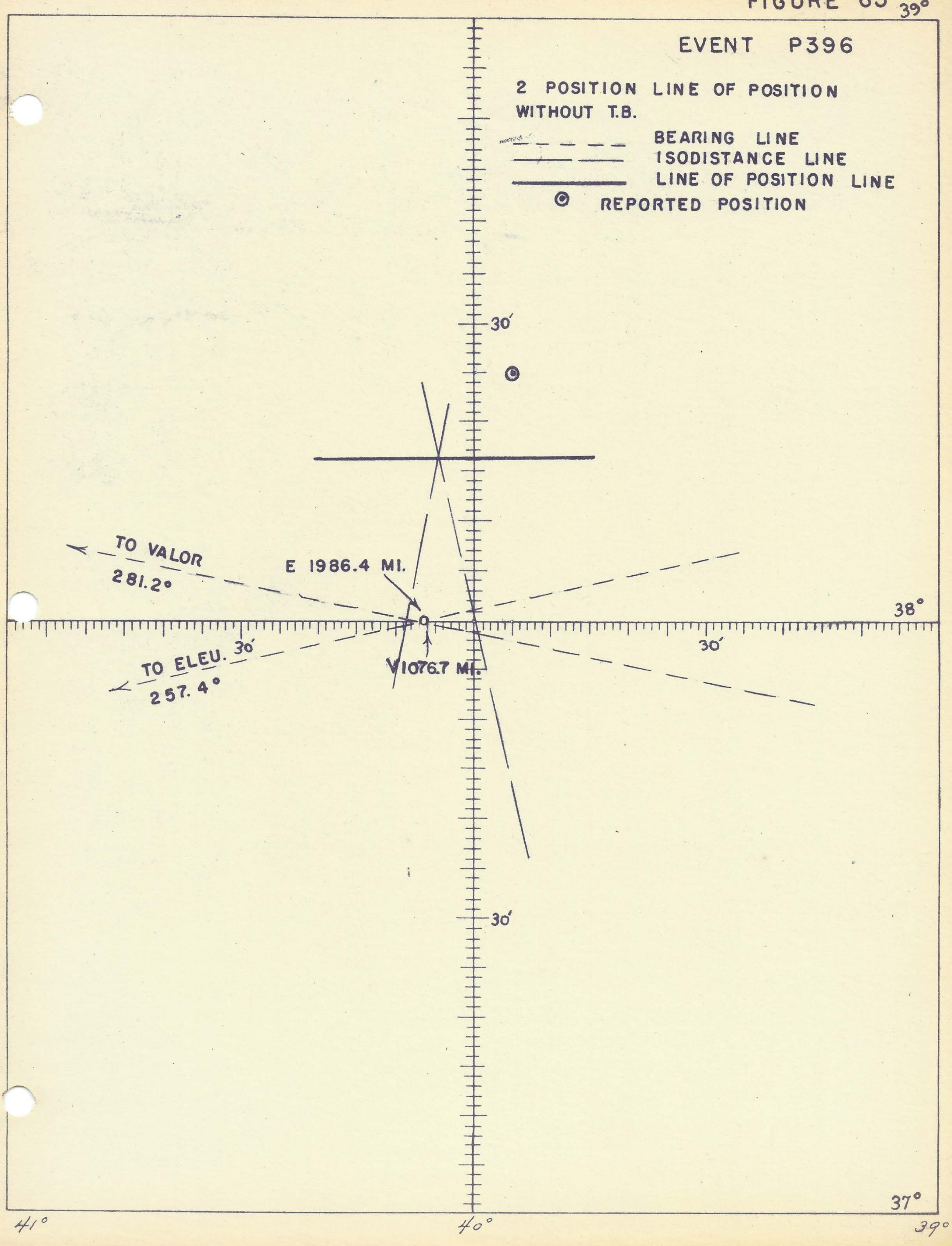

38°
EVENT P392
2 POSITION LINE OF POSITION
WITHOUT TB
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
TO VALOR
285.1°
E1649.0 MI.
30'
30'
37°
TO ELEU.
252.3°
V765.3 MI.
30'
36°
48°
47°
46°

FIGURE 63



39°
EVENT P396
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
TO VALOR
281.2°
E 1986.4 MI.
38°
TO ELEU.
30′
257.4°
V1076.7 MI.
30′
30′
37°
41°
40°
39°

FIGURE 64

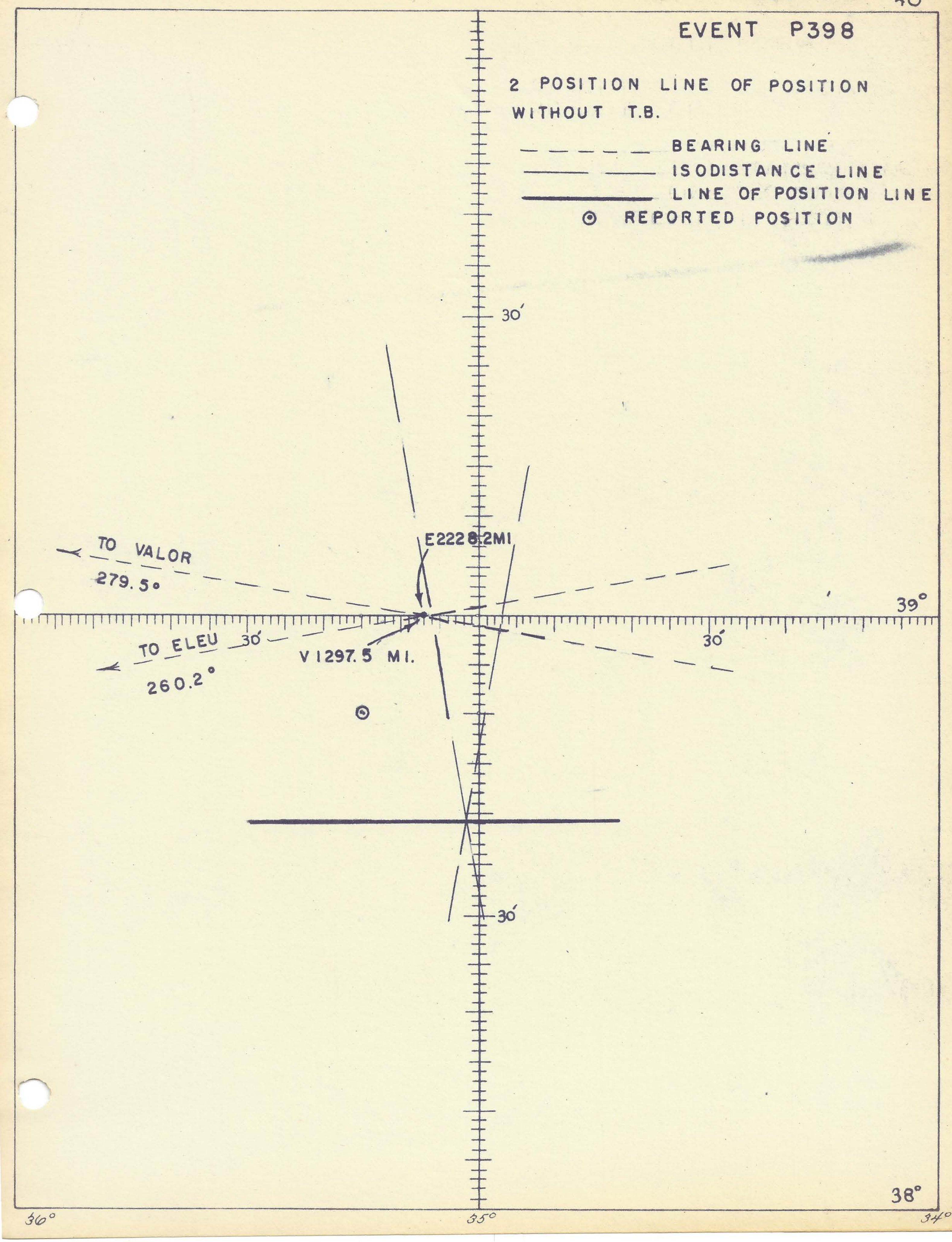
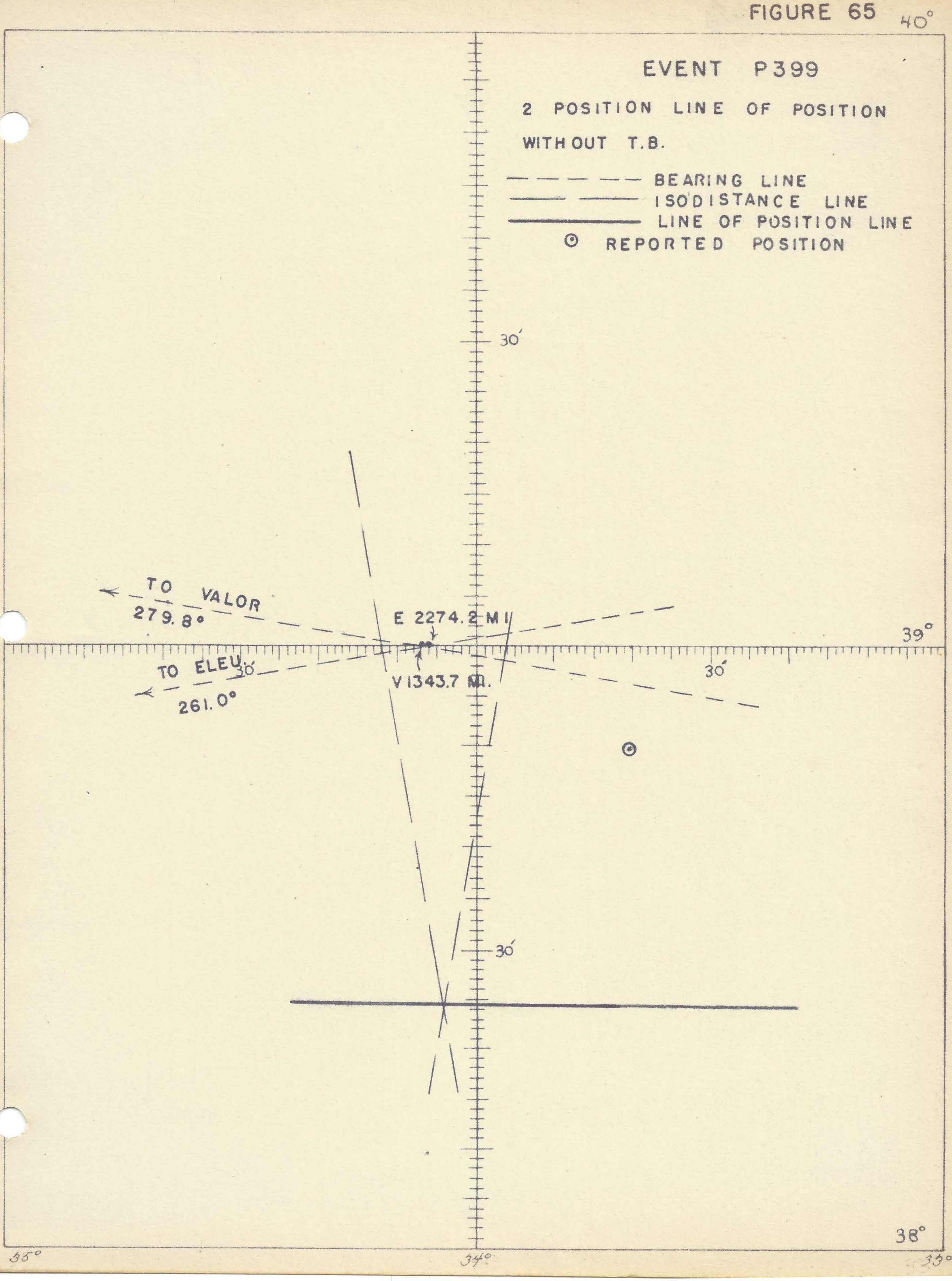

40°
EVENT P398
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
E2228.2MI
TO VALOR
279.5°
39°
TO ELEU
30′
260.2°
V 1297.5 MI.
30′
30′
38°
36°
35°
34°

FIGURE 65



40°
EVENT P399
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
TO VALOR
279.8°
E 2274.2 MI
39°
TO ELEV.
261.0°
30'
V 1343.7 MI.
30'
30'
38°
35°
34°
33°

FIGURE 66

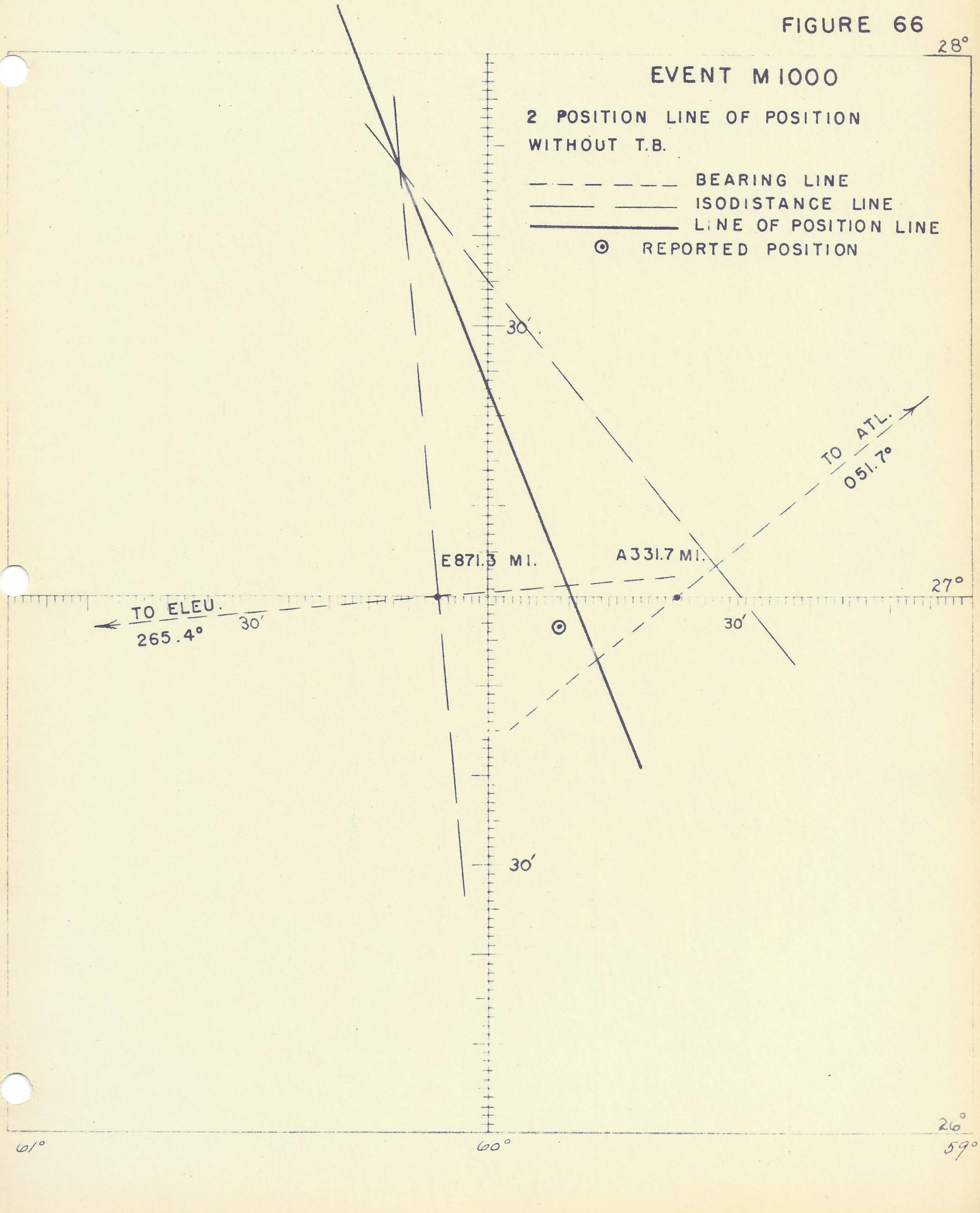

EVENT M 1000
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
28°
27°
26°
30′
30′
30′
30′
E871.3 MI.
A331.7 MI.
TO ELEU.
265.4°
TO ATL.
051.7°
61°
60°
59°

FIGURE 67

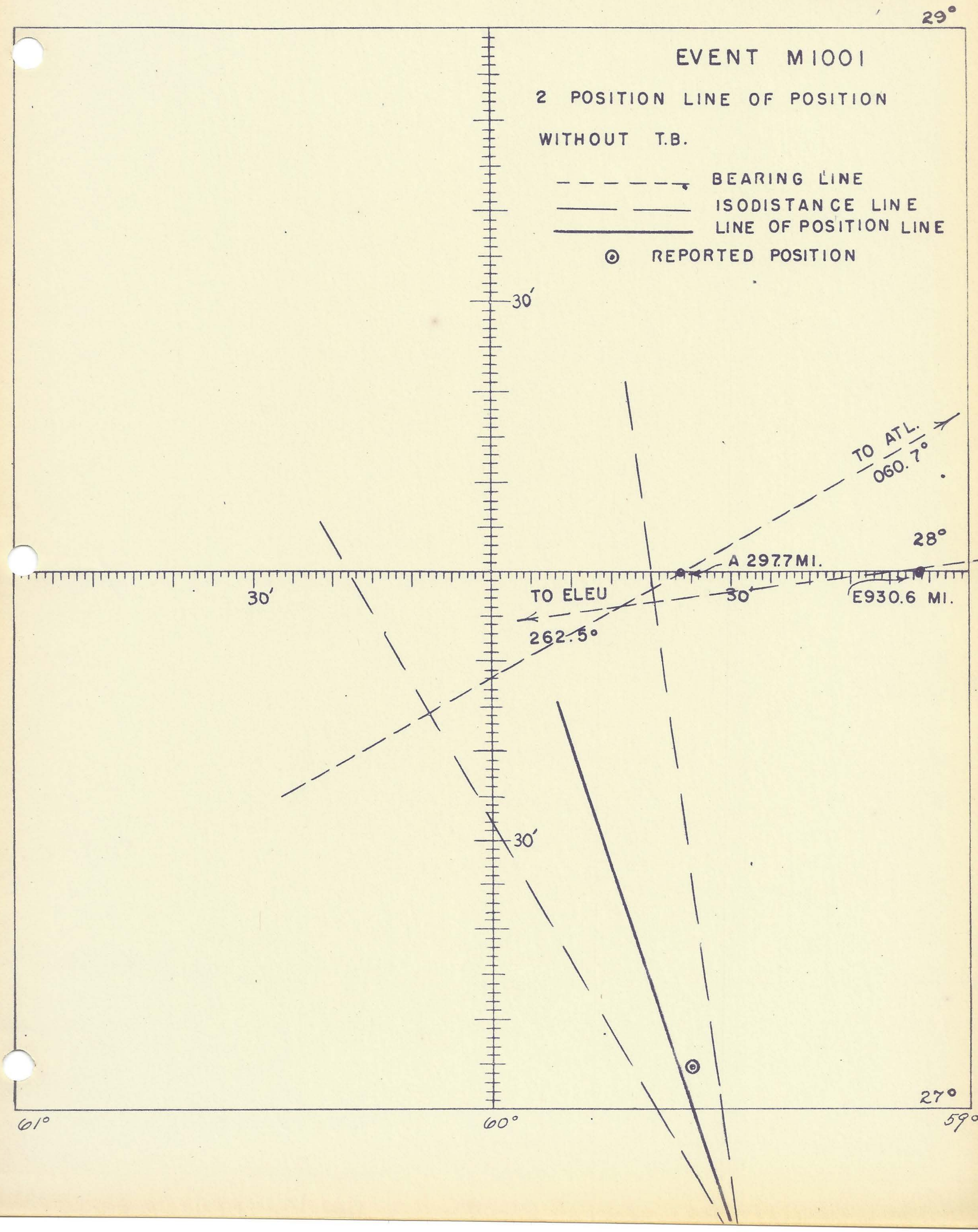
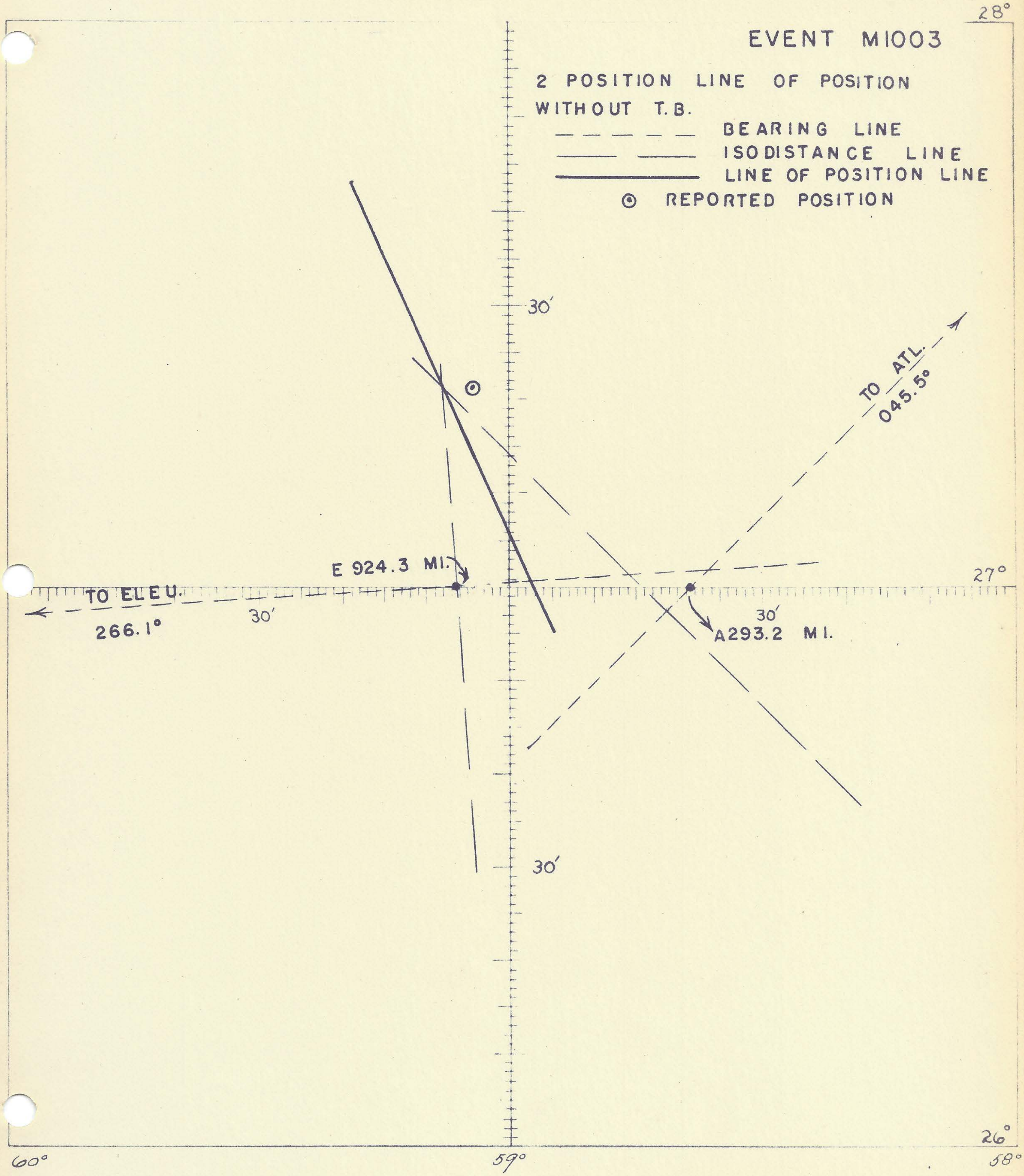

29°
EVENT M1001
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
TO ATL.
060.7°
28°
A 2977MI.
30'
TO ELEU
30'
E930.6 MI.
262.5°
30'
27°
61°
60°
59°

FIGURE 68



28°
EVENT M1003
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
TO ATL.
045.5°
E 924.3 MI.
27°
TO ELEU.
266.1°
30′
30′
A293.2 MI.
30′
26°
60°
59°
58°

FIGURE 69

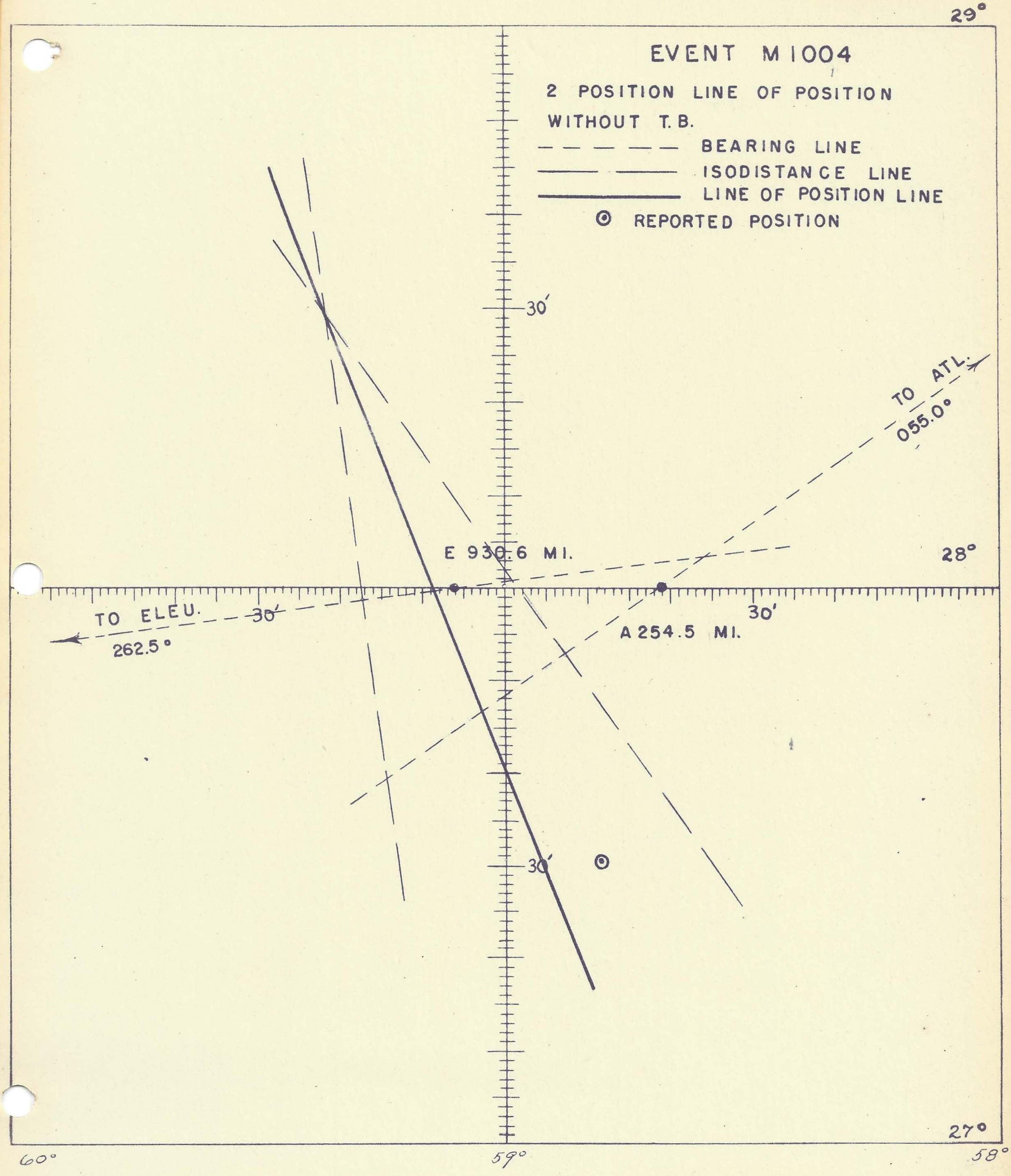
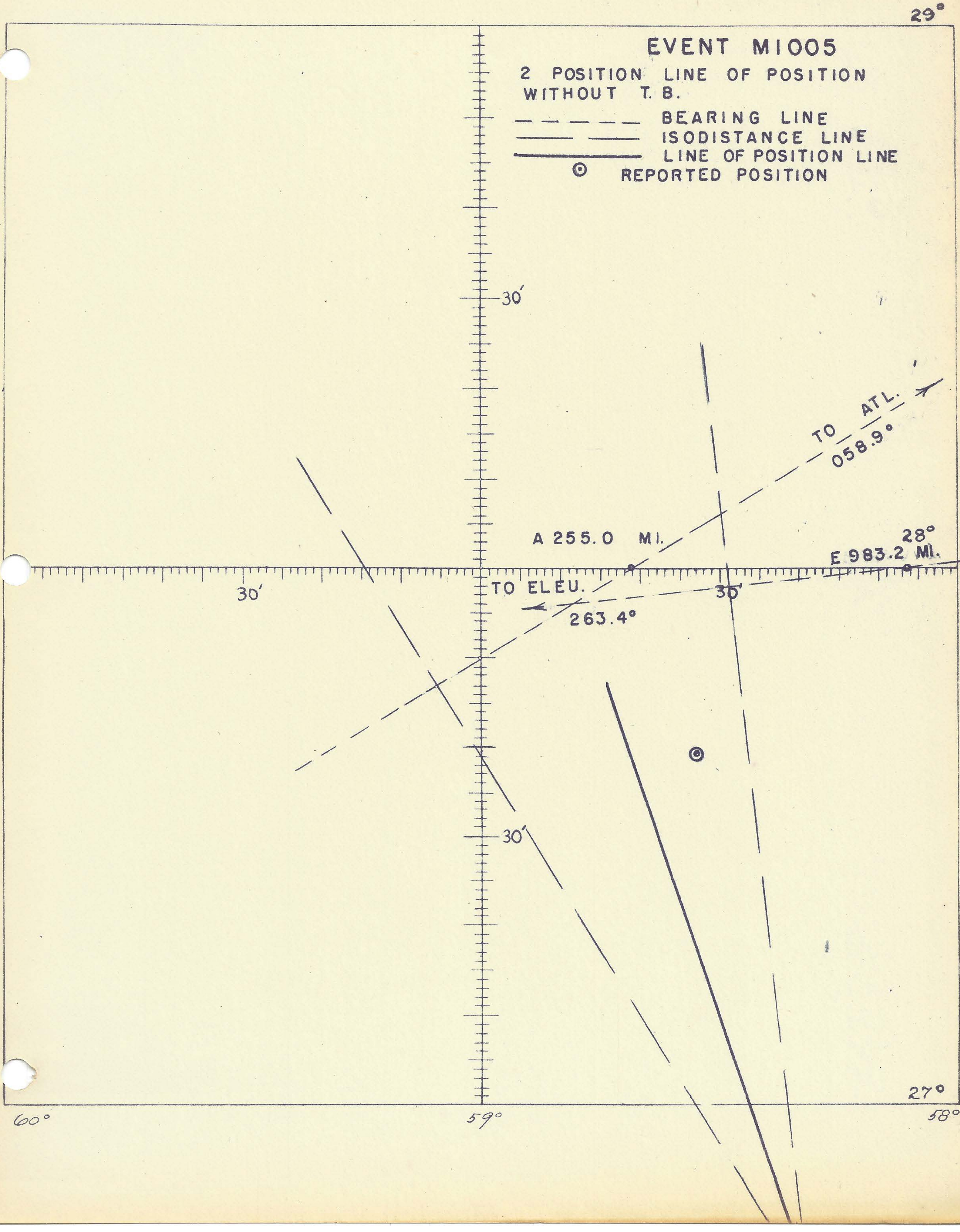

29°
EVENT M 1004
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
TO ATL.
055.0°
E 930.6 MI.
28°
TO ELEU.
262.5°
30'
30'
A 254.5 MI.
30'
27°
60°
59°
58°

FIGURE 70


29°
EVENT M1005
2 POSITION LINE OF POSITION
WITHOUT T. B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
TO ATL.
058.9°
A 255.0 MI.
28°
E 983.2 MI.
30′
TO ELEU.
30′
263.4°
30′
27°
60°
59°
58°

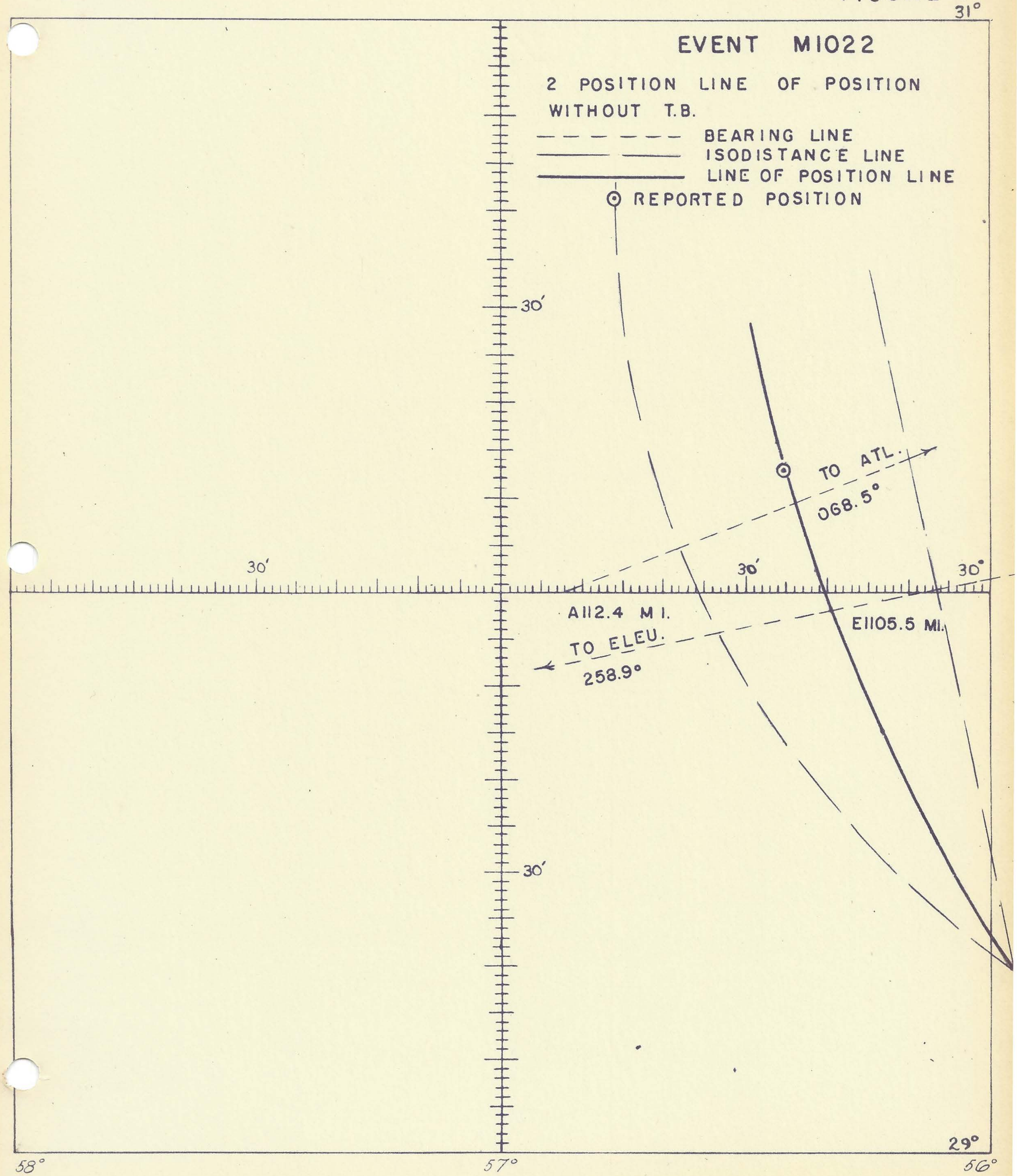

31°
EVENT M1022
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
TO ATL.
068.5°
30′
30′
30°
A112.4 MI.
E1105.5 MI.
TO ELEU.
258.9°
30′
29°
58°
57°
56°

FIGURE 72

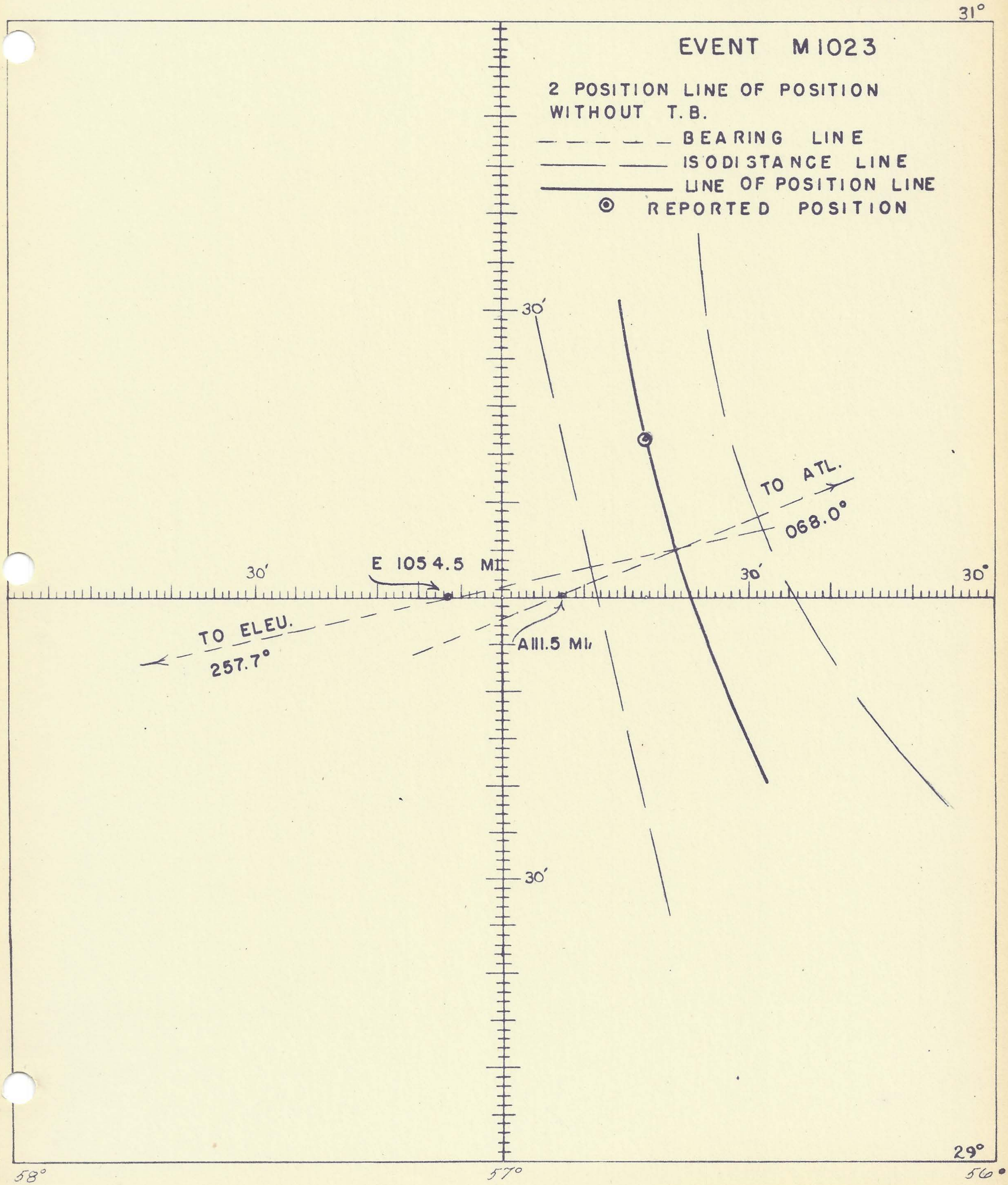
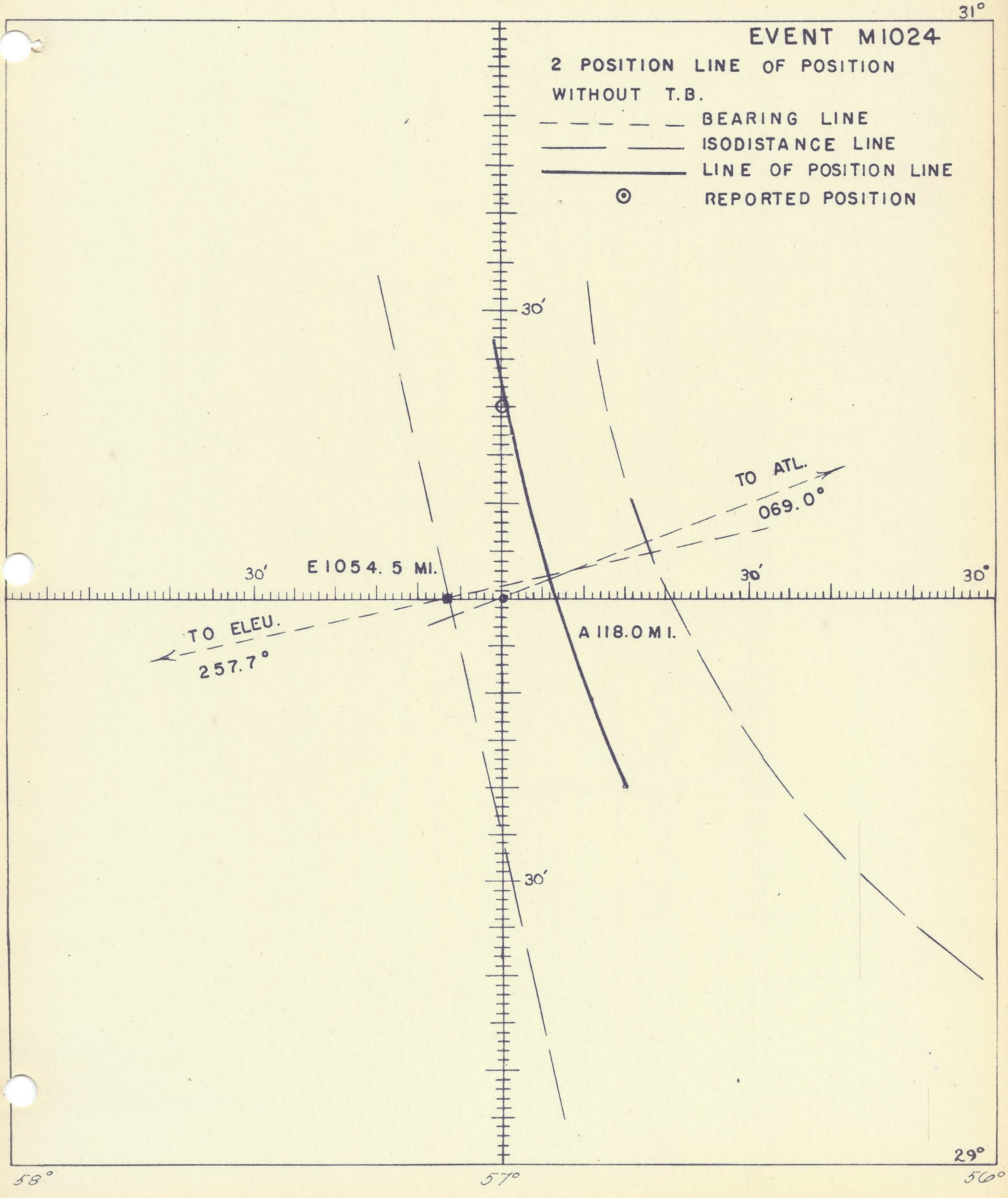

31°
EVENT M1023
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
30′
E 1054.5 MI
30′
30°
TO ATL.
068.0°
TO ELEU.
257.7°
A111.5 MI.
30′
29°
58°
57°
56°

FIGURE 73



31°
EVENT M1024
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
TO ATL.
069.0°
30′
E 1054. 5 MI.
30′
30°
TO ELEU.
257.7°
A 118.0 MI.
30′
29°
58°
57°
56°

FIGURE 74

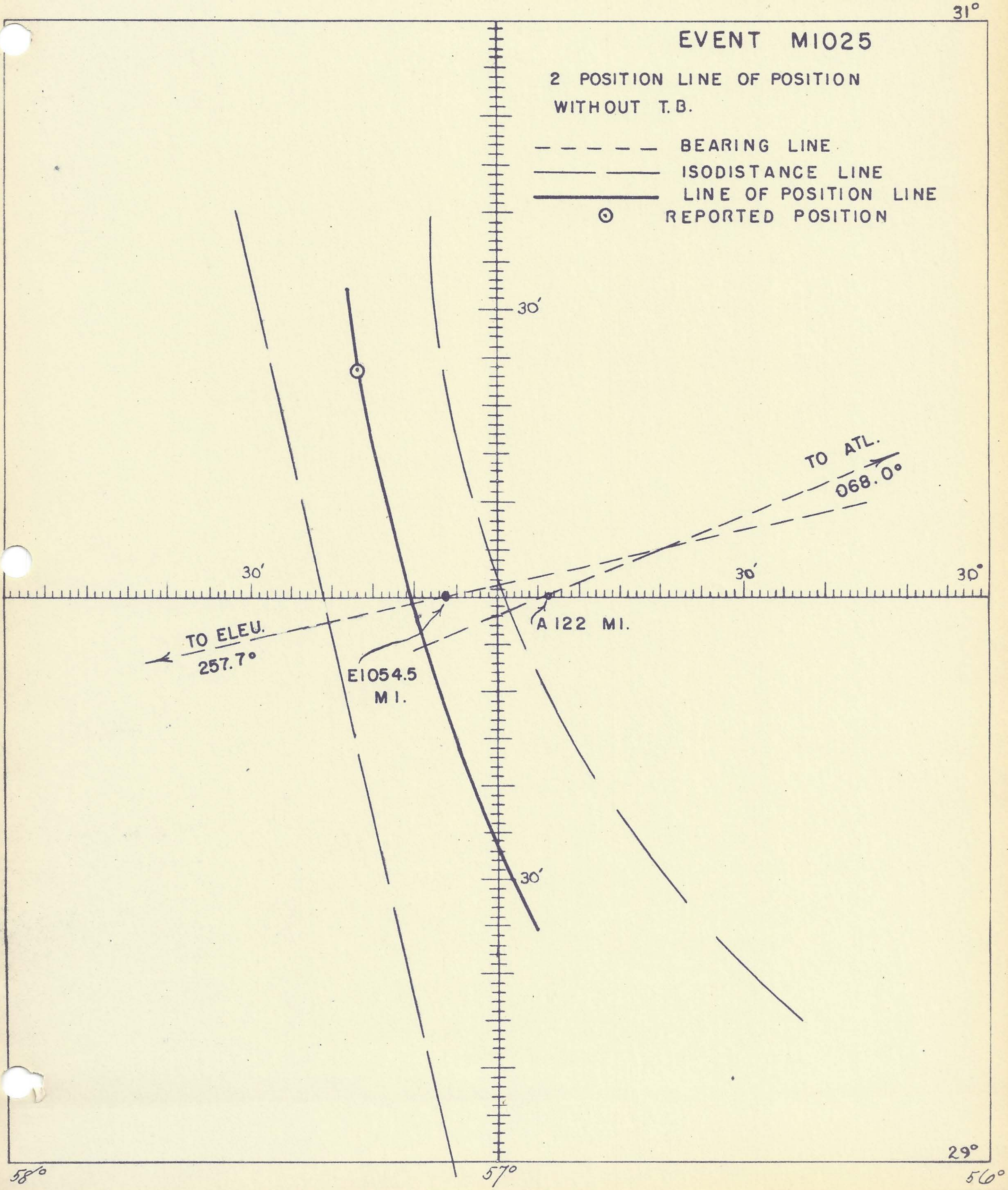
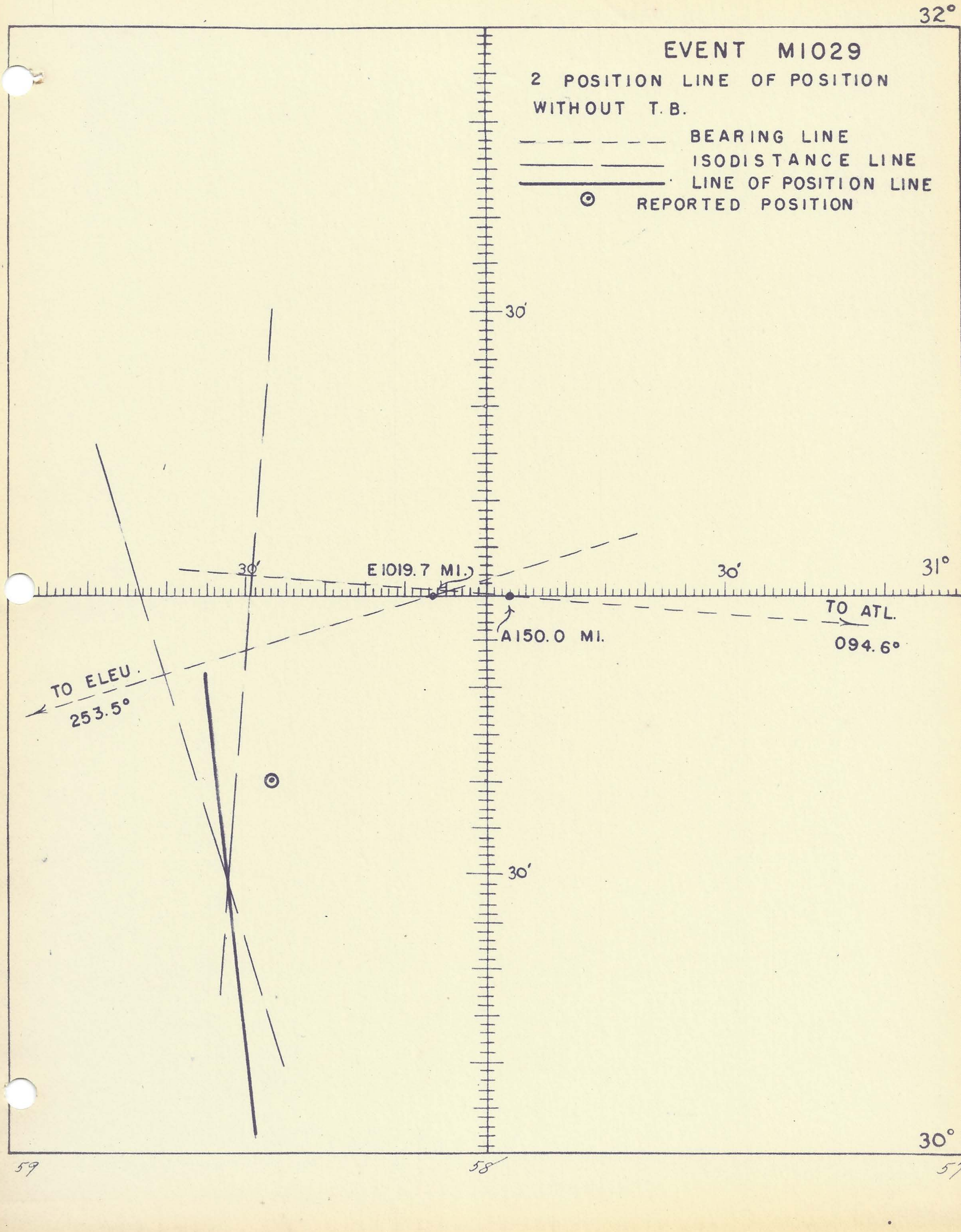

31°
EVENT M1025
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30′
TO ATL.
068.0°
30′
30′
30°
TO ELEU.
257.7°
A 122 MI.
E1054.5
MI.
30′
29°
58°
57°
56°

FIGURE 75



32°
EVENT M1029
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
30'
E1019.7 MI.
30'
31°
TO ATL.
094.6°
A150.0 MI.
TO ELEU.
253.5°
30'
30°
59
58
57

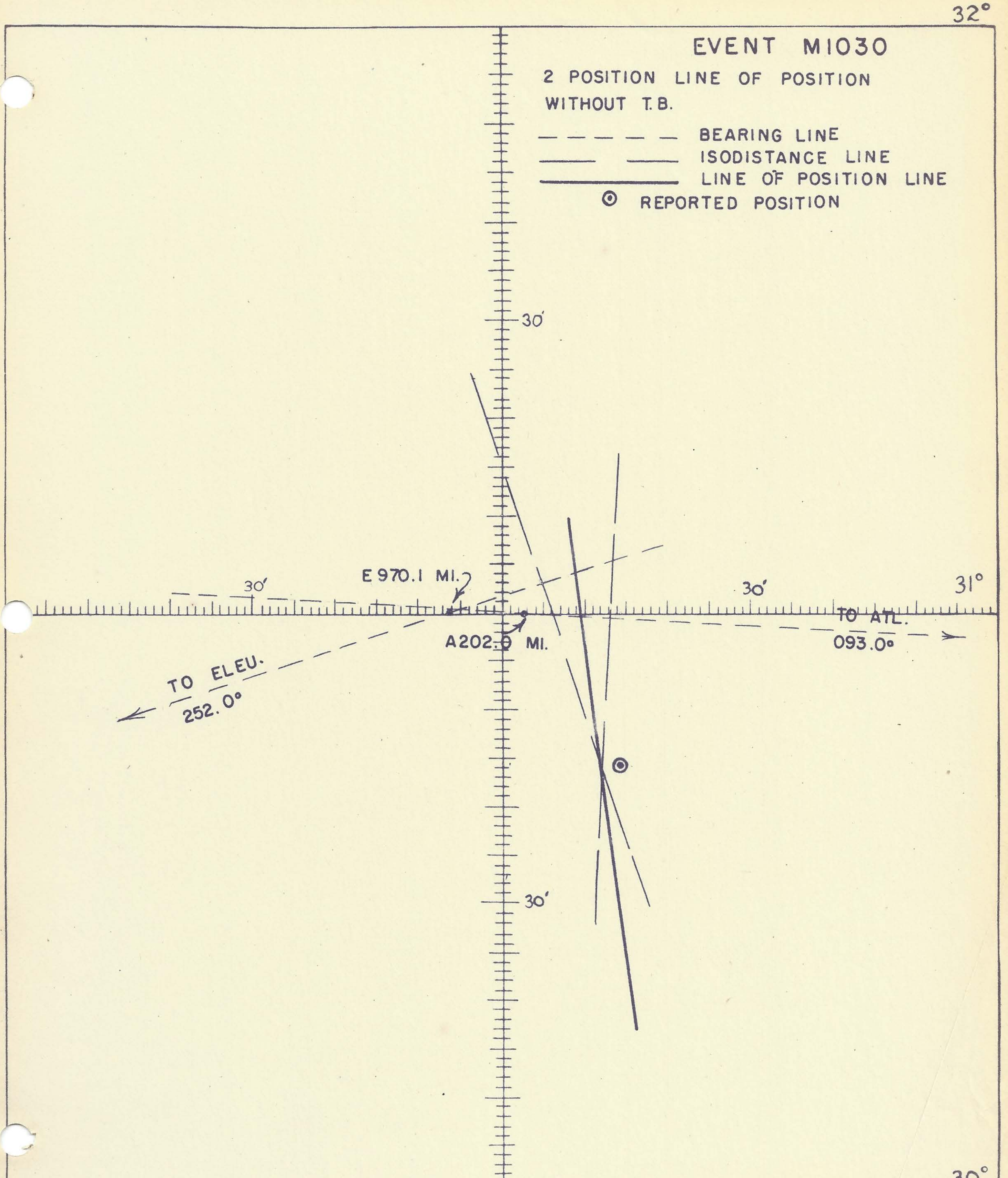

32°
EVENT M1030
2 POSITION LINE OF POSITION
WITHOUT T.B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
E 970.1 MI.
30'
30'
31°
TO ATL.
093.0°
A202.0 MI.
TO ELEU.
252.0°
30'
30°
60°
59°
58°

FIGURE 77

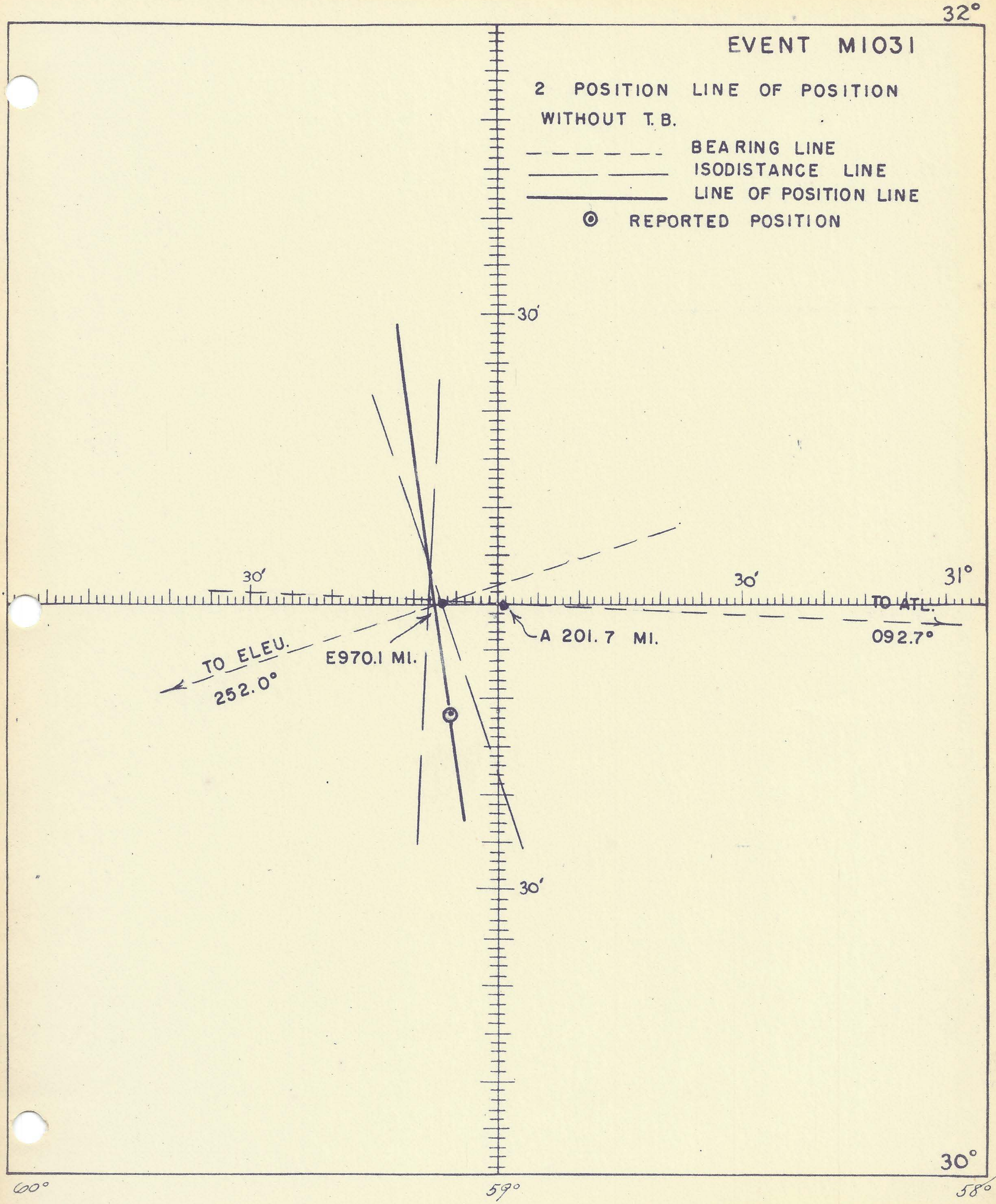

32°
EVENT M1031
2 POSITION LINE OF POSITION
WITHOUT T. B.
BEARING LINE
ISODISTANCE LINE
LINE OF POSITION LINE
REPORTED POSITION
30'
30'
30'
30'
31°
TO ATL.
092.7°
A 201.7 MI.
E970.1 MI.
TO ELEU.
252.0°
30°
60°
59°
58°

FIGURE 78

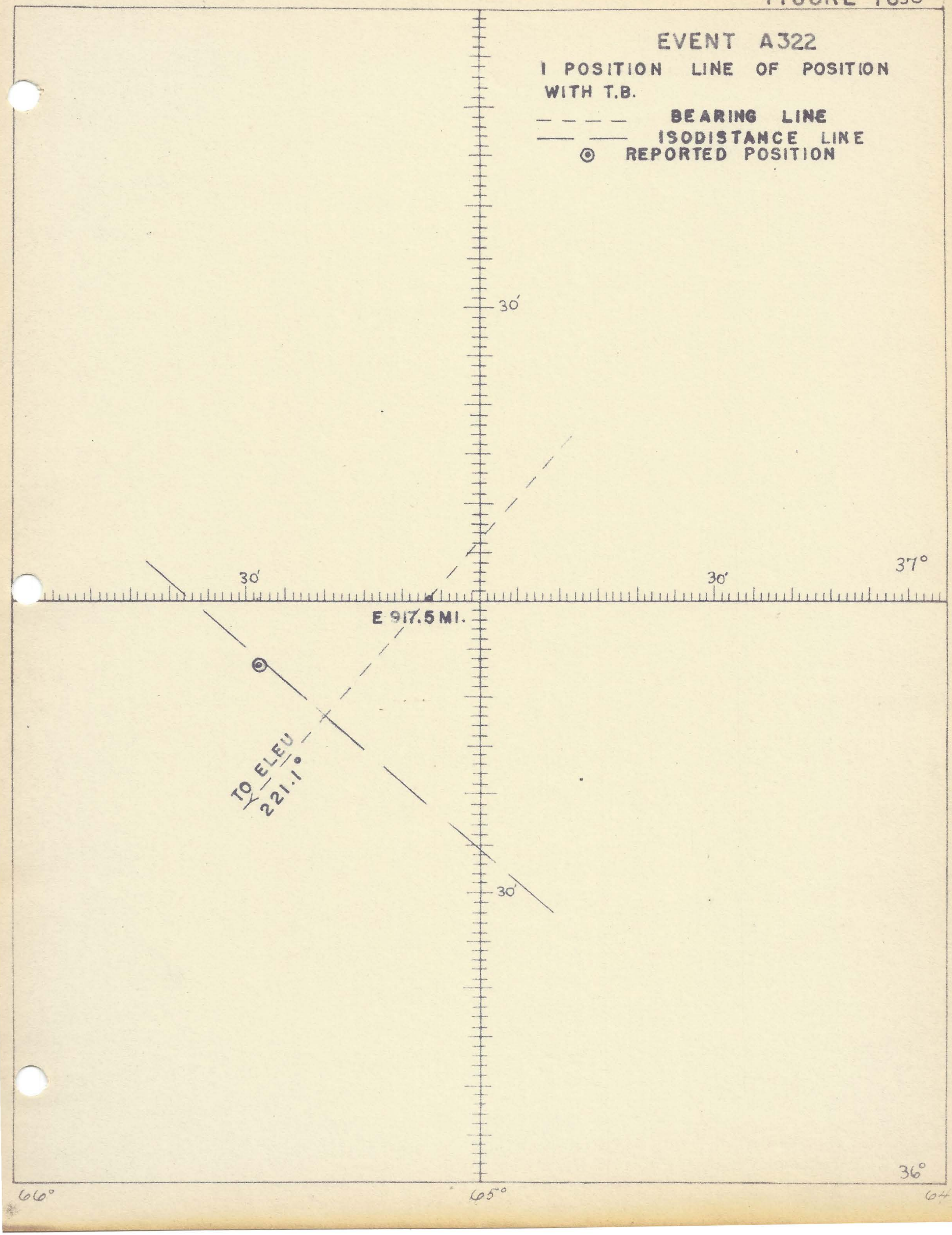

38°
EVENT A322
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
30'
30'
37°
E 917.5 MI.
TO ELEU
221.1°
30'
36°
66°
65°
64°

FIGURE 79

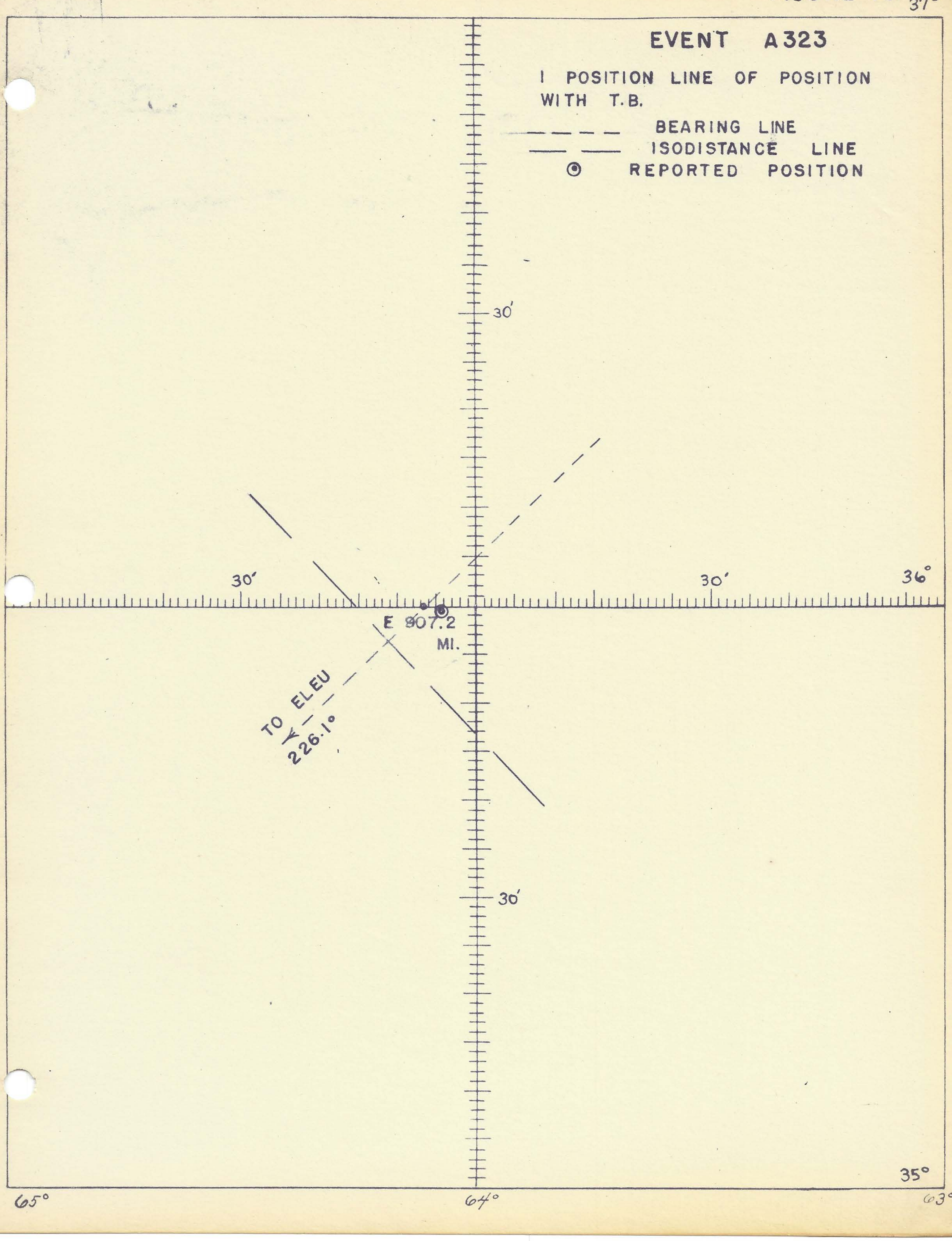

37°
EVENT A323
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
30'
30'
36°
E 907.2
MI.
TO ELEU
226.1°
30'
35°
65°
64°
63°

FIGURE 80

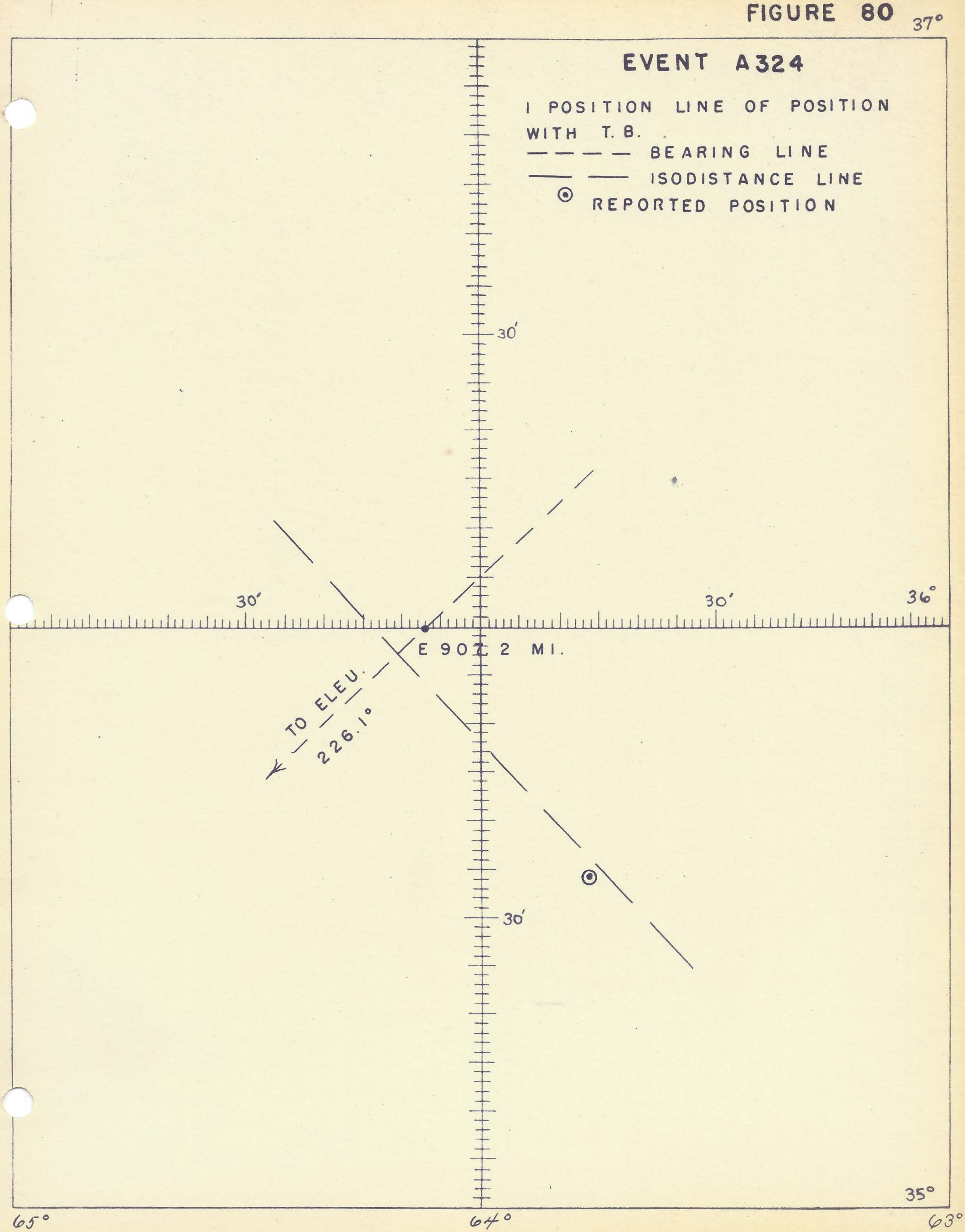

37°
EVENT A324
I POSITION LINE OF POSITION WITH T. B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
30′
30′
36°
E 907.2 MI.
TO ELEU.
226.1°
30′
35°
65°
64°
63°

FIGURE 81

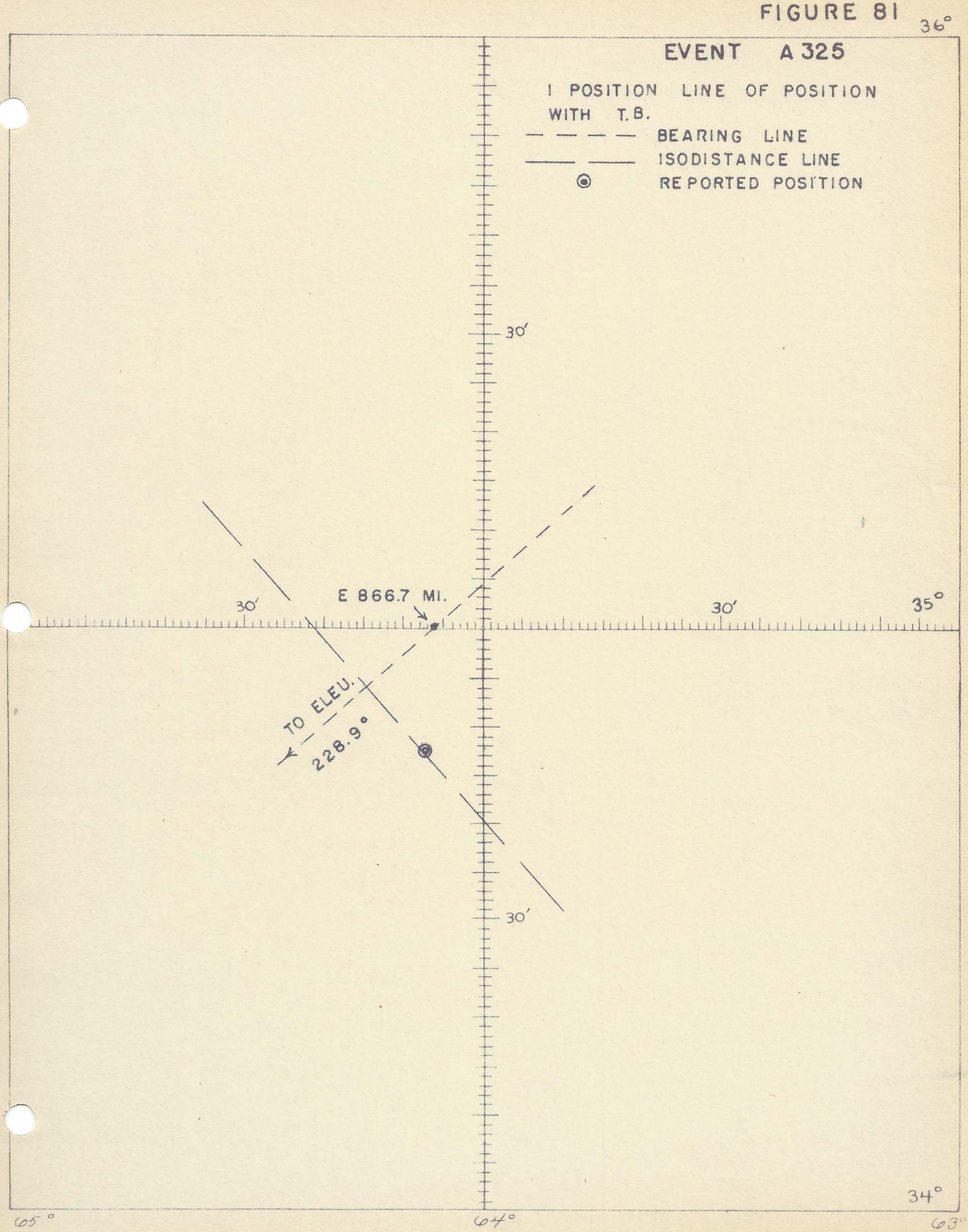
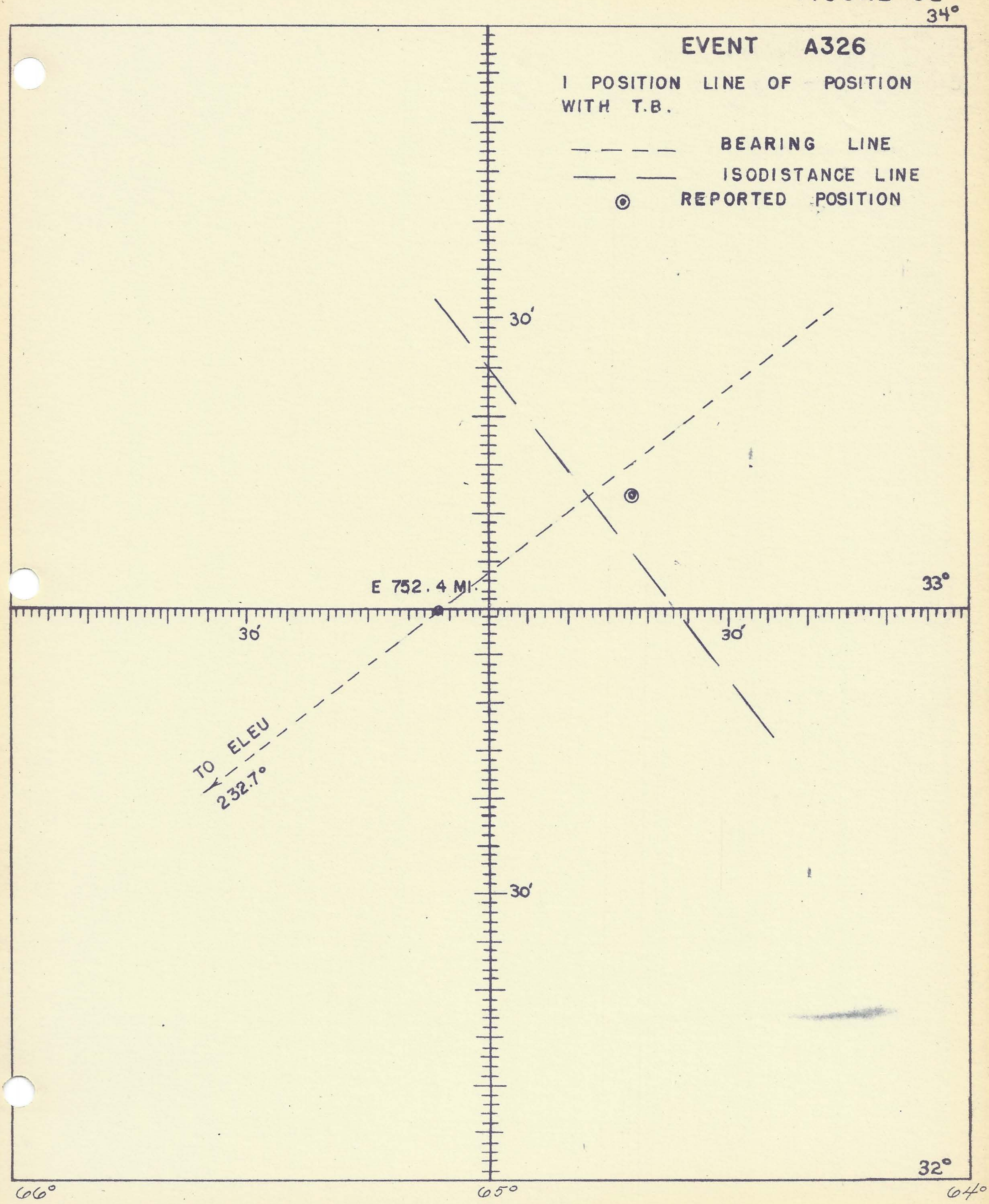
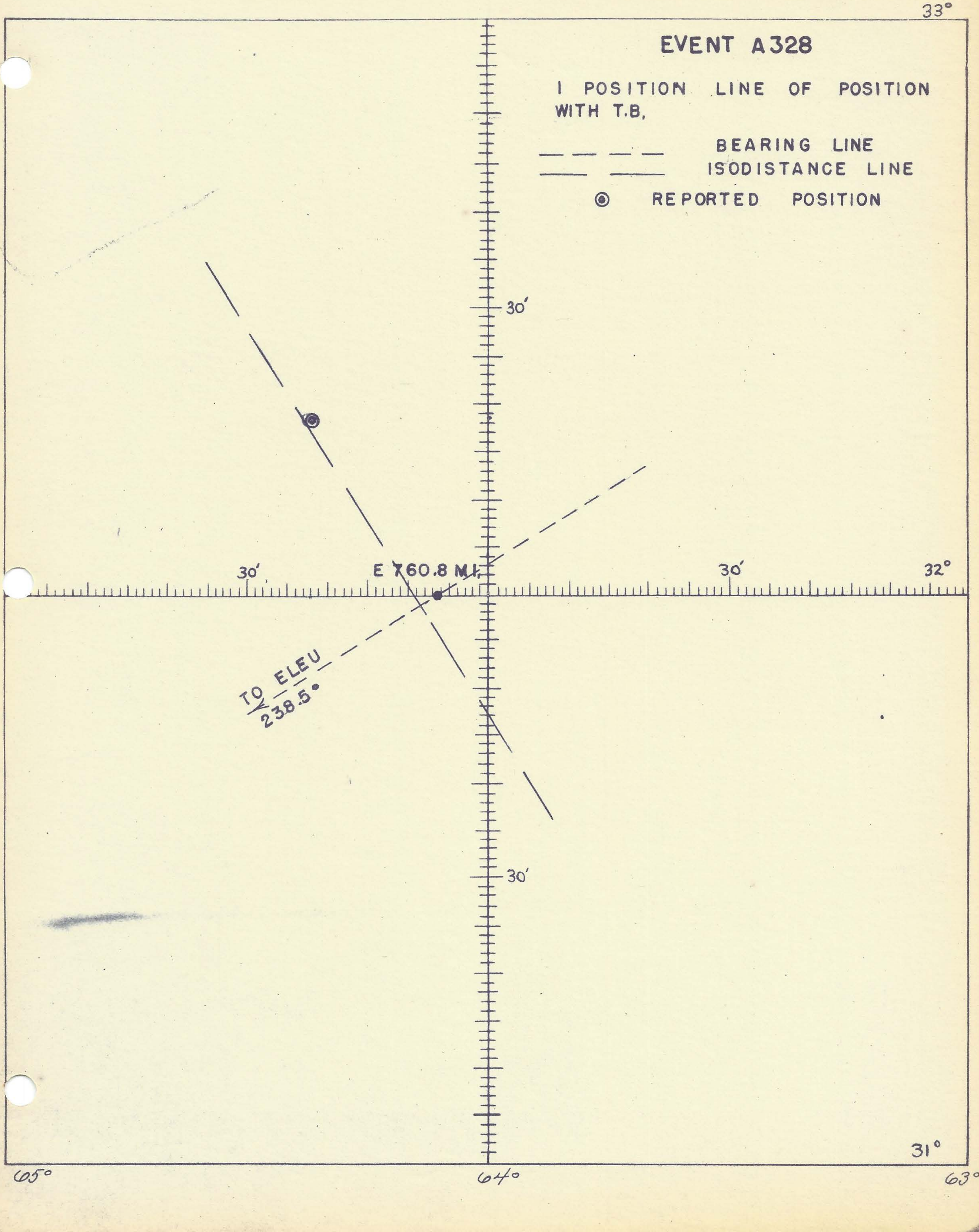

36°
EVENT A 325
I POSITION LINE OF POSITION WITH T. B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
30′
E 866.7 MI.
30′
35°
TO ELEU.
228.9°
30′
34°
65°
64°
63°

FIGURE 82



34°
EVENT A326
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
E 752.4 MI.
33°
30′
30′
TO ELEU
232.7°
30′
32°
66°
65°
64°

FIGURE 83



33°
EVENT A328
1 POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
30′
E 760.8 MI.
30′
32°
TO ELEU
238.5°
30′
31°
65°
64°
63°

FIGURE 84

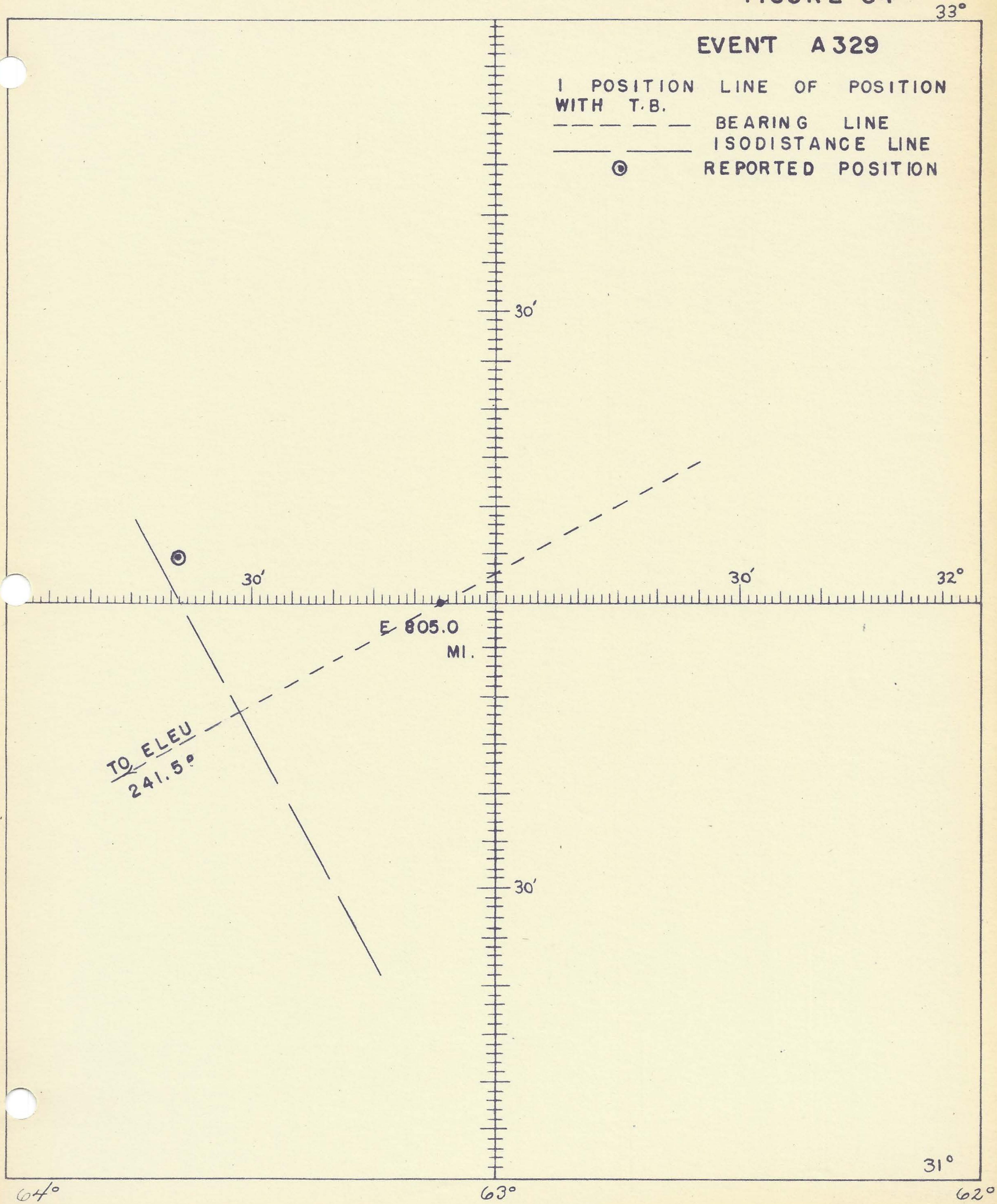
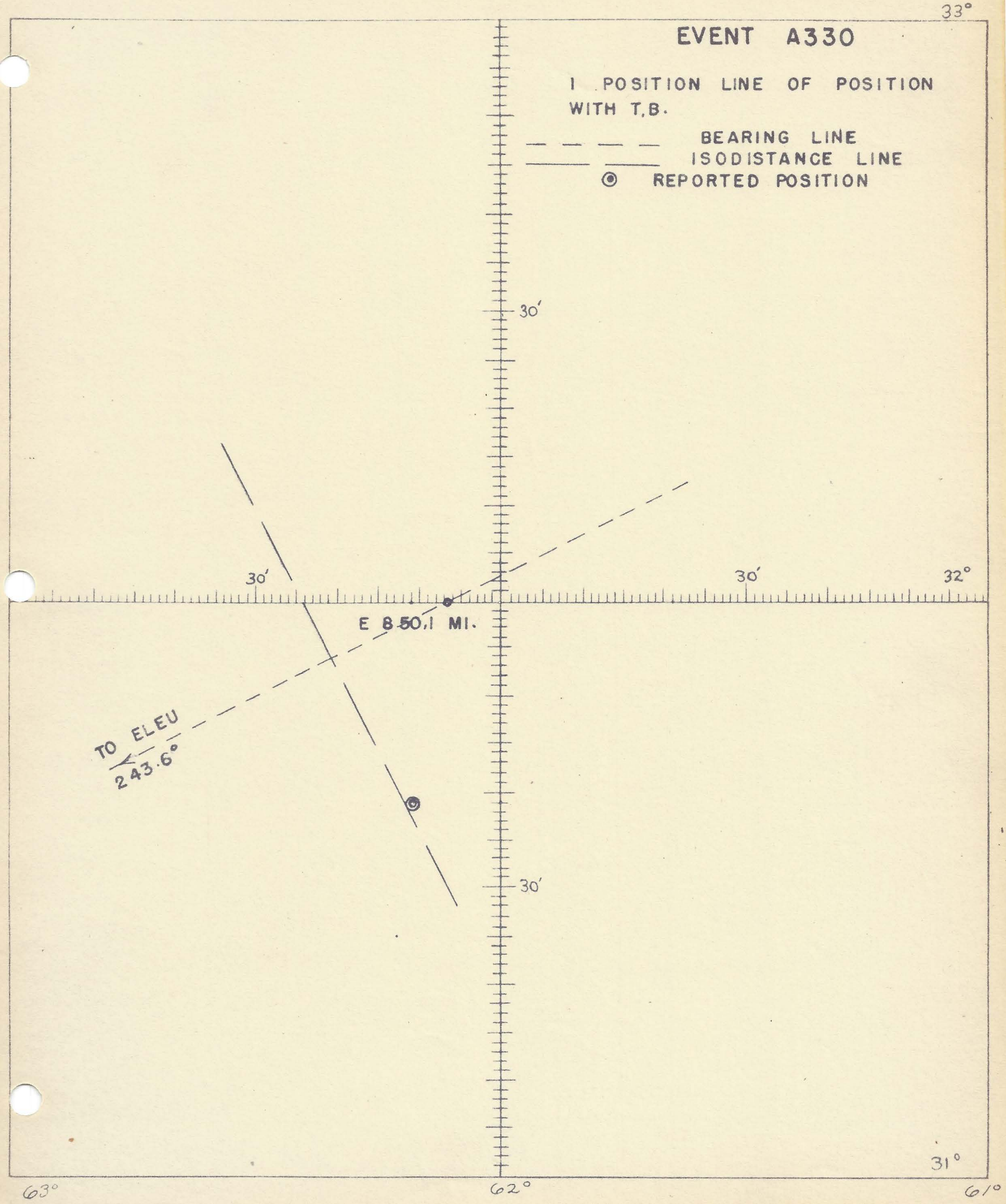

33°
EVENT A329
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
30′
30′
32°
E 805.0 MI.
TO ELEU 241.5°
30′
31°
64°
63°
62°

FIGURE 85



EVENT A330
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
33°
30′
30′
32°
30′
30′
E 8 50.1 MI.
TO ELEU
243.6°
31°
63°
62°
61°

FIGURE 86

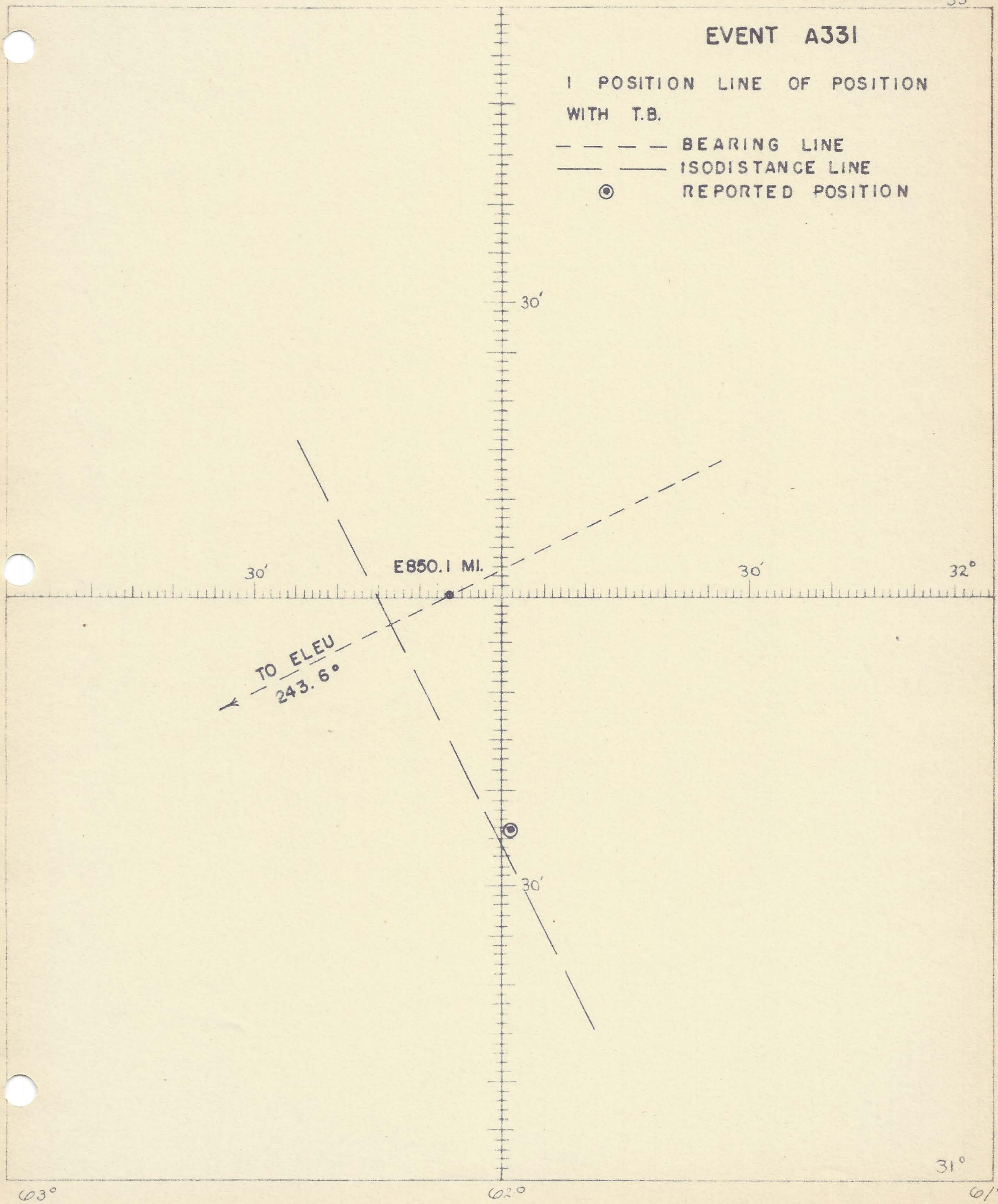

EVENT A331
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
E850.1 MI.
TO ELEU
243.6°
30'
30'
30'
30'
32°
33°
31°
63°
62°
61°

FIGURE 87

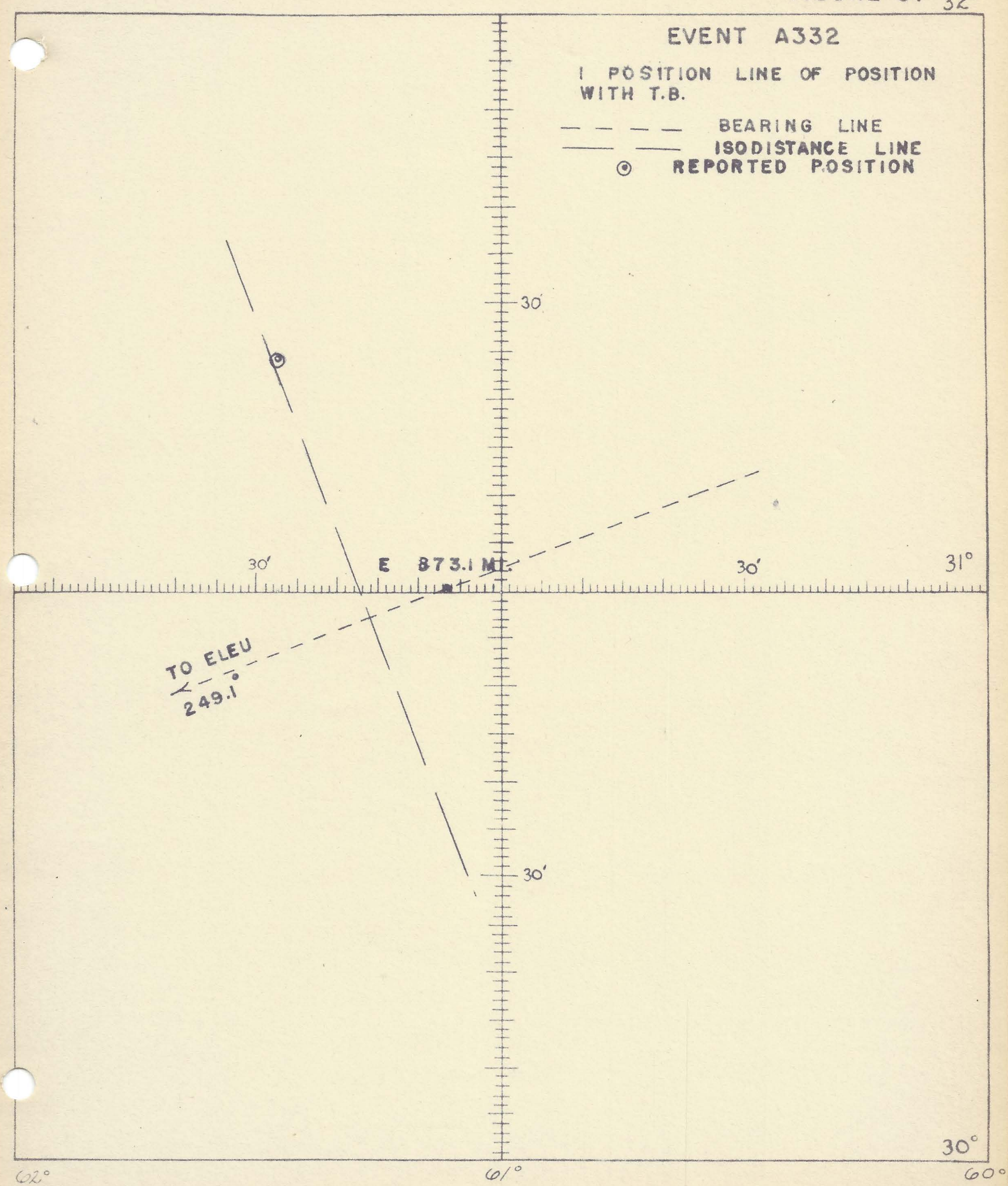
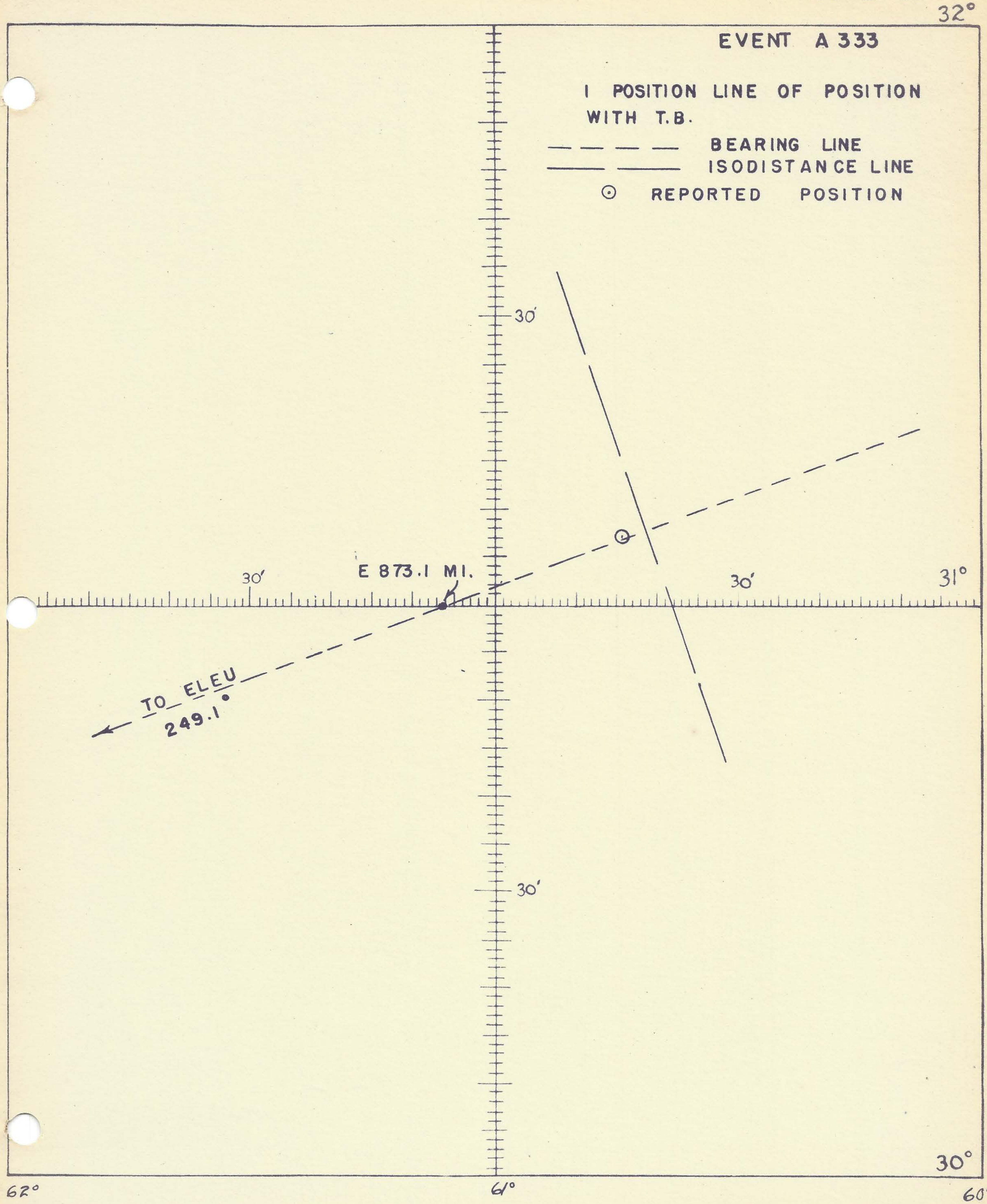

32°
EVENT A332
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
30'
E 873.1 MI.
30'
31°
TO ELEU
249.1
30'
30°
62°
61°
60°

FIGURE 88



32°
EVENT A 333
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
30'
E 873.1 MI.
30'
31°
TO ELEU
249.1°
30'
30°
62°
61°
60°

FIGURE 89

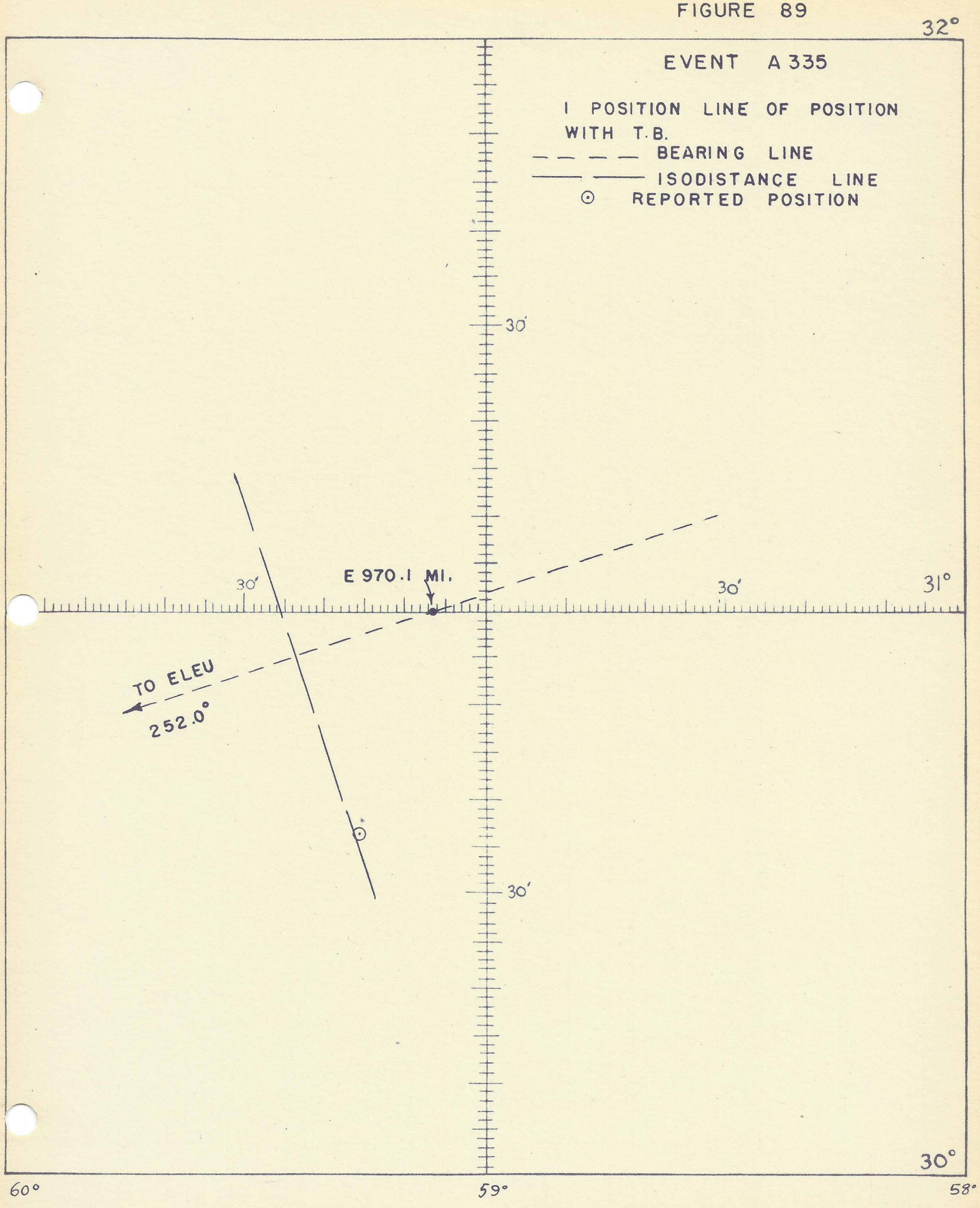

EVENT A 335
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
32°
30'
E 970.1 MI.
30'
30'
31°
TO ELEU
252.0°
30'
30°
60°
59°
58°

FIGURE 90

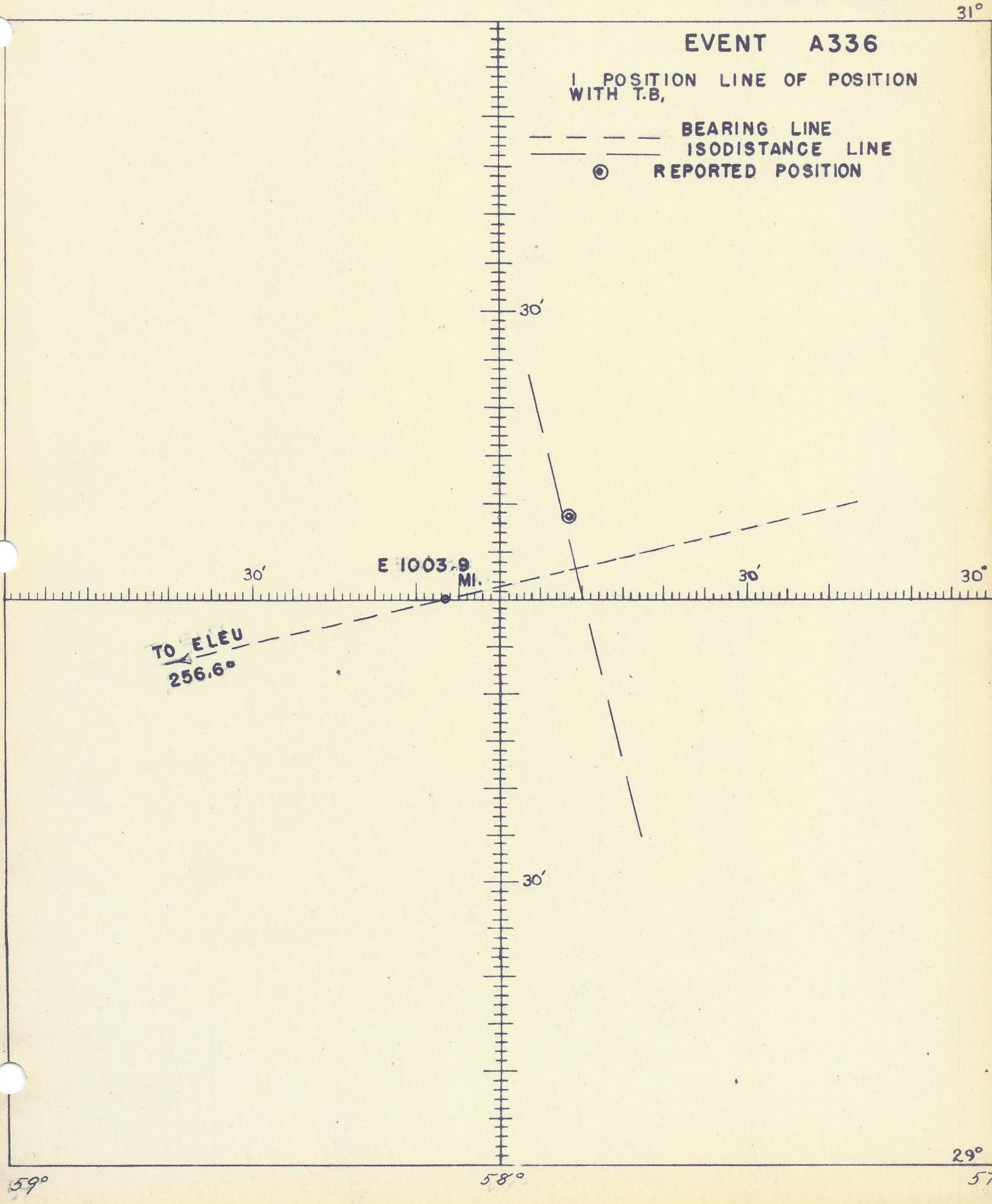
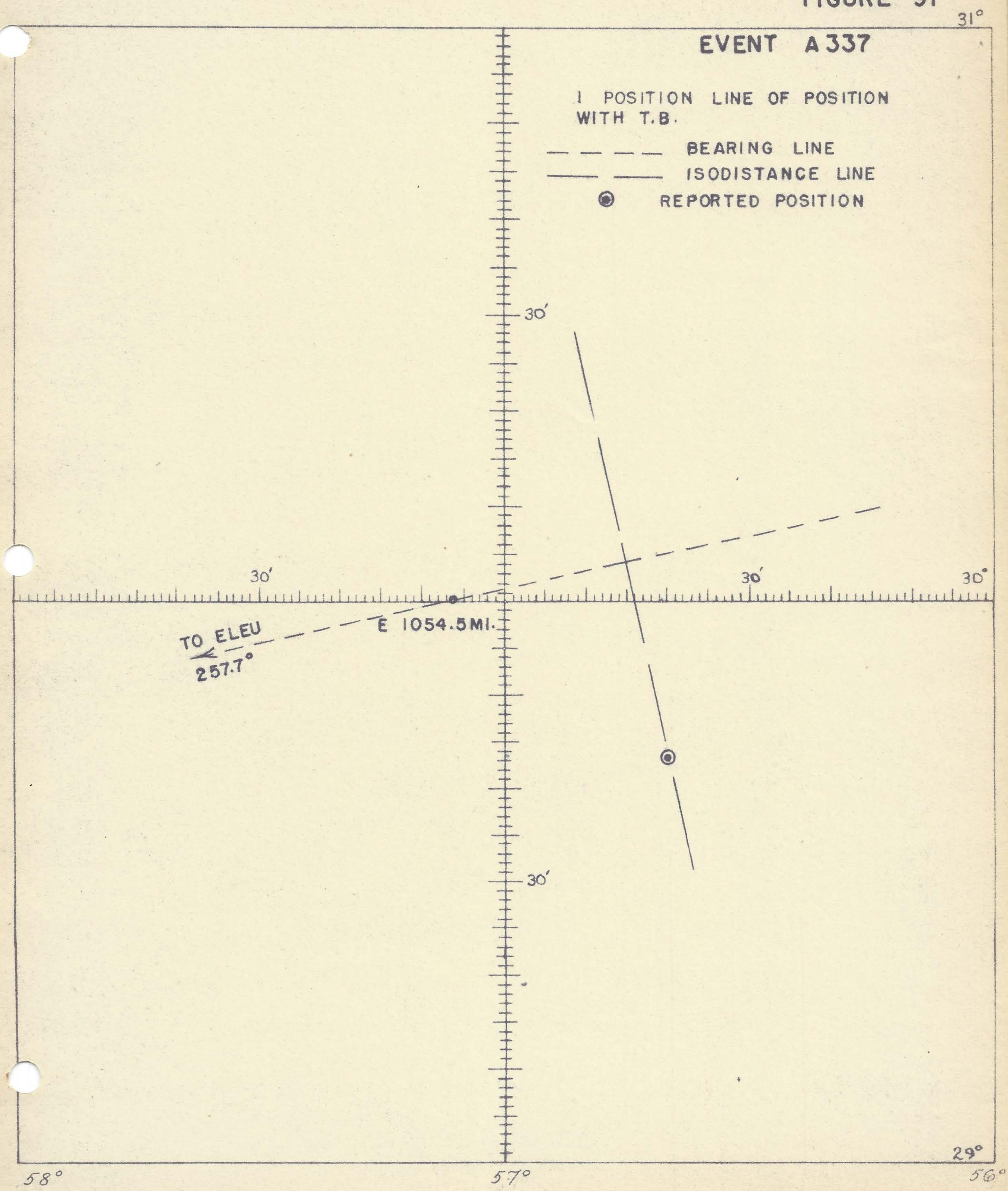

31°
EVENT A336
1 POSITION LINE OF POSITION WITH T.B,
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
30′
E 1003.9 MI.
30′
30°
TO ELEU
256.6°
30′
29°
59°
58°
57°

FIGURE 91



31°
EVENT A337
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
30′
30′
30°
TO ELEU
257.7°
E 1054.5 MI.
30′
29°
58°
57°
56°

FIGURE 92

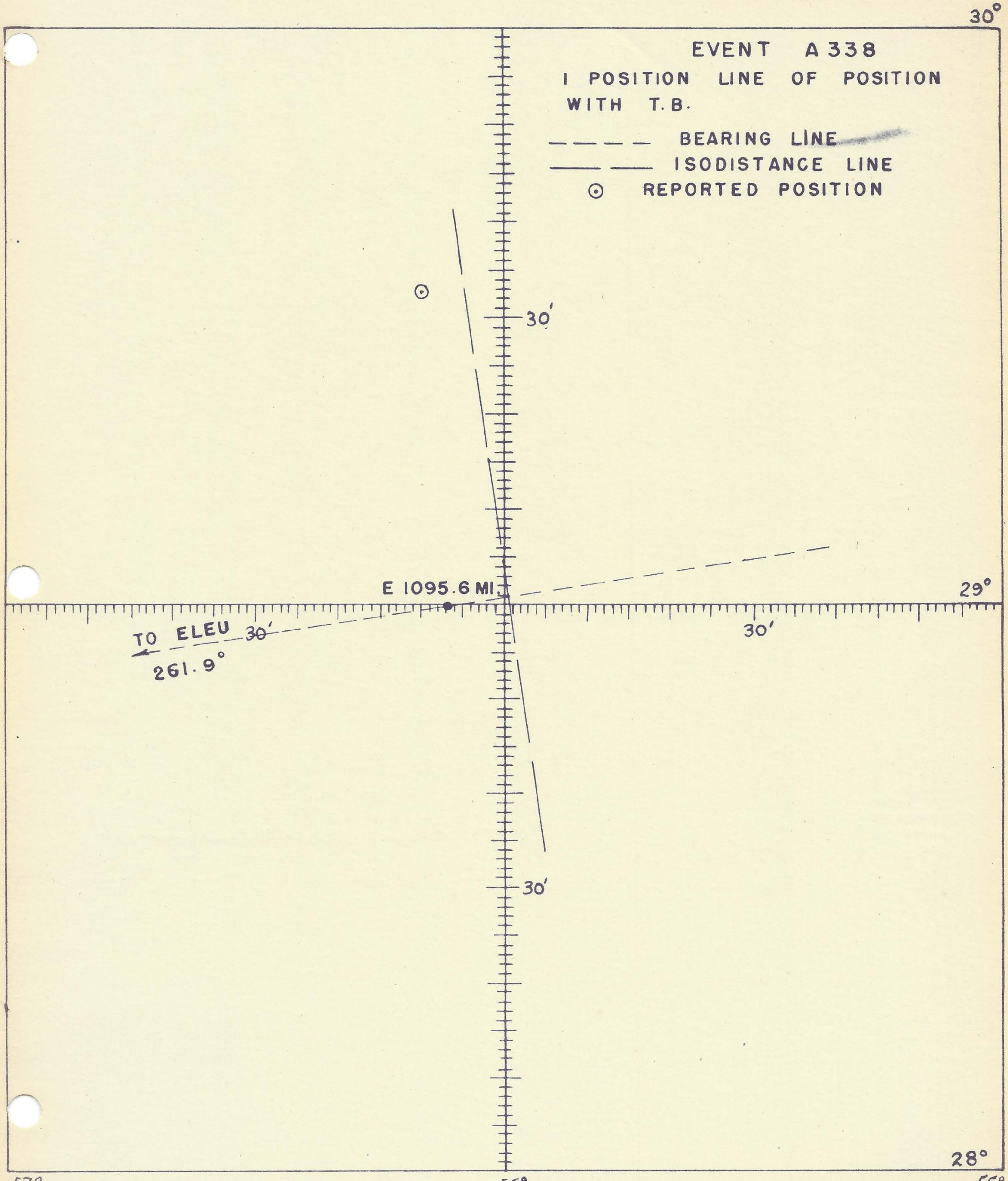
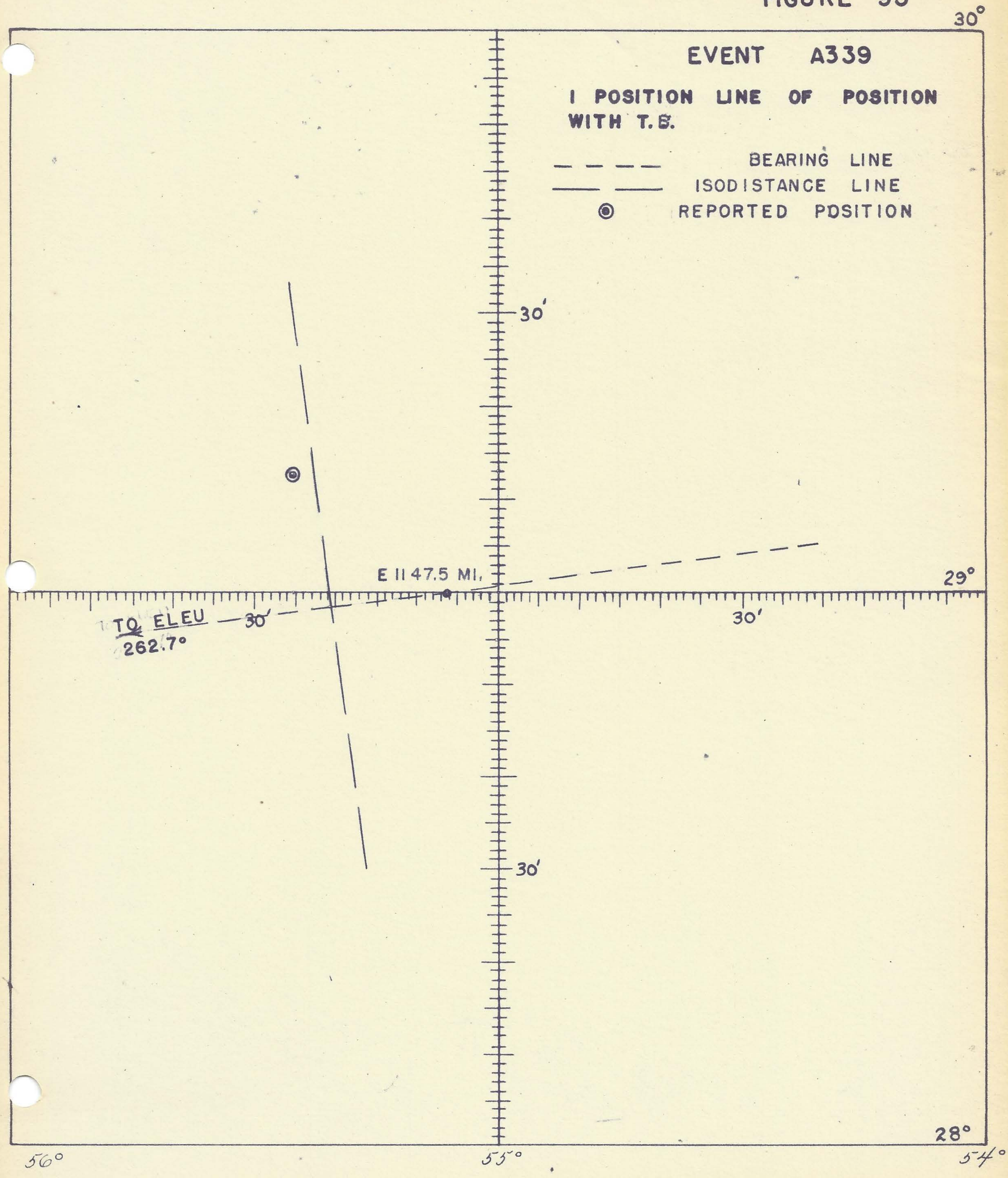

30°
EVENT A 338
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
E 1095.6 MI.
29°
TO ELEU
30'
261.9°
30'
30'
28°
57°
56°
55°

FIGURE 93



30°
EVENT A339
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
E II 47.5 MI.
29°
TO ELEU
262.7°
30′
30′
30′
28°
56°
55°
54°

FIGURE 94

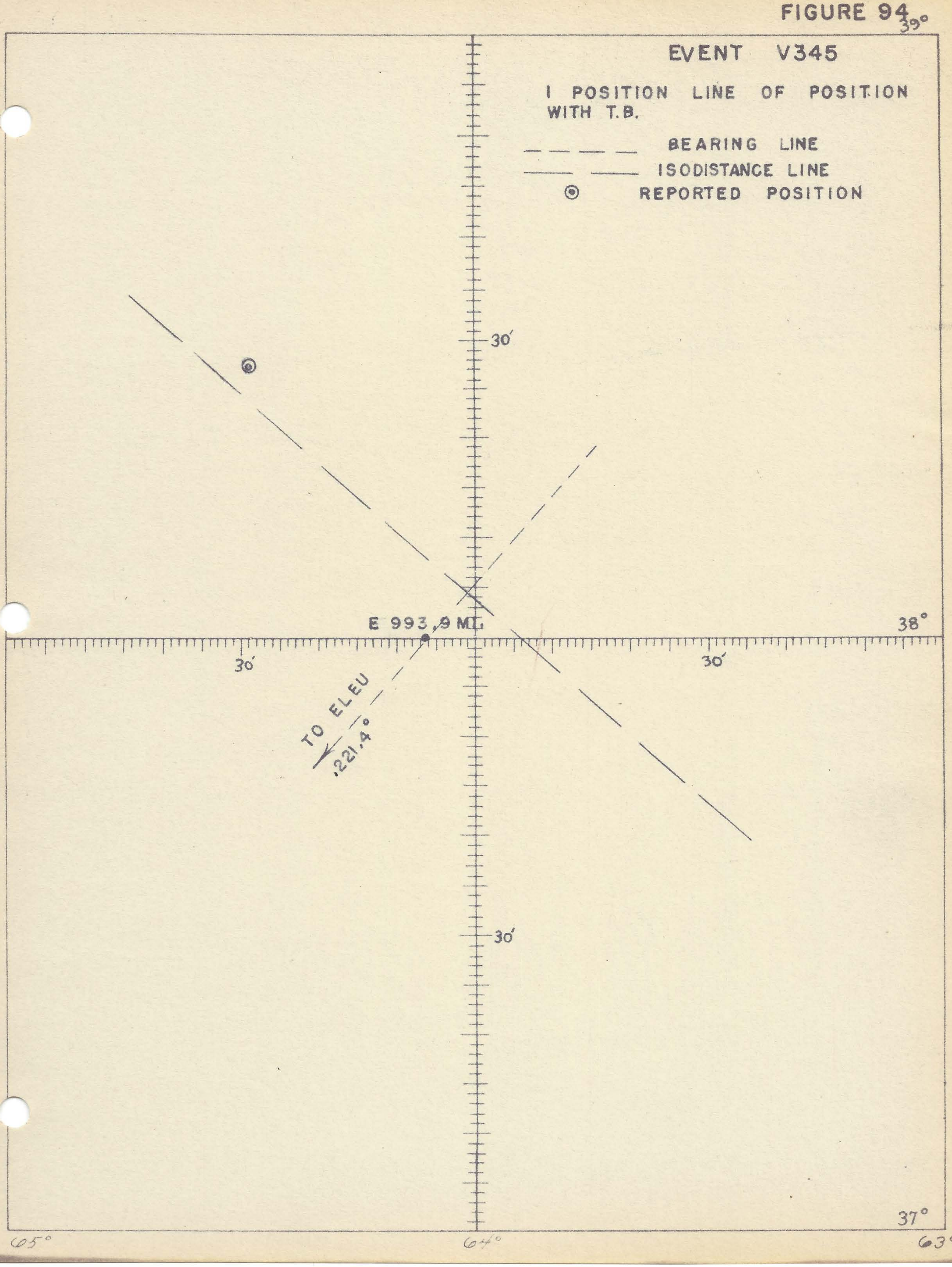
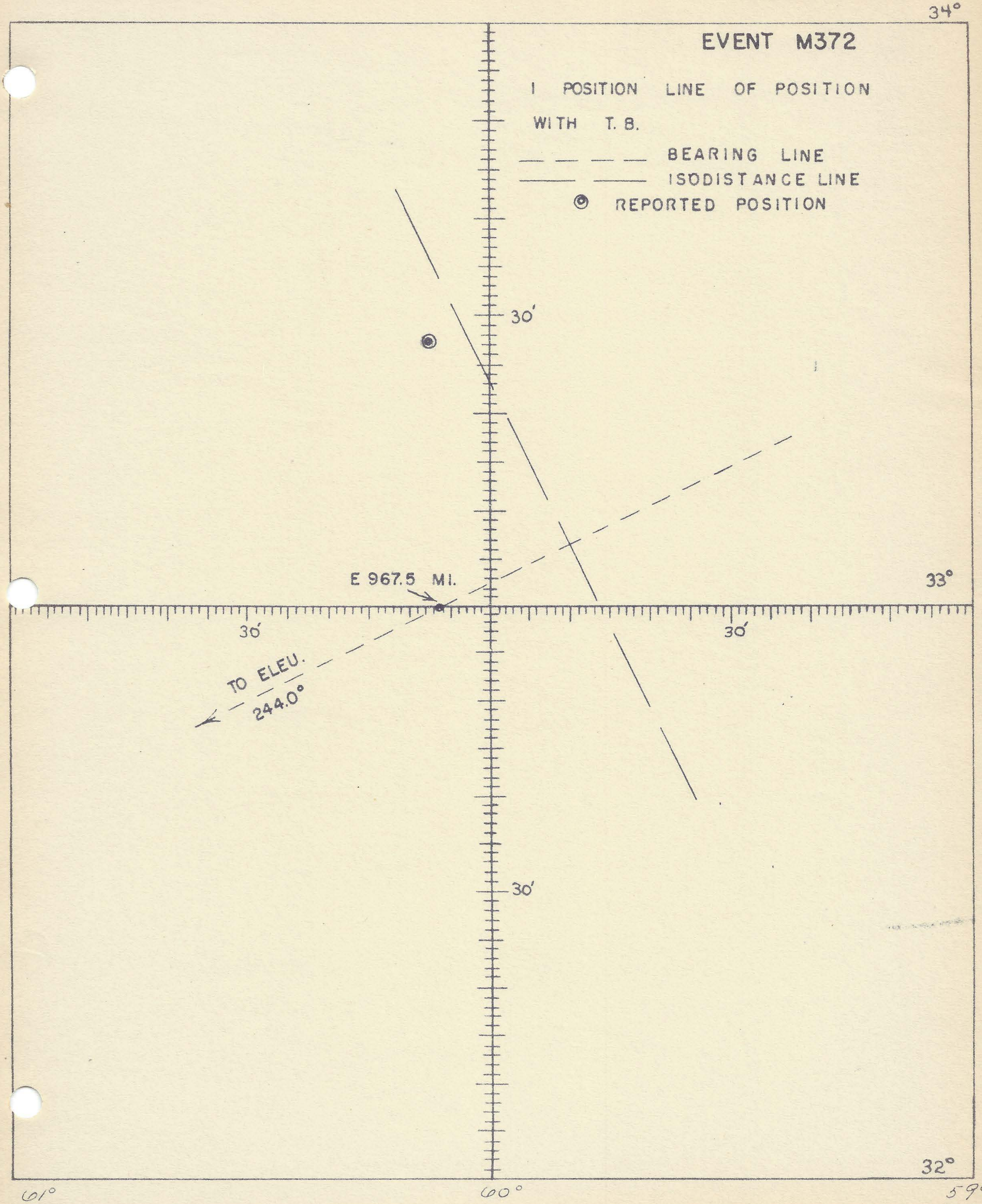
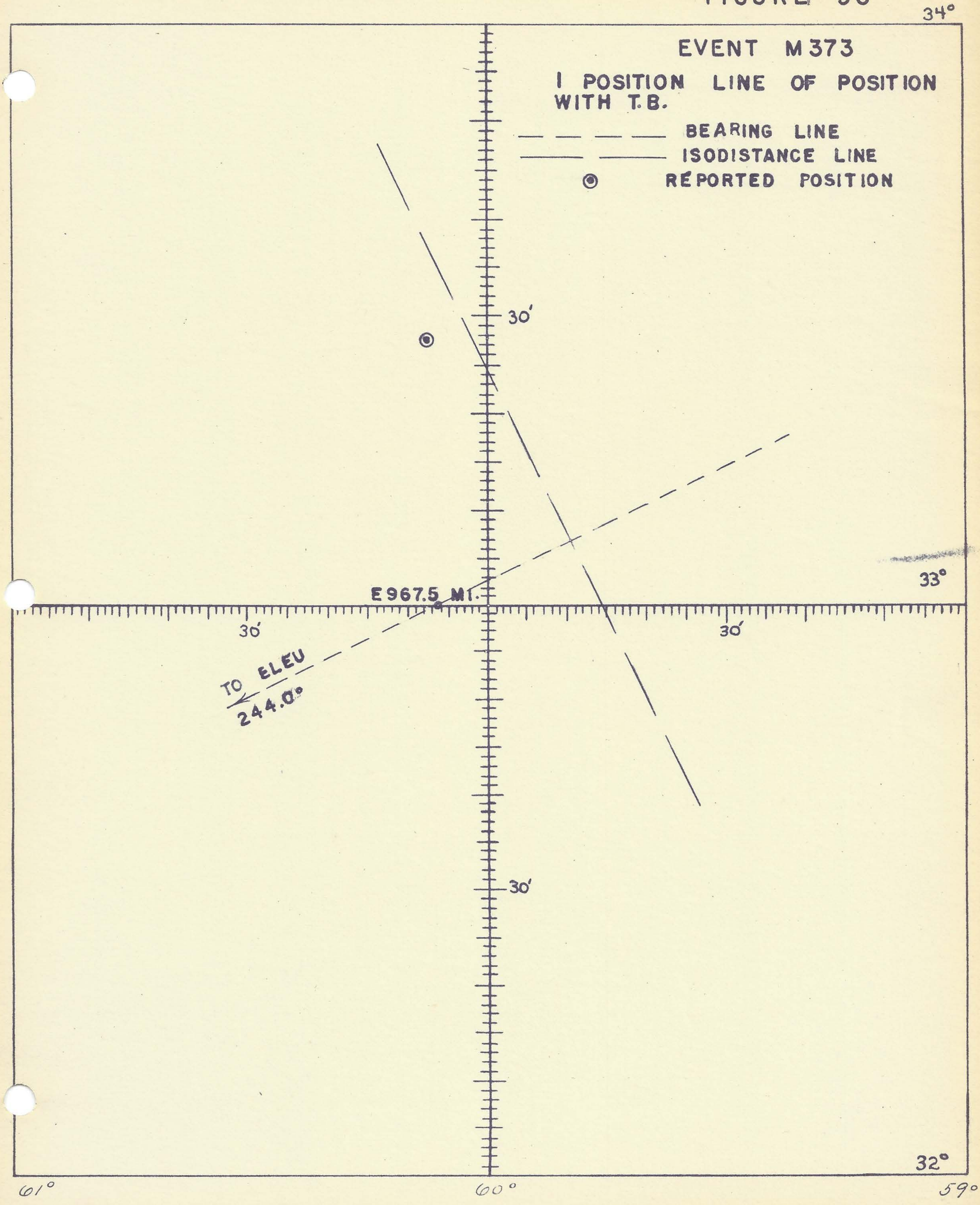

39°
EVENT V345
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
E 993.9 MI.
38°
30′
30′
TO ELEU
221.4°
30′
37°
65°
64°
63°

FIGURE 95



34°
EVENT M372
I POSITION LINE OF POSITION
WITH T. B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
E 967.5 MI.
33°
30′
30′
TO ELEV.
244.0°
30′
32°
61°
60°
59°

FIGURE 96



34°
EVENT M373
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
30′
30′
30′
33°
E967.5 MI.
TO ELEU
244.0°
32°
61°
60°
59°

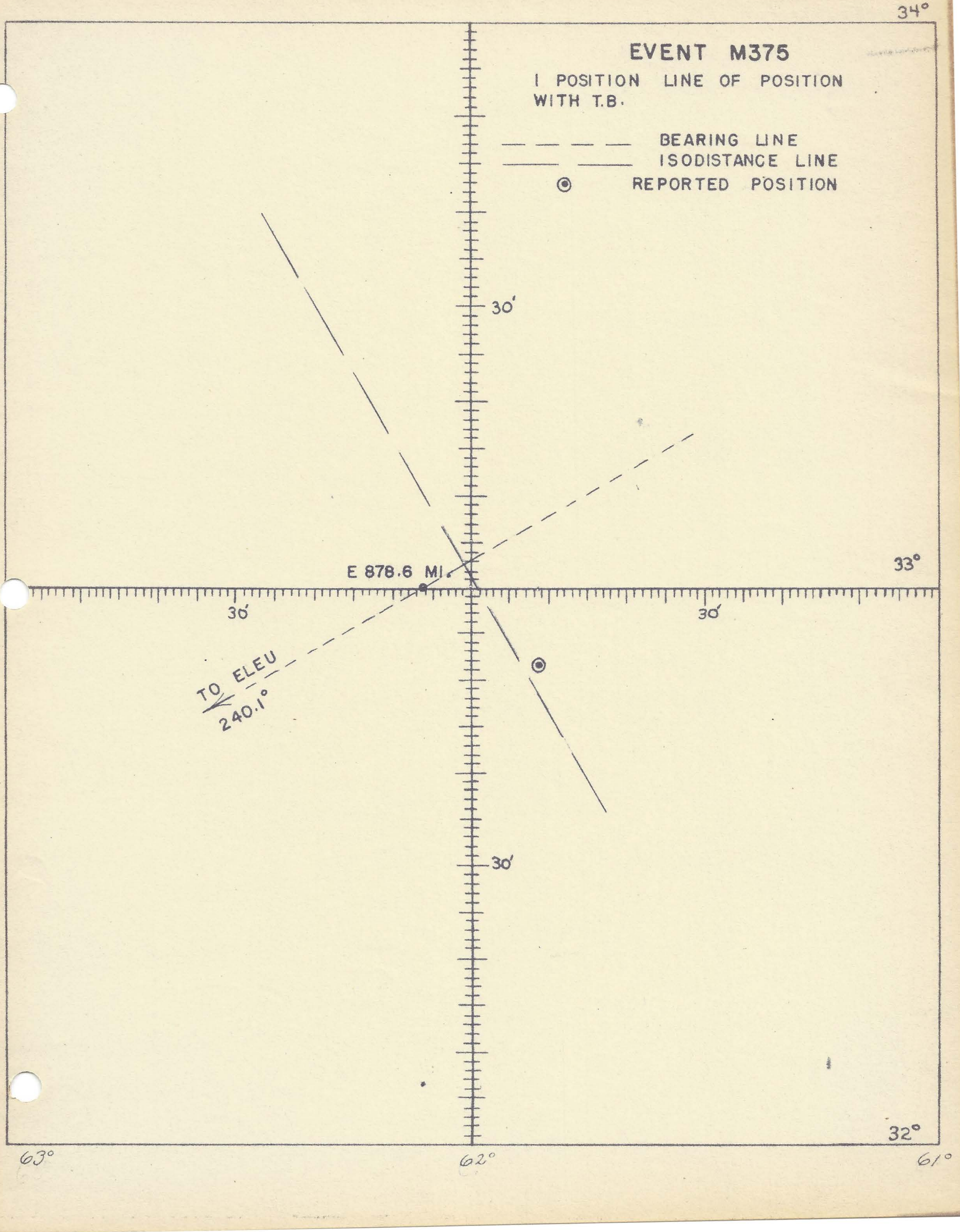
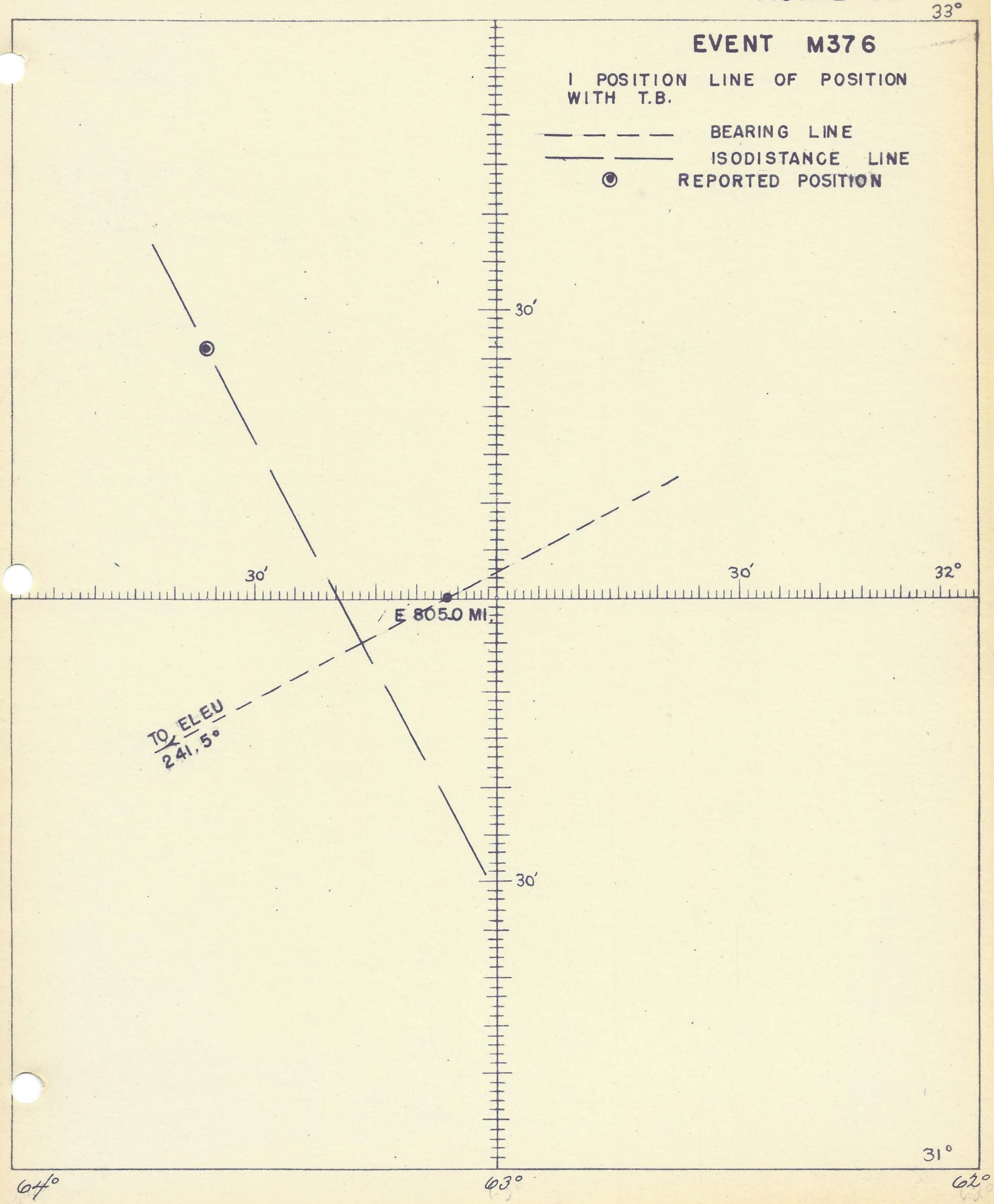

34°
EVENT M375
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
E 878.6 MI.
33°
30'
30'
TO ELEU
240.1°
30'
32°
63°
62°
61°

FIGURE 98



33°
EVENT M376
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
30'
30'
32°
E 805.0 MI.
TO ELEU
241.5°
30'
31°
64°
63°
62°

FIGURE 99

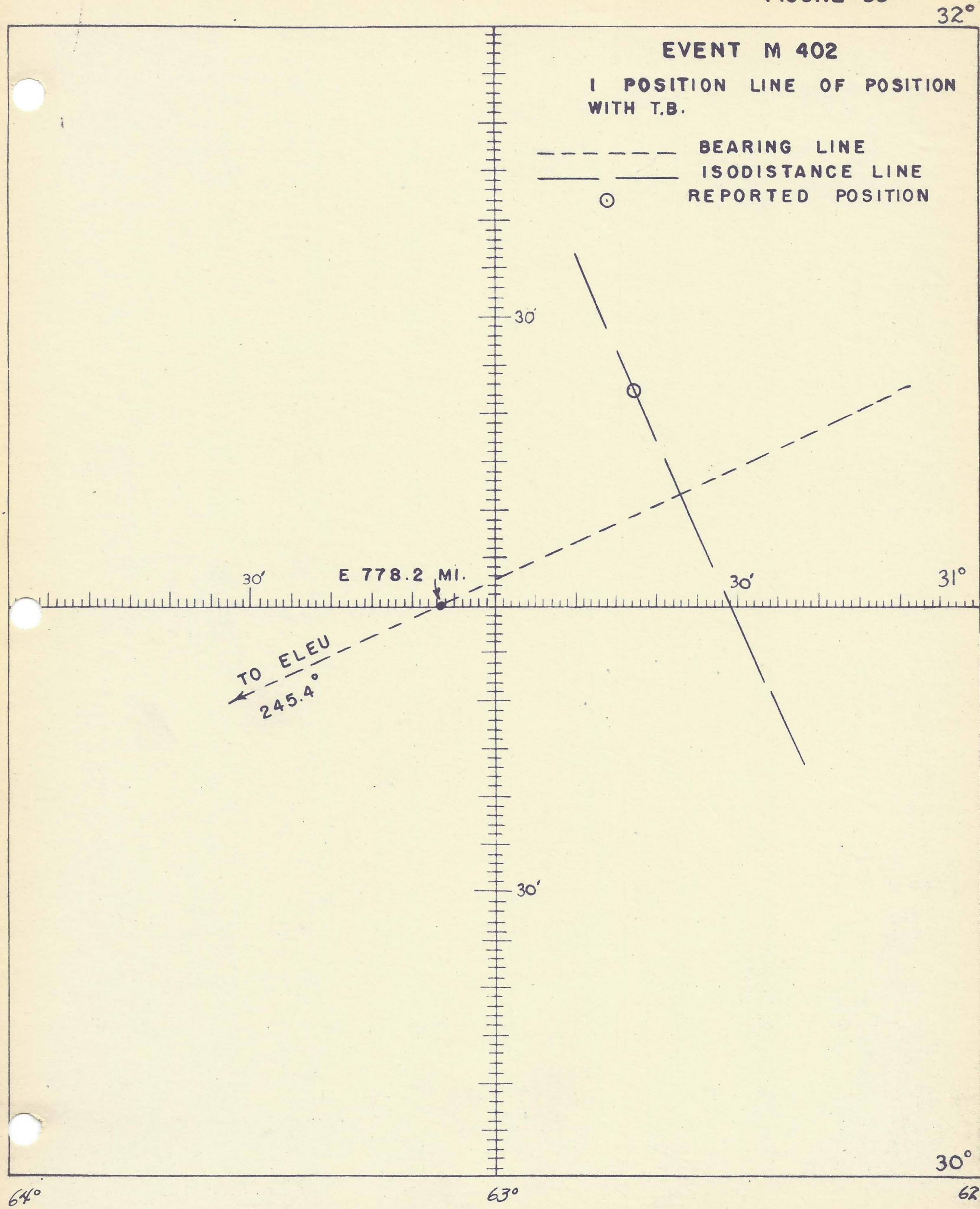

32°
EVENT M 402
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
30′
E 778.2 MI.
30′
31°
TO ELEU
245.4°
30′
30°
64°
63°
62°

FIGURE 100

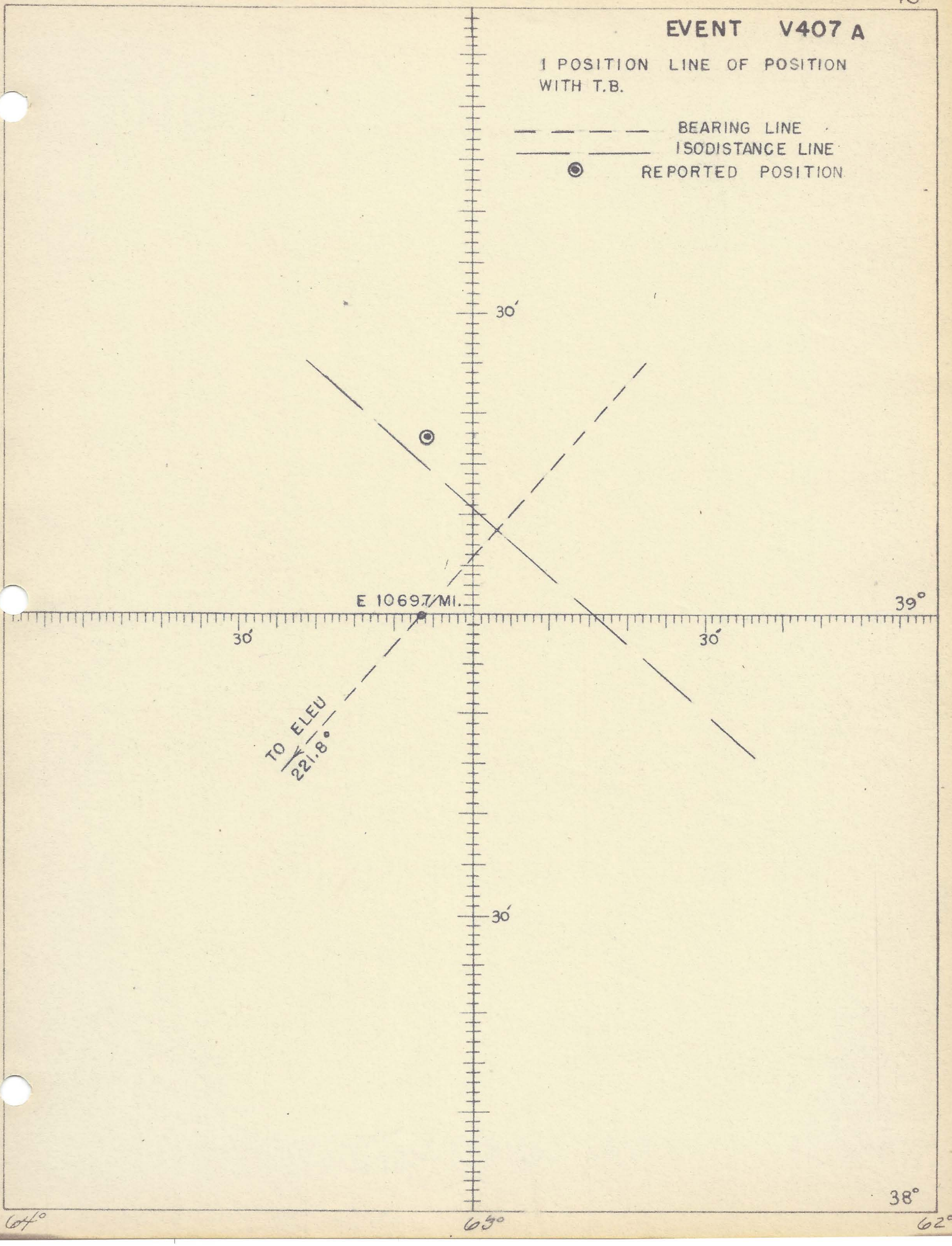

40°
EVENT V407 A
1 POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
E 10697 MI.
39°
30'
30'
TO ELEU
221.8°
30'
38°
64°
63°
62°

FIGURE 101

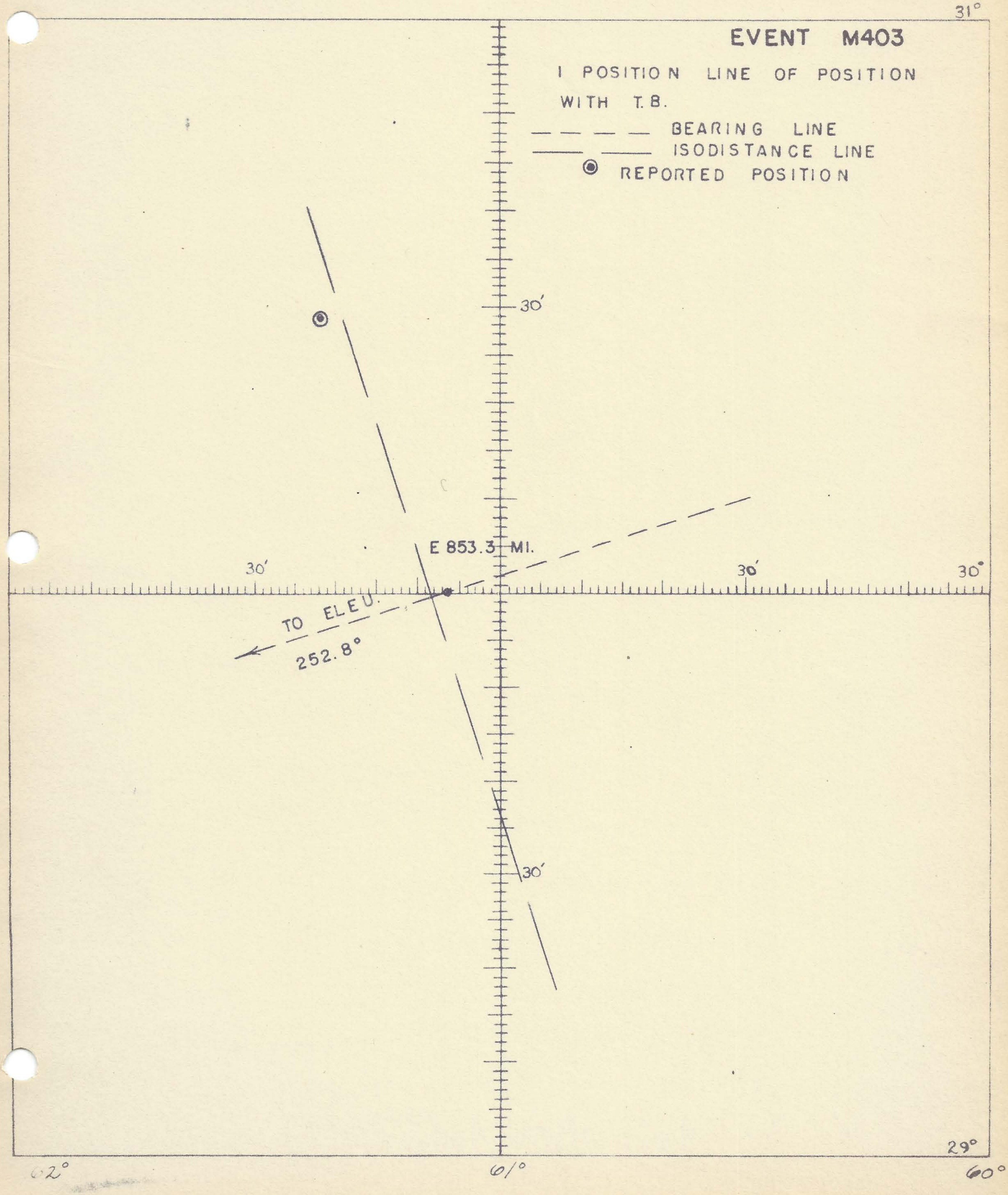

31°
EVENT M403
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
E 853.3 MI.
30′
30′
30°
TO ELEU.
252.8°
30′
29°
62°
61°
60°

FIGURE 102

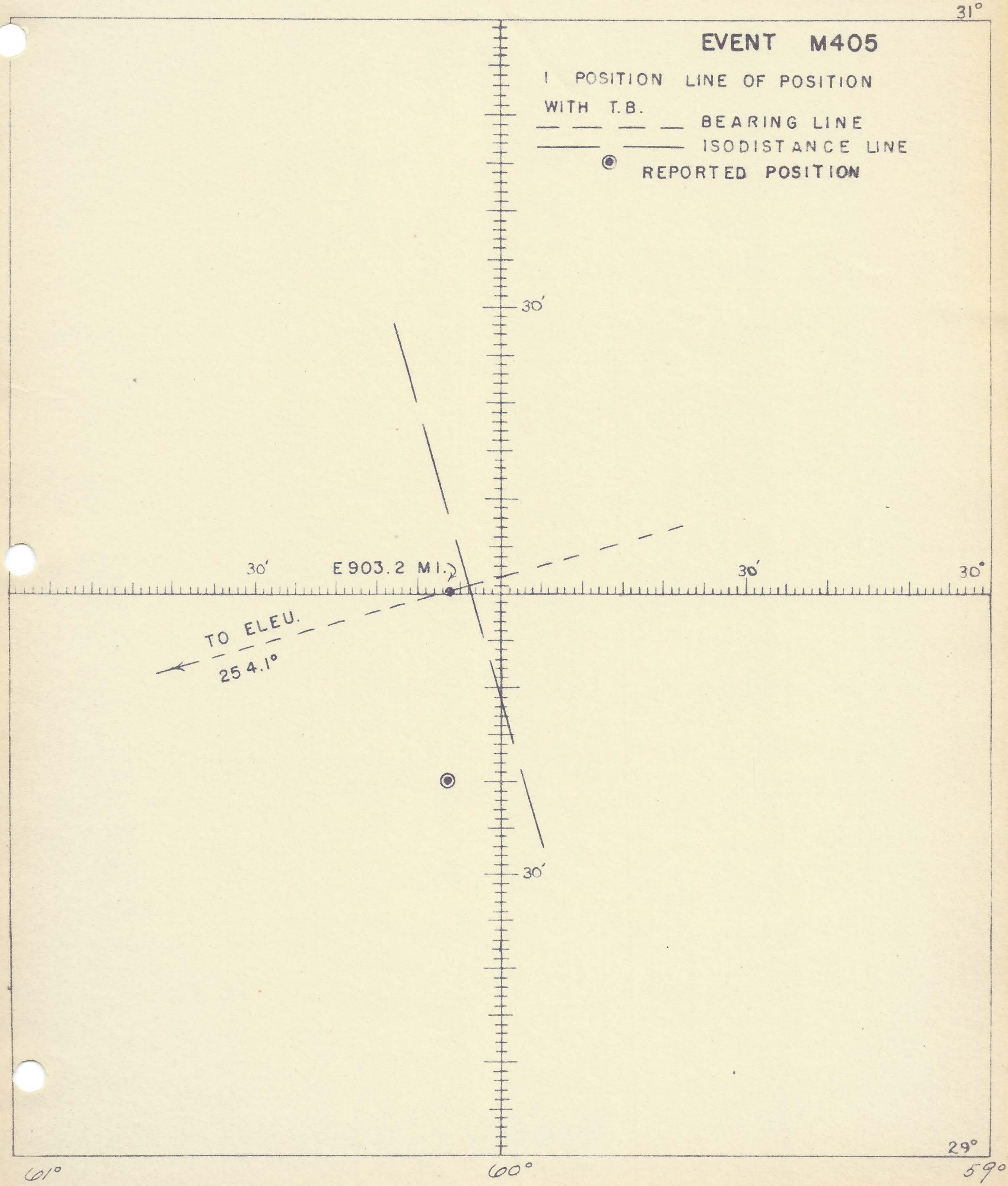
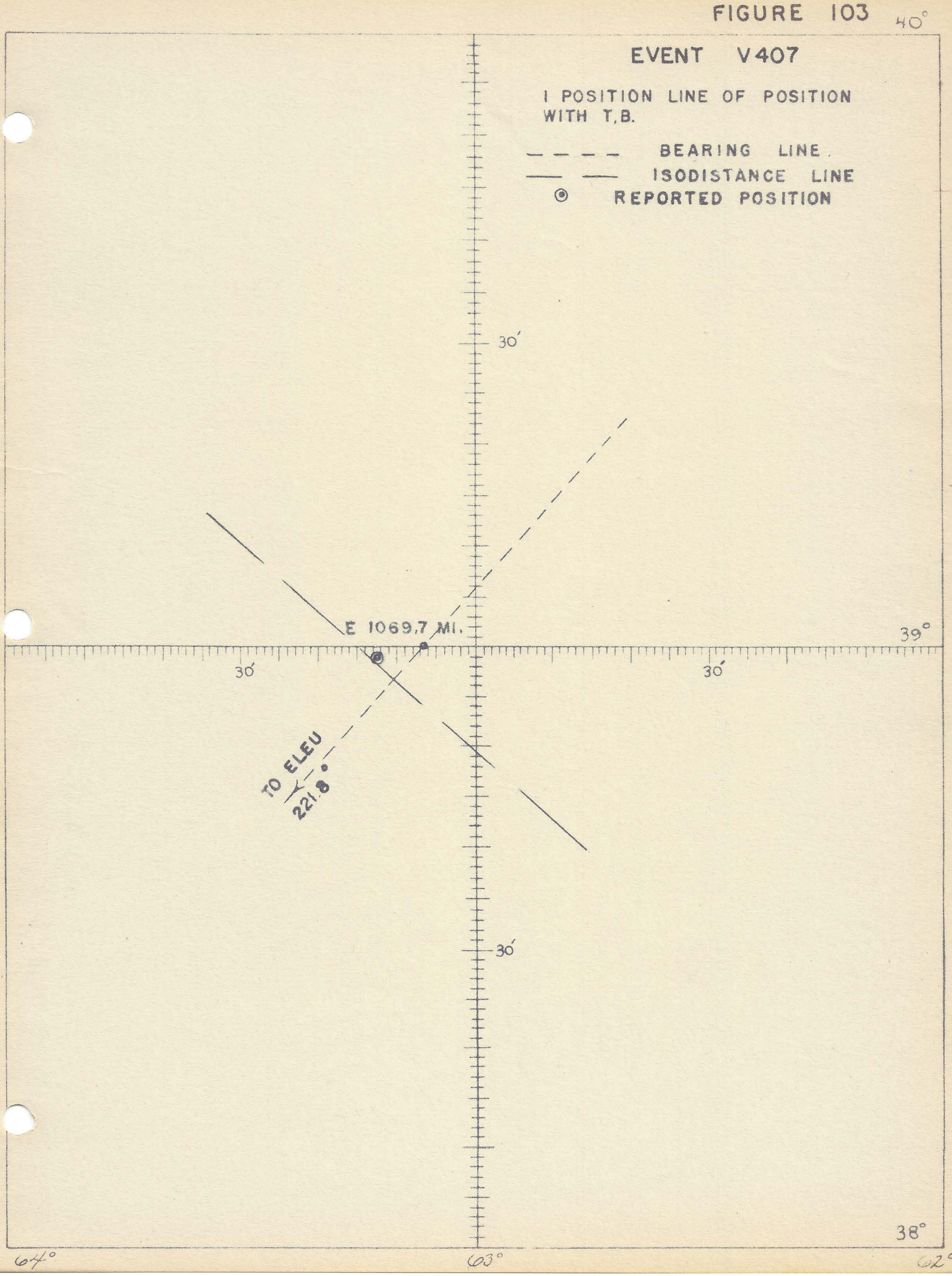

31°
EVENT M405
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
30'
E903.2 MI.
30'
30°
TO ELEU.
254.1°
30'
29°
61°
60°
59°

FIGURE 103



40°
EVENT V407
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE.
ISODISTANCE LINE
REPORTED POSITION
30'
E 1069.7 MI.
39°
30'
30'
TO ELEU
221.8°
30'
38°
64°
63°
62°

FIGURE 104

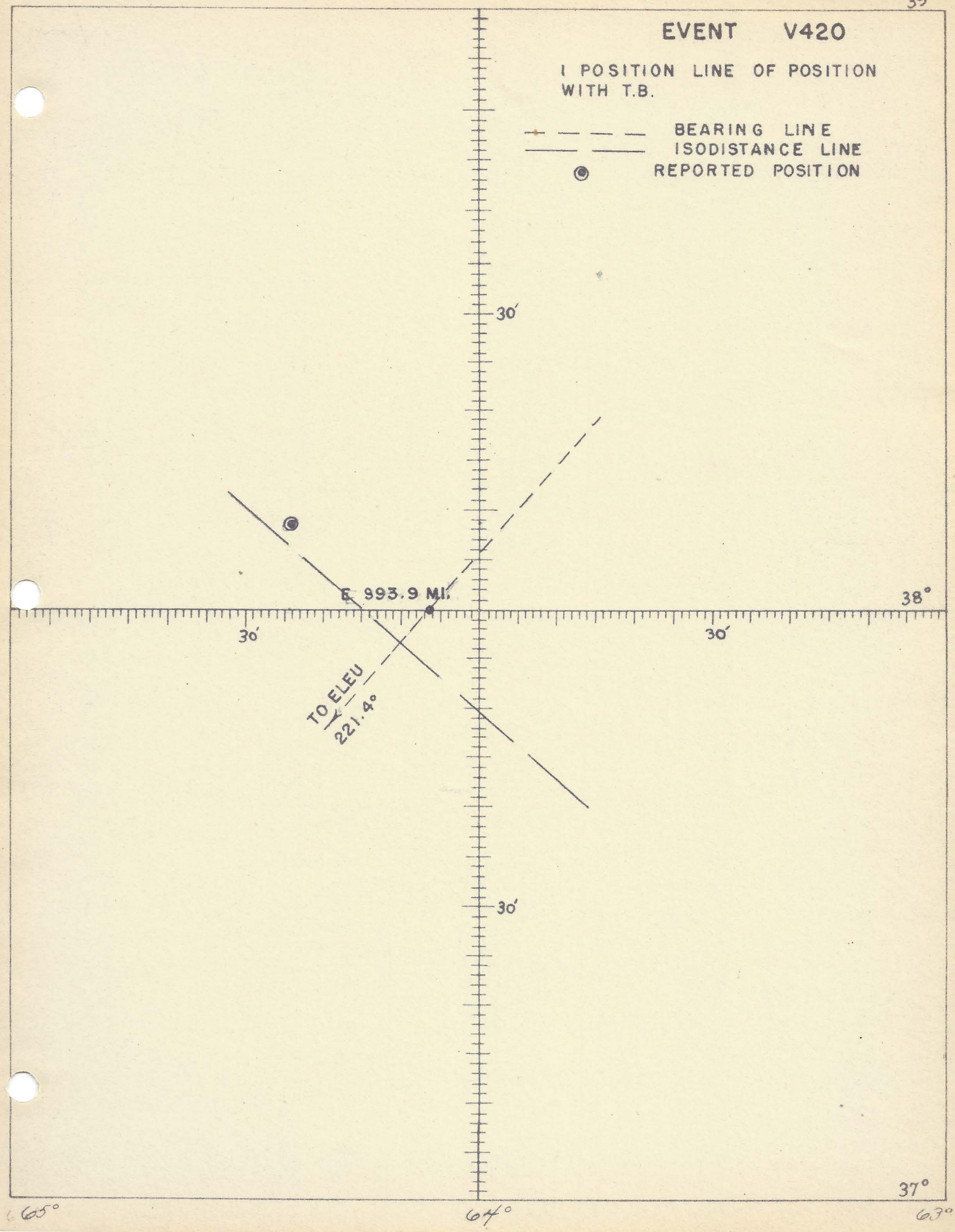
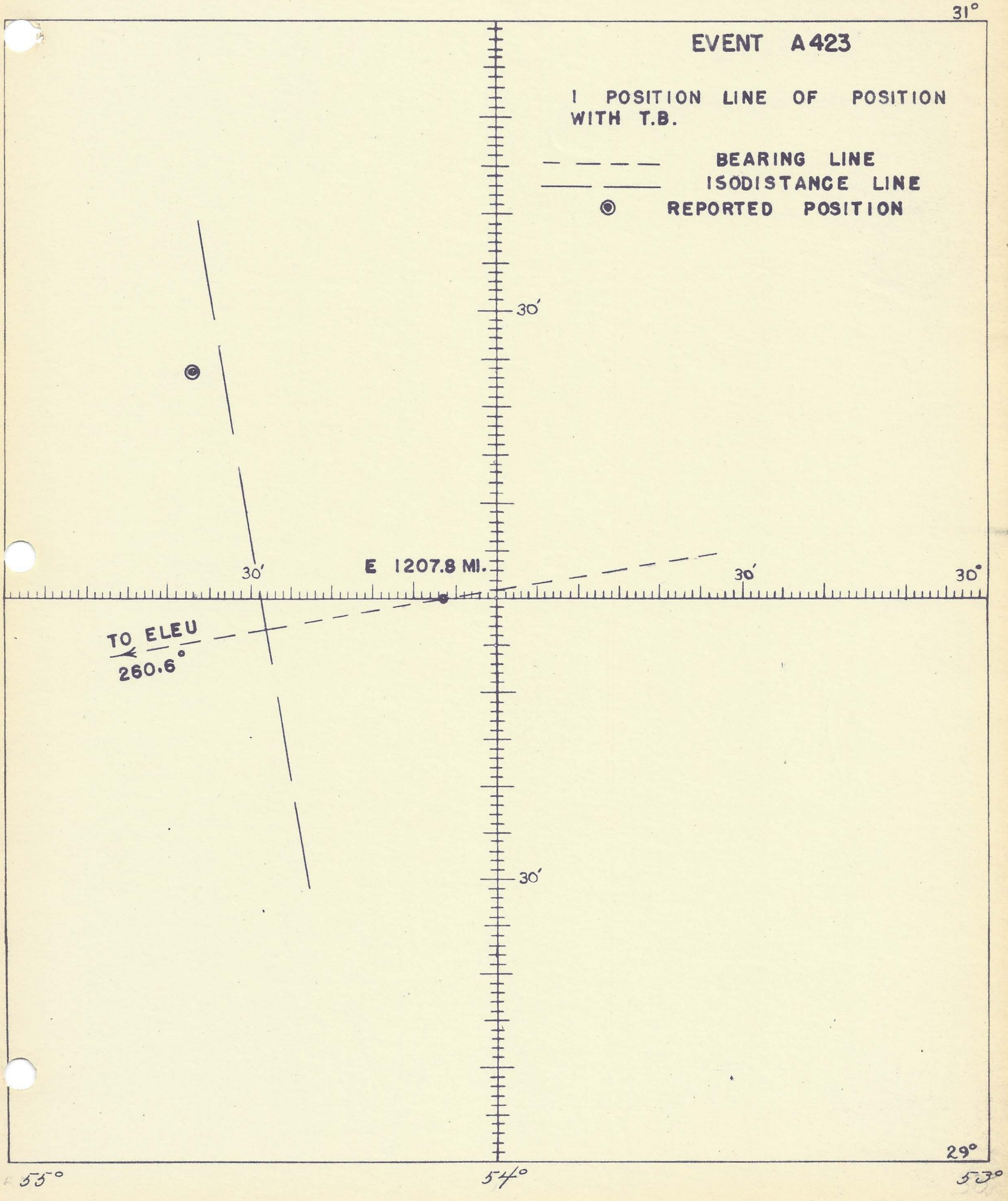
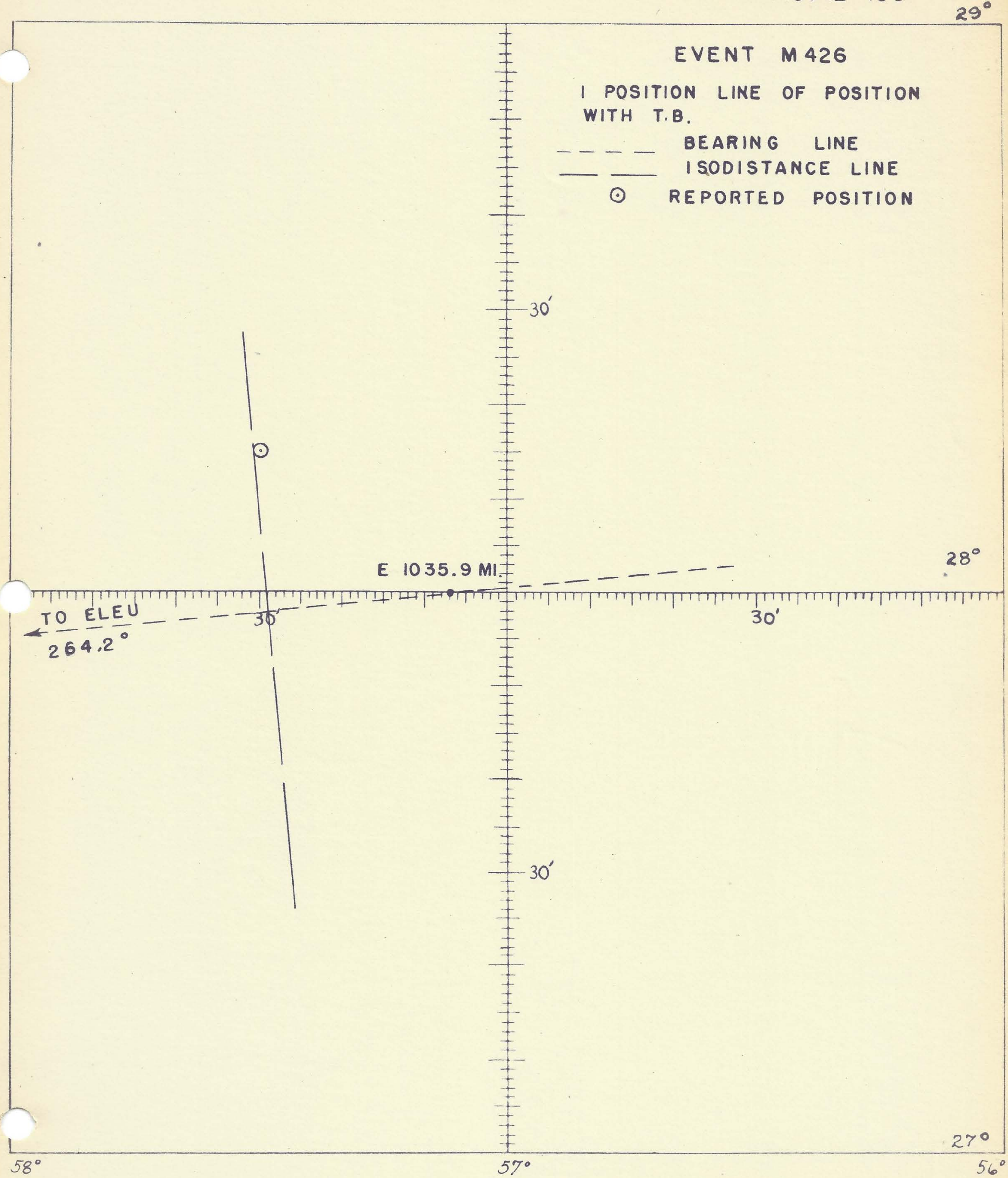

39°
EVENT V420
1 POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
E 993.9 MI.
38°
30′
30′
TO ELEU
221.4°
30′
37°
65°
64°
63°

FIGURE 105



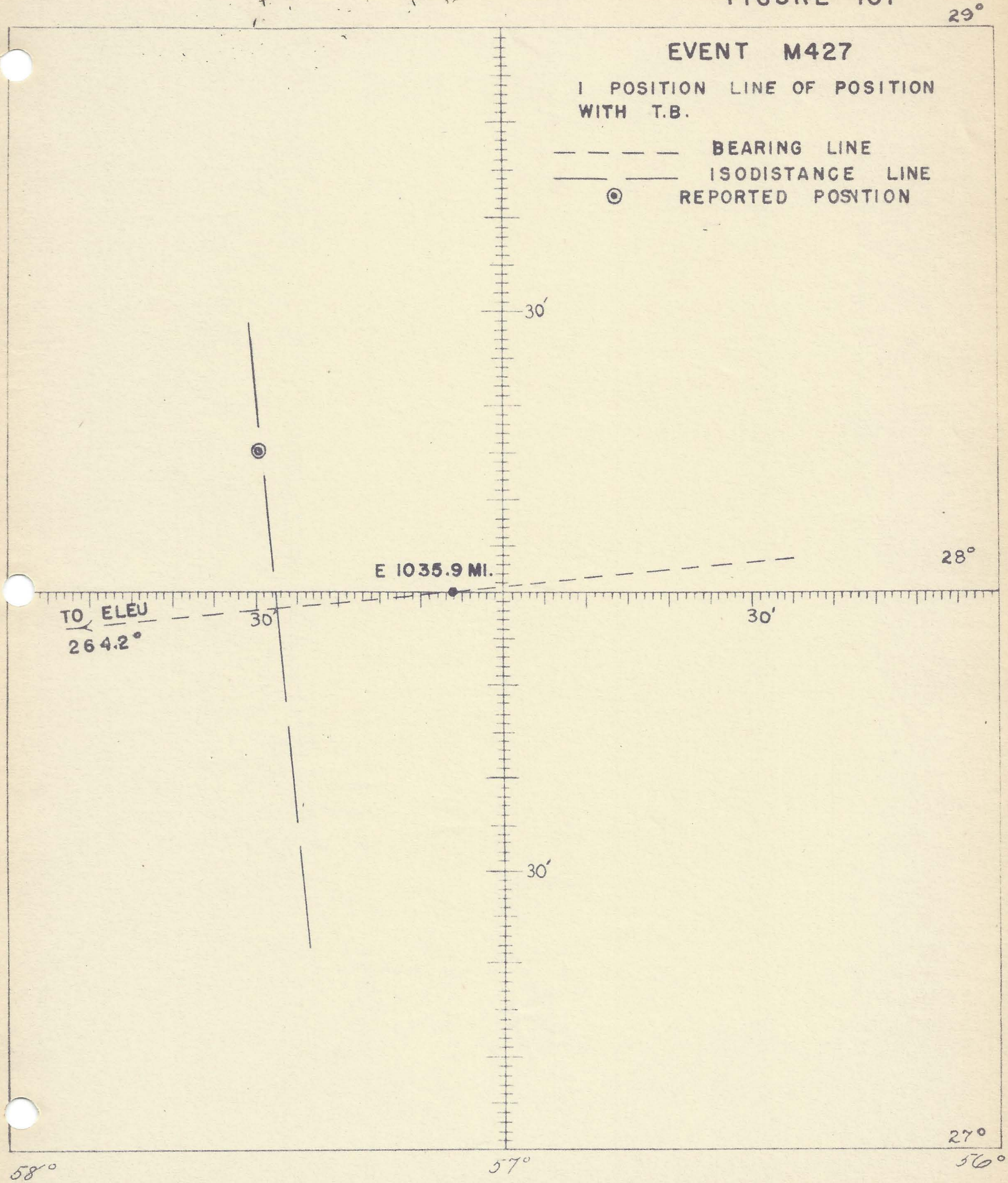
31°
EVENT A423
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
30′
E 1207.8 MI.
30′
30°
TO ELEU
260.6°
30′
29°
55°
54°
53°

FIGURE 106



29°
EVENT M 426
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
E 1035.9 MI.
28°
TO ELEU
264.2°
30'
30'
30'
27°
58°
57°
56°

FIGURE 107



29°
EVENT M427
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
28°
E 1035.9 MI.
TO ELEU
264.2°
30′
30′
30′
27°
58°
57°
56°

FIGURE 108

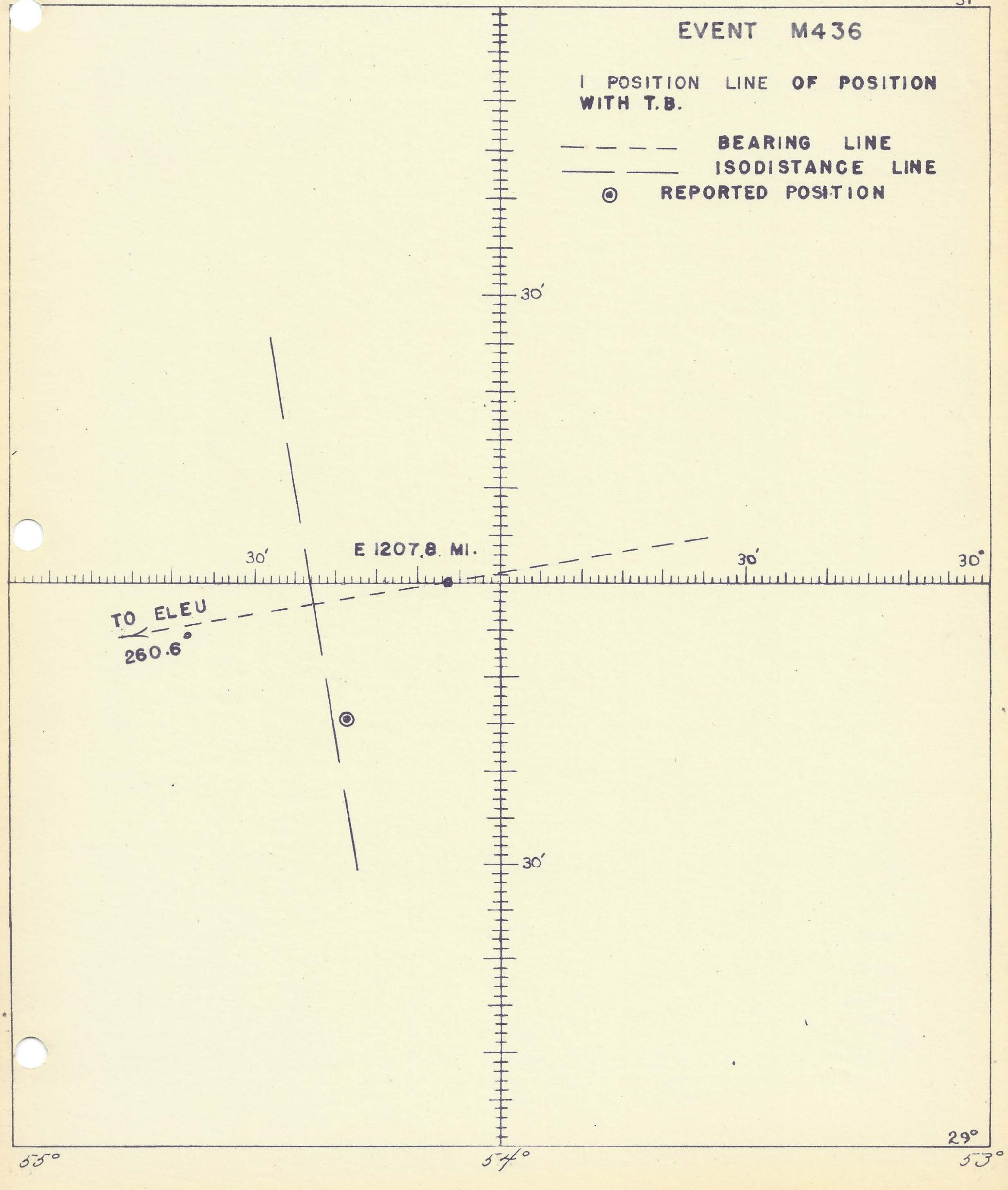

EVENT M436
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
31°
30'
30'
E 1207.8 MI.
30'
30°
TO ELEU
260.6°
30'
29°
55°
54°
53°

FIGURE 109

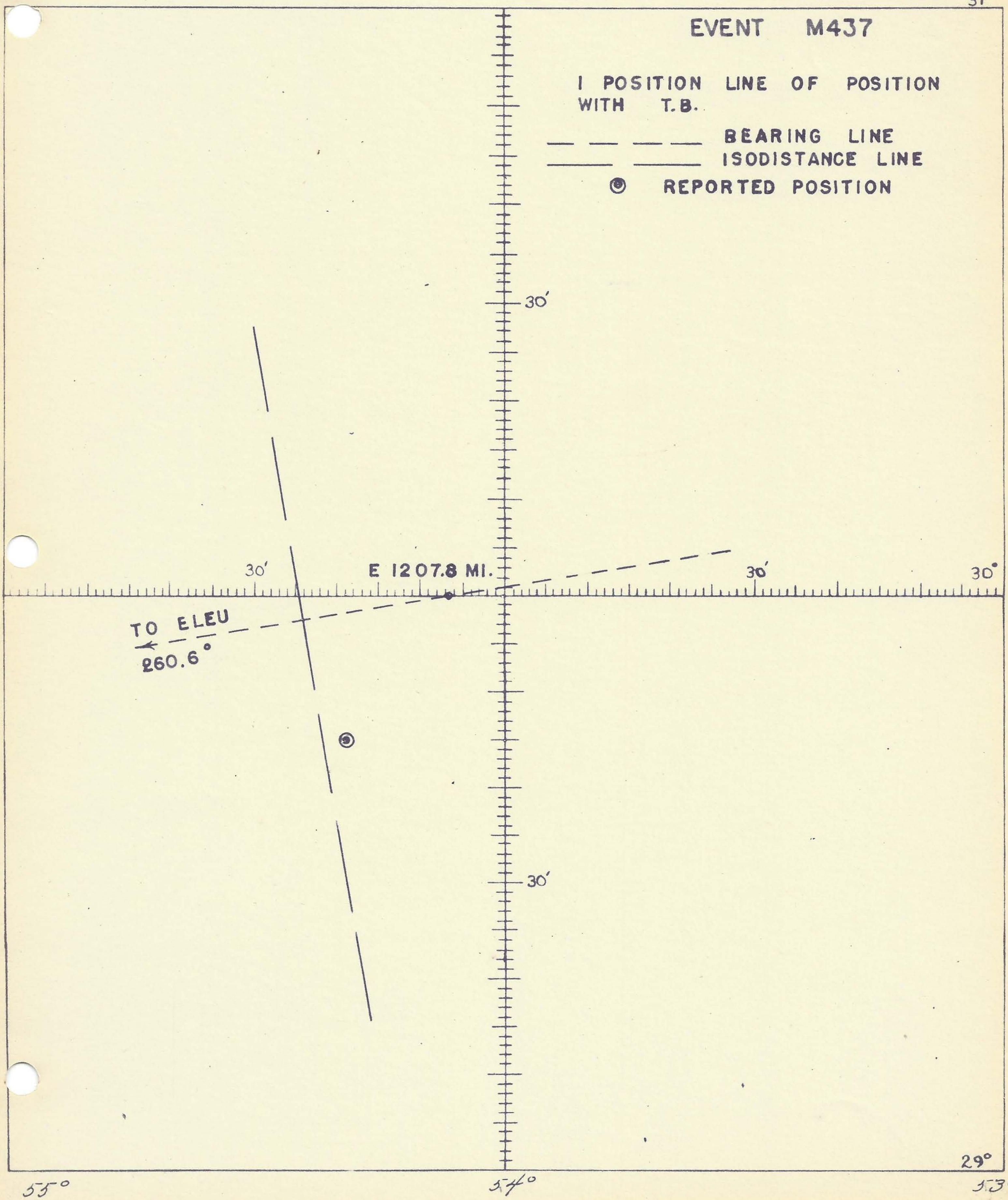

31°
EVENT M437
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
30'
E 1207.8 MI.
30'
30°
TO ELEU
260.6°
30'
29°
55°
54°
53°

FIGURE 110

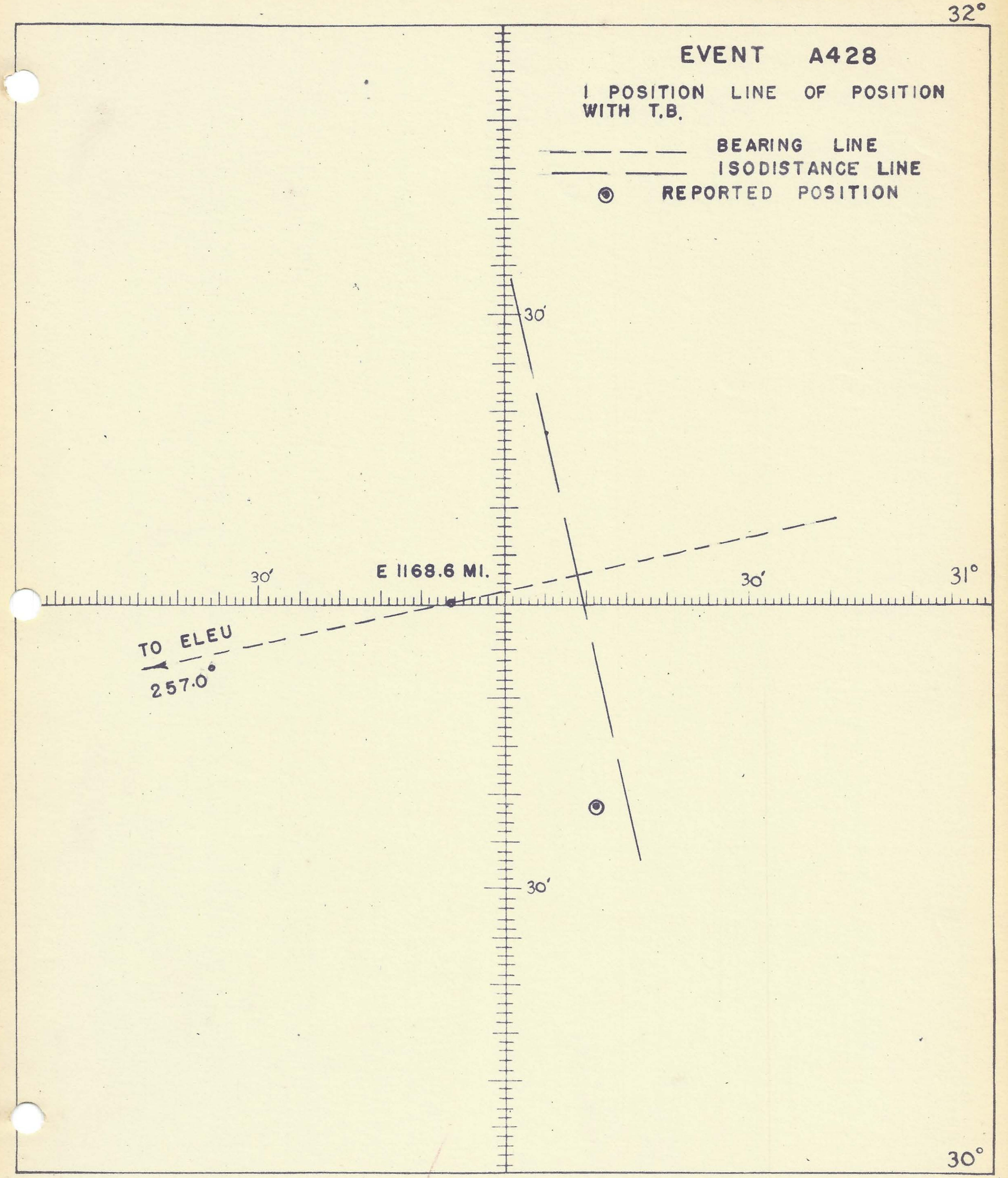
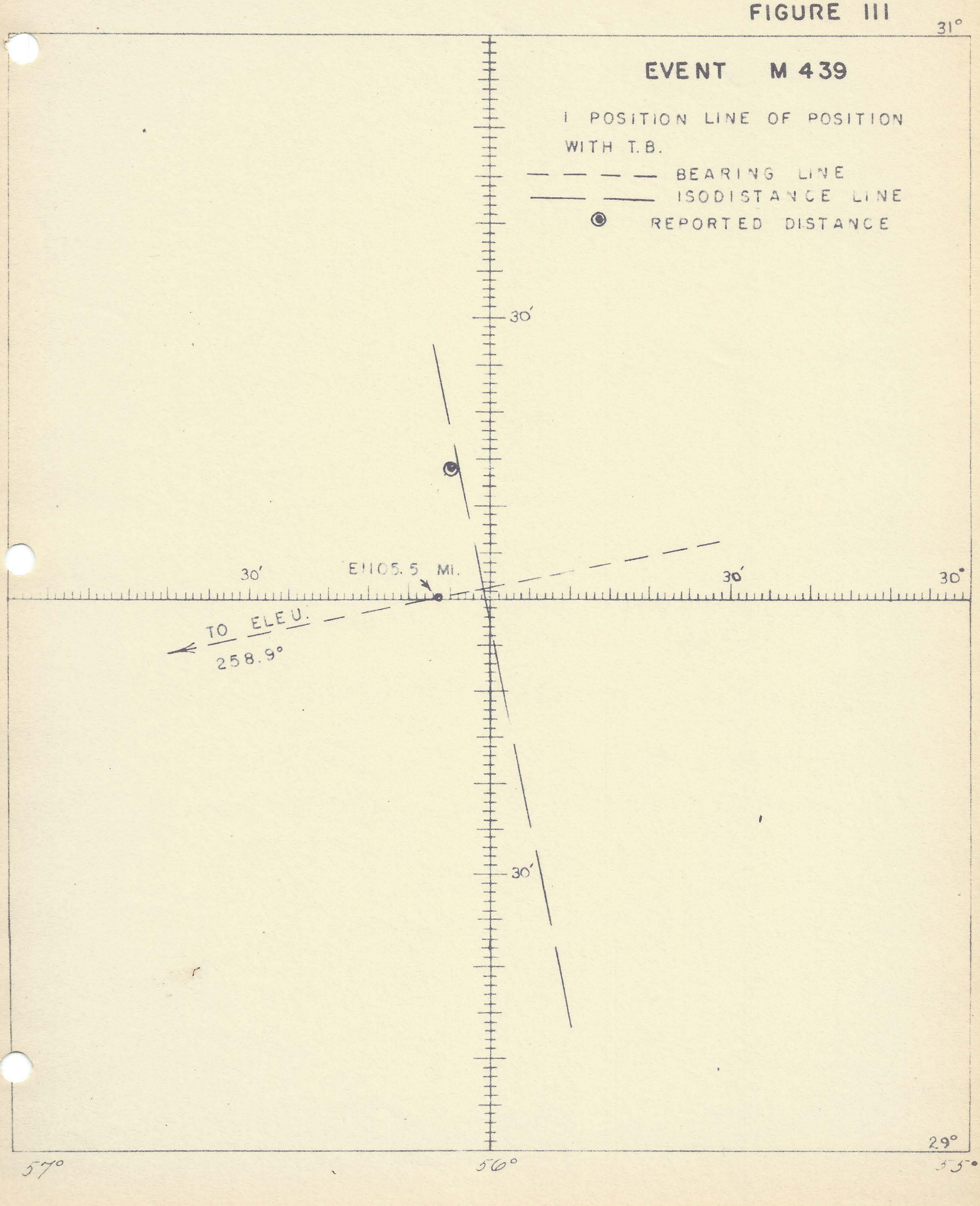

32°
EVENT A428
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
E 1168.6 MI.
30′
30′
31°
TO ELEU
257.0°
30′
30°
56°
55°
54°

FIGURE III



EVENT M 439
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED DISTANCE
31°
30′
30′
30′
30°
30′
E 105.5 MI.
TO ELEV.
258.9°
29°
57°
56°
55°

FIGURE 112

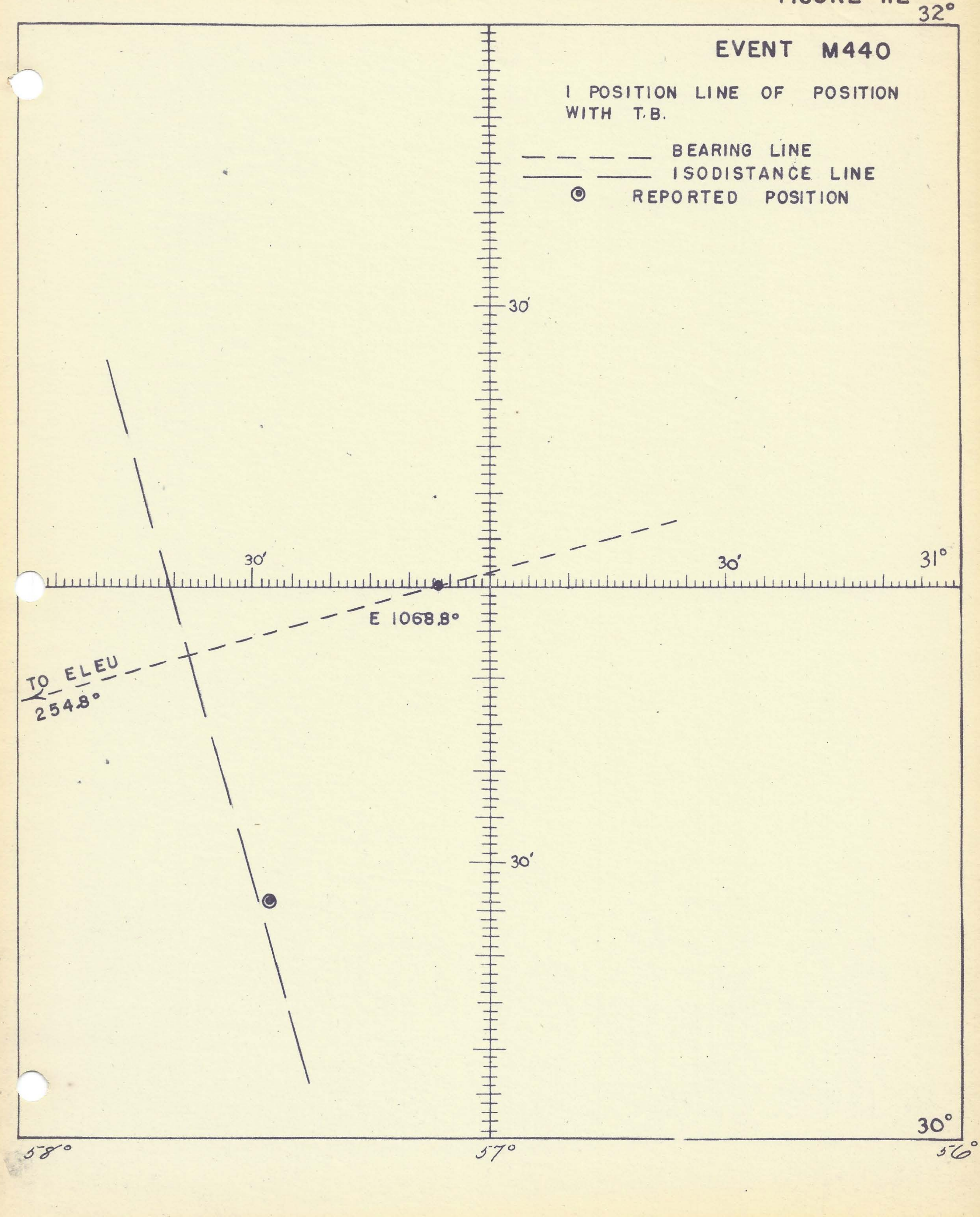

32°
EVENT M440
1 POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
30′
30′
31°
E 1068.8°
TO ELEU
254.8°
30′
30°
58°
57°
56°

FIGURE 113

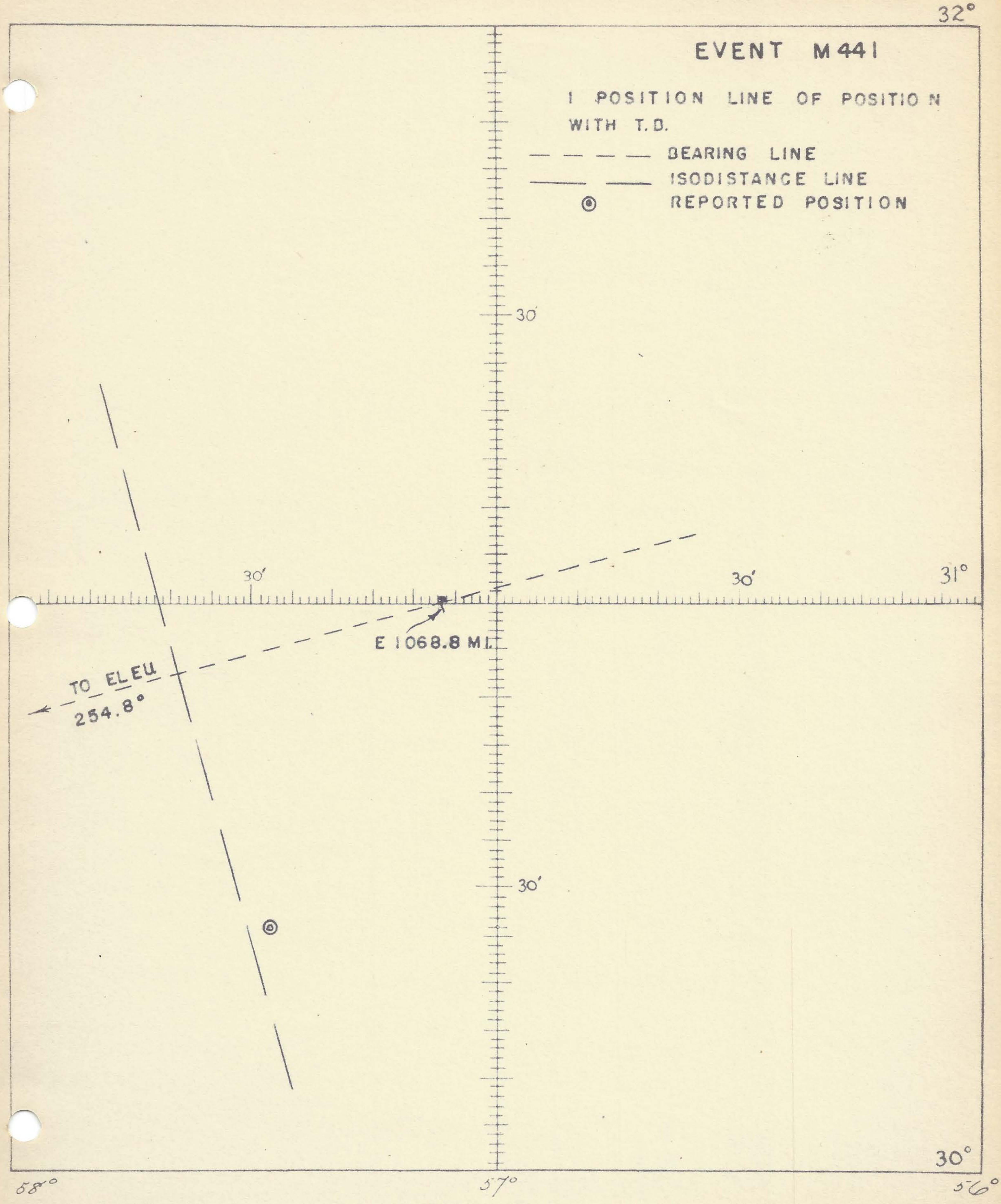
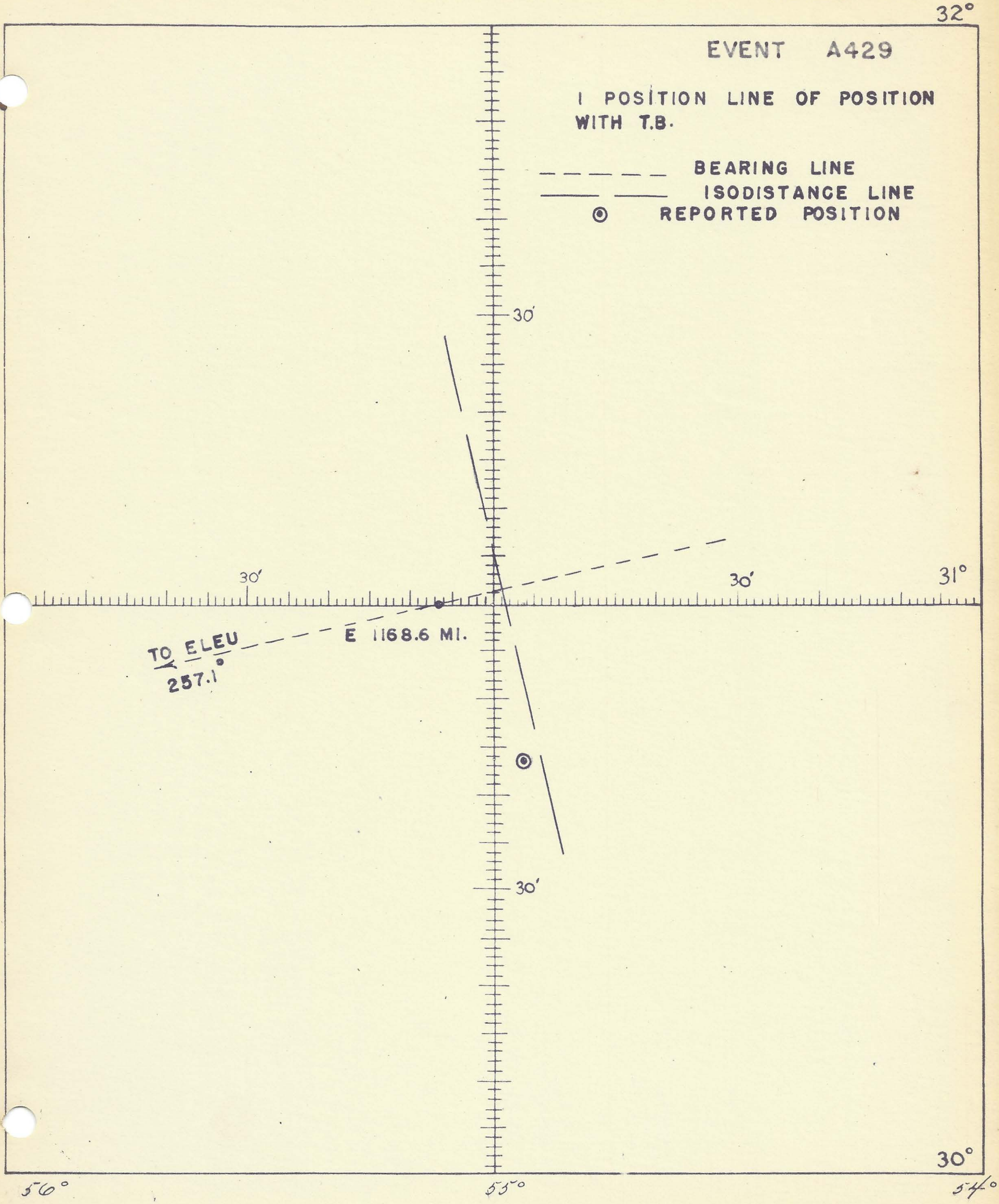

32°
EVENT M441
1 POSITION LINE OF POSITION WITH T.D.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
30'
30'
31°
E 1068.8 MI.
TO ELEU
254.8°
30'
30°
58°
57°
56°

FIGURE 114



EVENT A429
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
TO ELEU
257.1°
E 1168.6 MI.
30'
30'
30'
30'
32°
31°
30°
56°
55°
54°

FIGURE 115

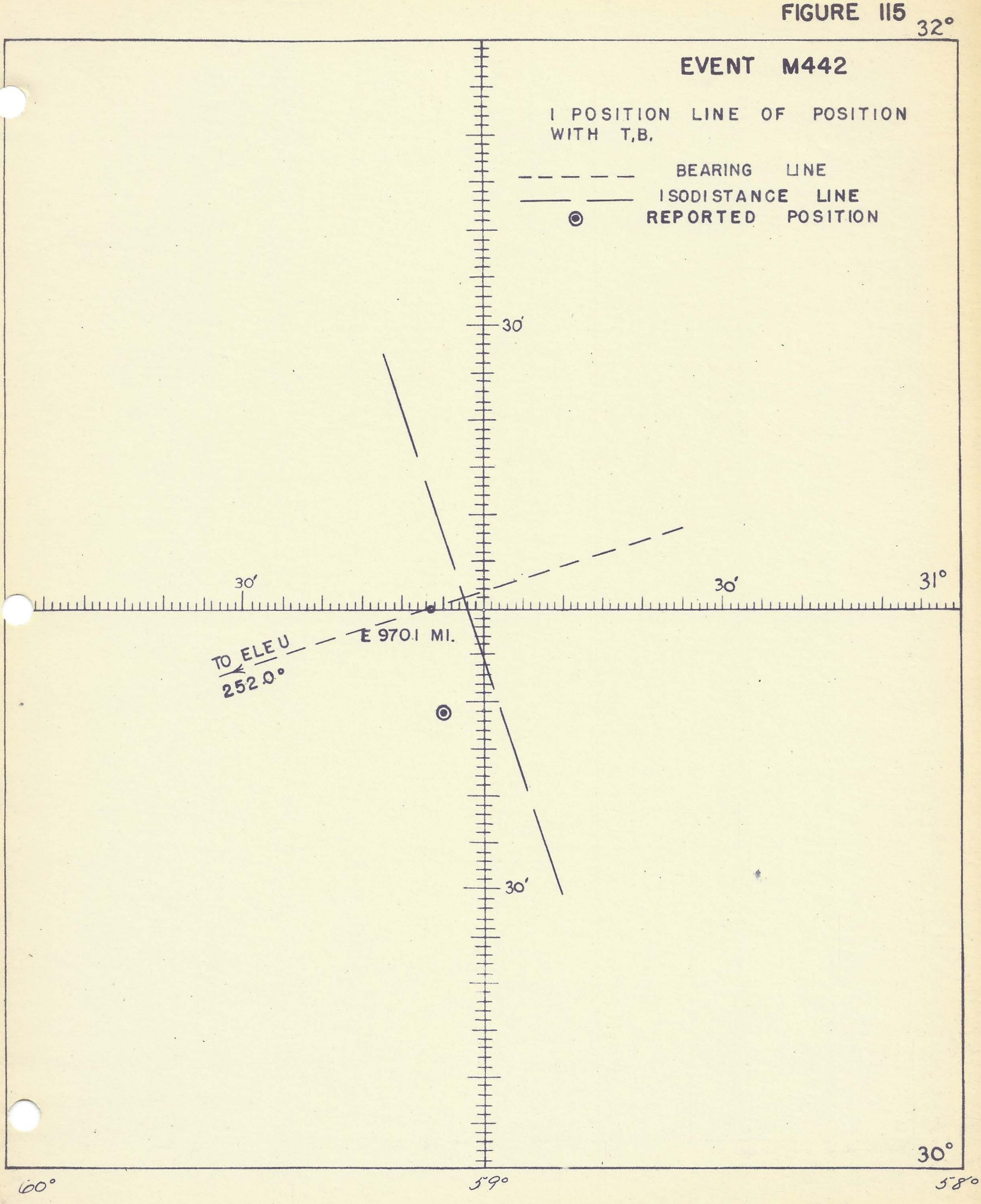

32°
EVENT M442
1 POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
30'
30'
31°
TO ELEU
252.0°
E 970.1 MI.
30'
30°
60°
59°
58°

FIGURE 116

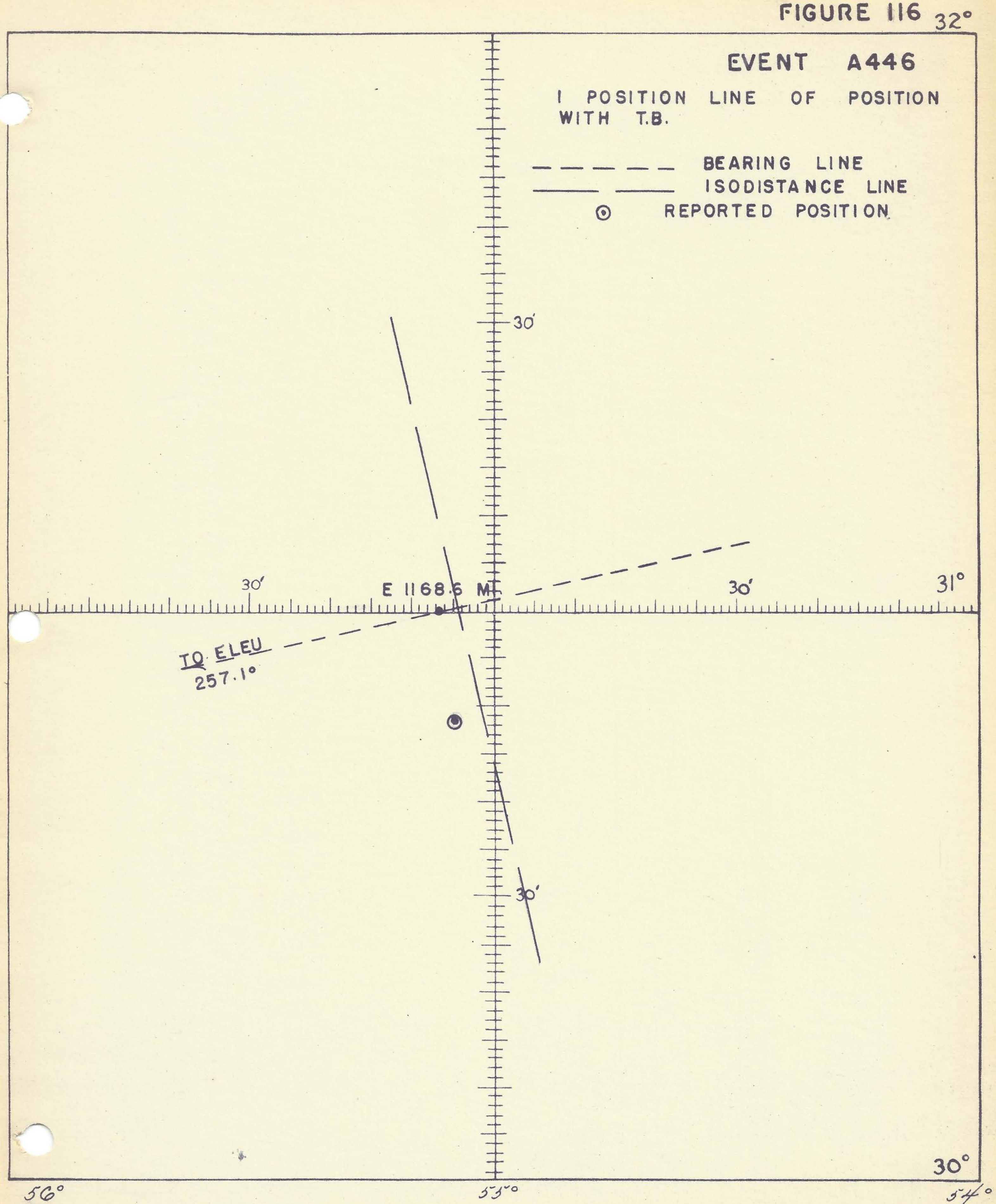

32°
EVENT A446
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
E 1168.6 M
30'
30'
31°
TO ELEU
257.1°
30'
30°
56°
55°
54°

FIGURE 117

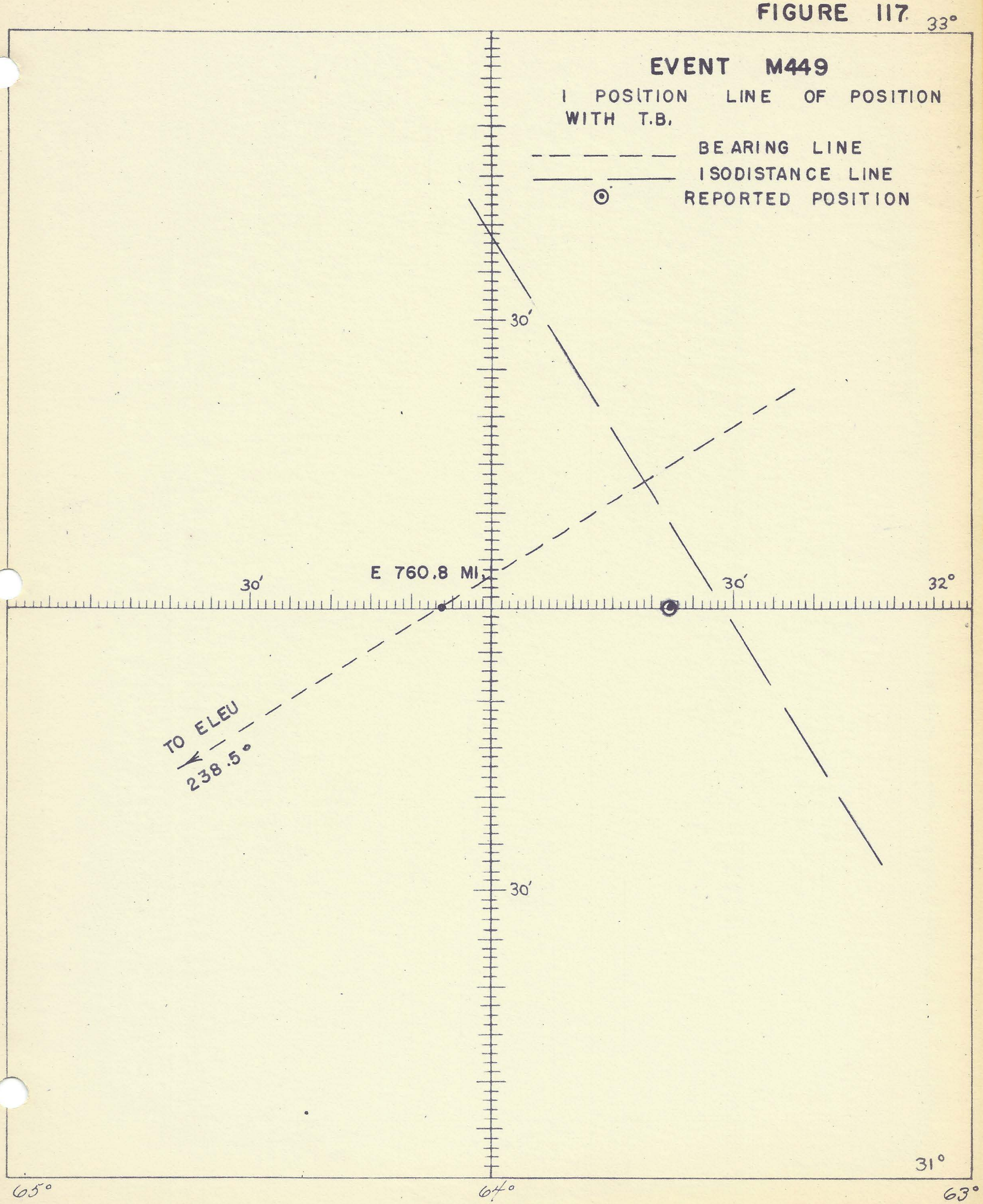

33°
EVENT M449
I POSITION LINE OF POSITION WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30′
E 760.8 MI.
30′
30′
32°
TO ELEU
238.5°
30′
31°
65°
64°
63°

FIGURE 118

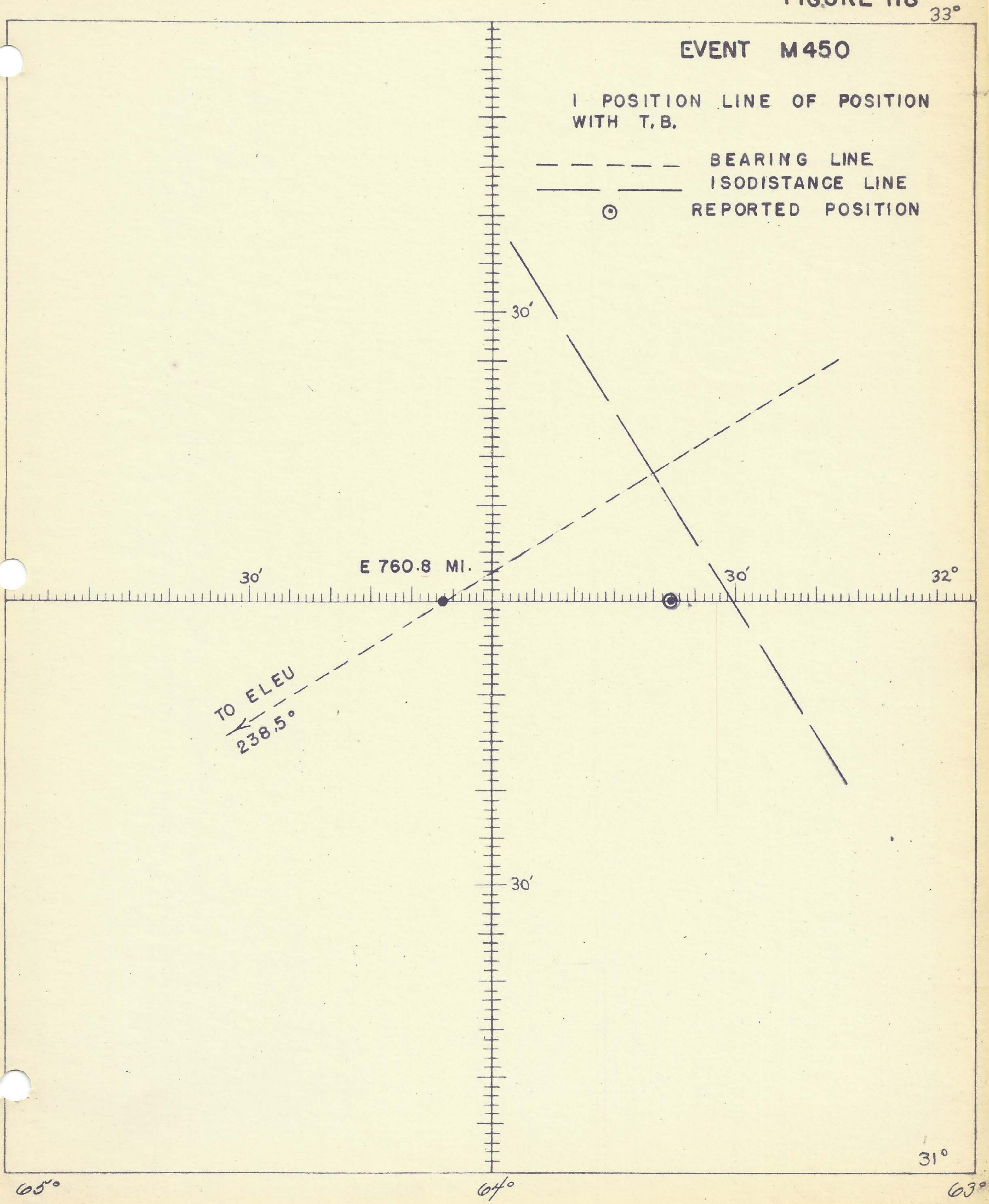

33°
EVENT M450
I POSITION LINE OF POSITION
WITH T.B.
BEARING LINE
ISODISTANCE LINE
REPORTED POSITION
30'
30'
E 760.8 MI.
30'
32°
TO ELEU
238.5°
30'
31°
65°
64°
63°

FIGURE 119

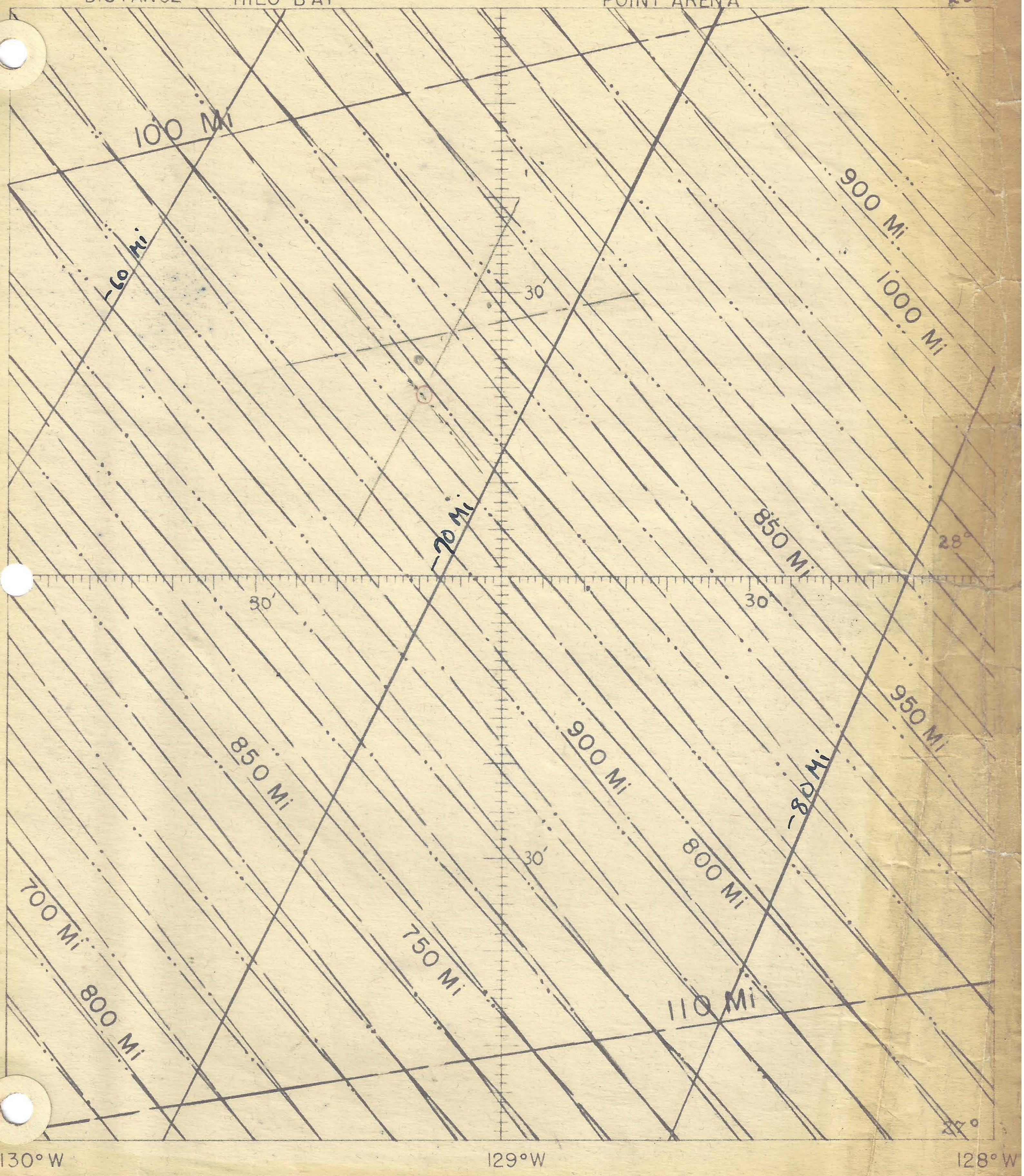
COMBINED RESULTS OF SOFAR FIXES

AUGUST 20th - SEPTEMBER 5th, 1945

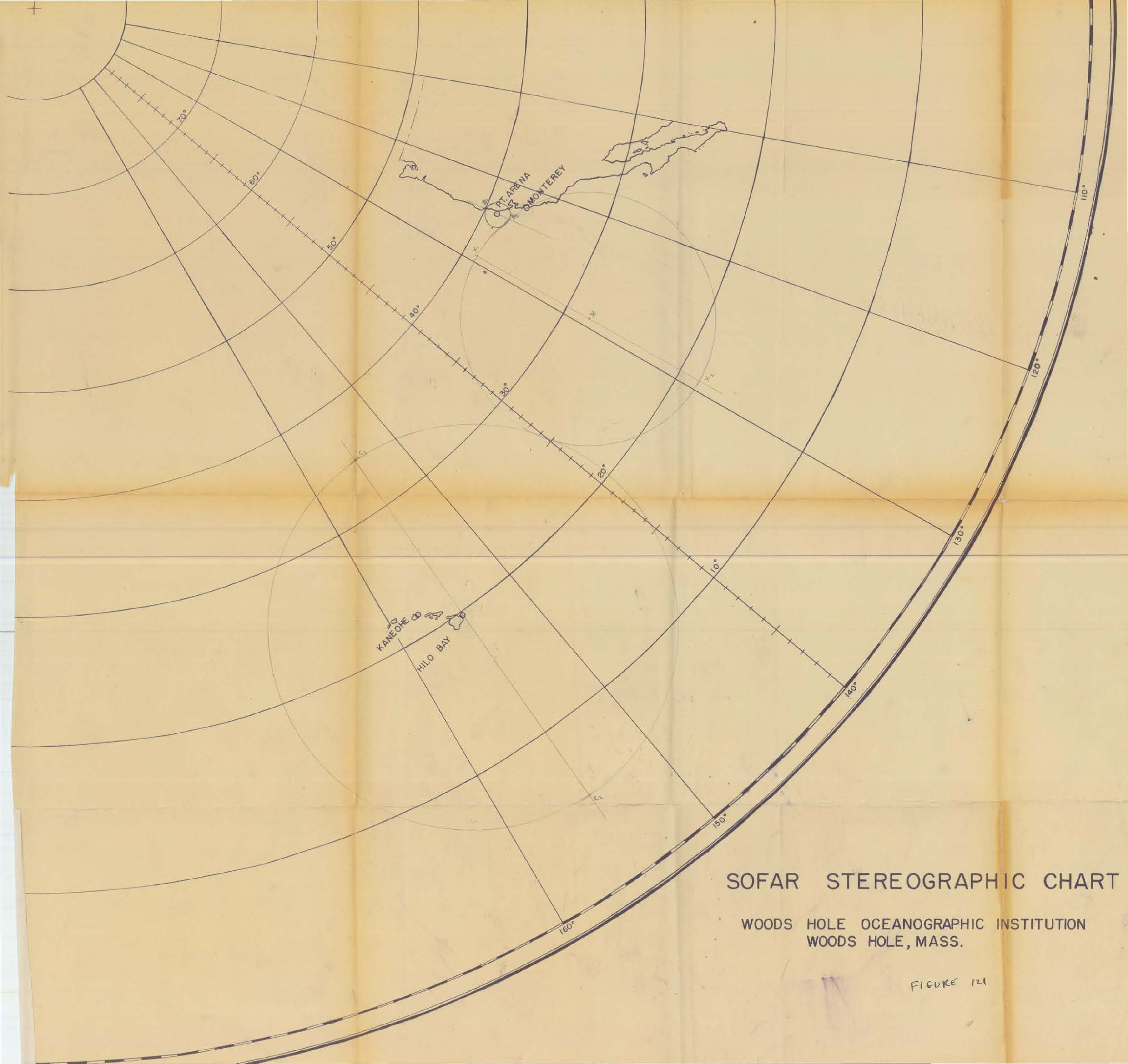
FIGURE 120

ELLIPTICAL DISTANCE DIFFERENCE CURVES:

DISTANCE TO MONTEREY LESS DISTANCE TO POINT ARENA
DISTANCE " KANEOHE " " " HILO BAY
DISTANCE " KANEOHE " " " POINT ARENA
DISTANCE " HILO BAY " " " POINT ARENA



SOFAR POSITION PLOTTING SHEET V-PA



SOFAR STEREOGRAPHIC CHART

WOODS HOLE OCEANOGRAPHIC INSTITUTION
WOODS HOLE, MASS.

FIGURE 121

FIGURE 122

EVENT M409

---- BEARING LINE

—.—.— ISODISTANCE LINE AT .8040 M/S (4888.5 F/S)

— — — ISODISTANCE LINE AT .8017 M/S (4874.6 F/S)

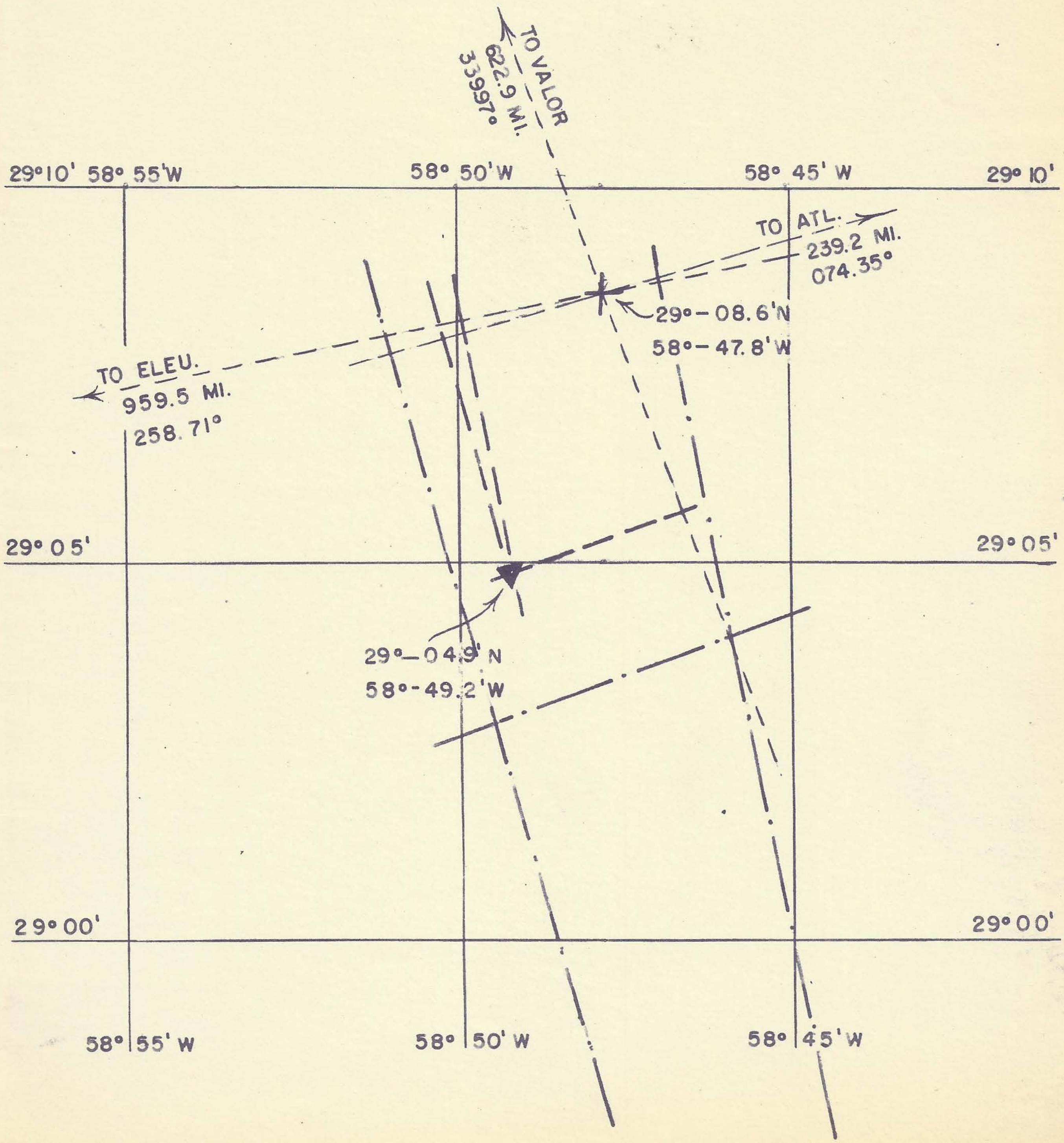


FIGURE 123

EVENT M410

- - - - - - BEARING LINE

—·—·— ISODISTANCE LINE AT .8040 M/S (4888.5 F/S)

— — — ISODISTANCE LINE AT .8015 M/S (4873.3 F/S)

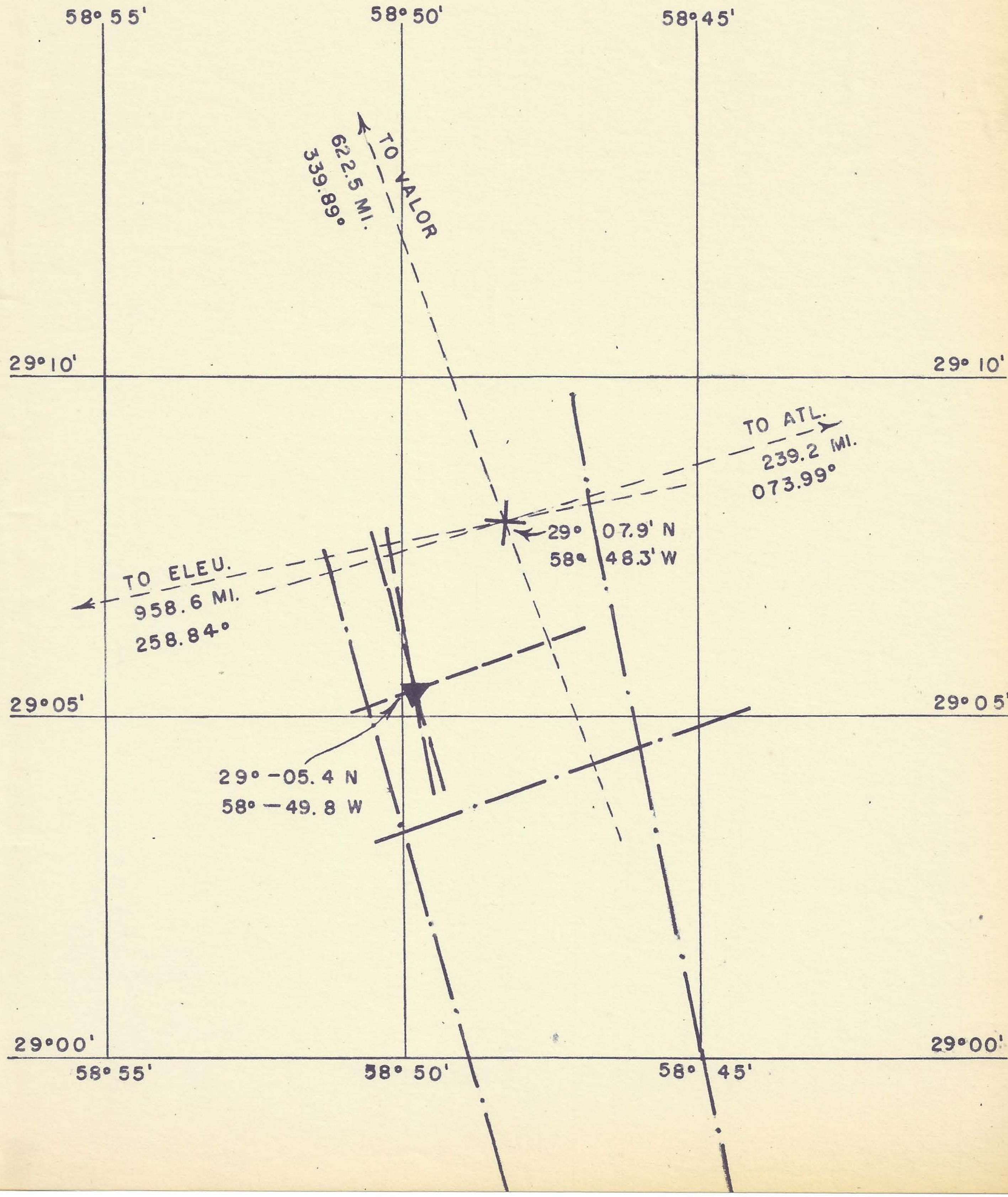


FIGURE 124
EVENT 412

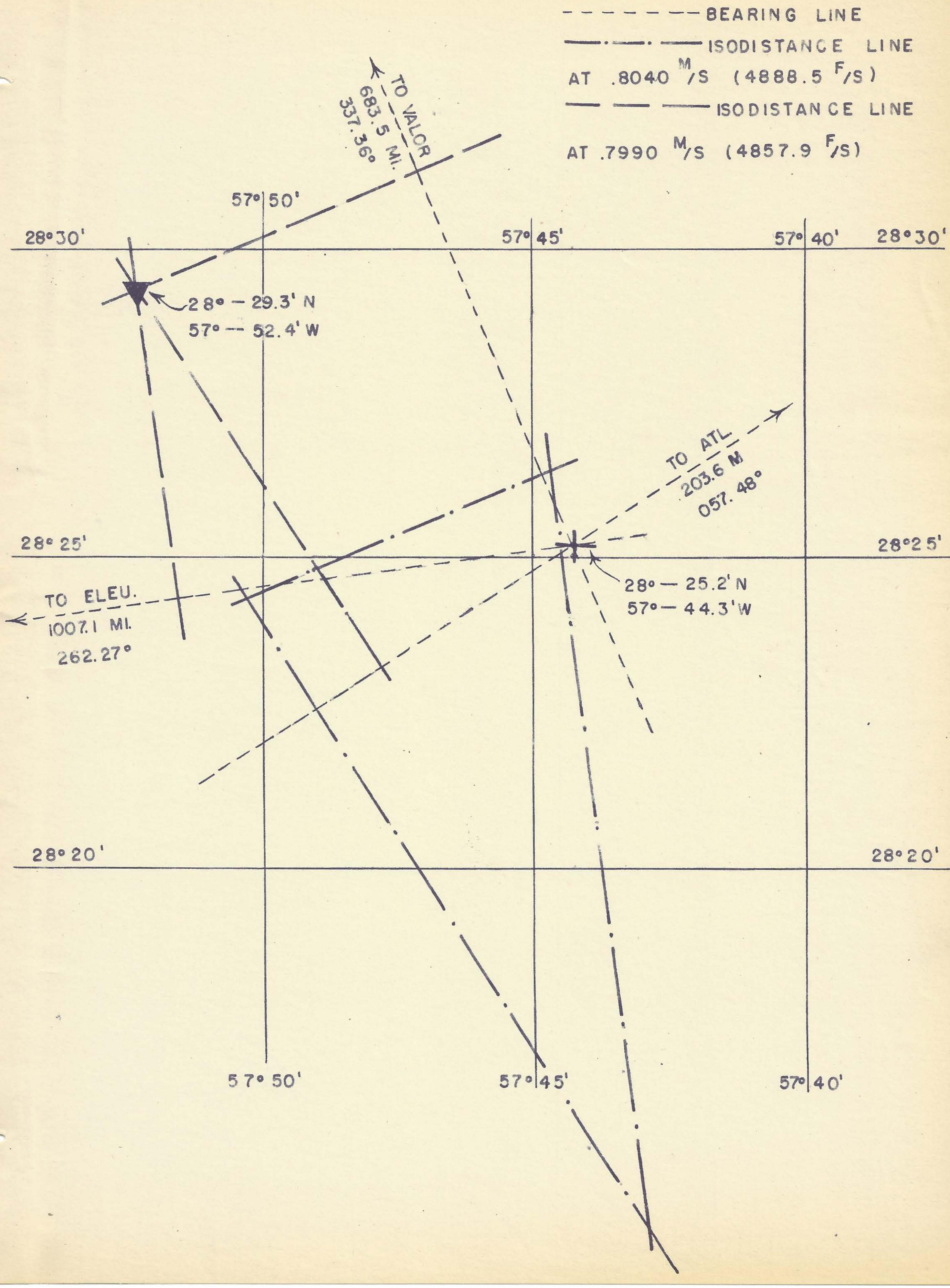
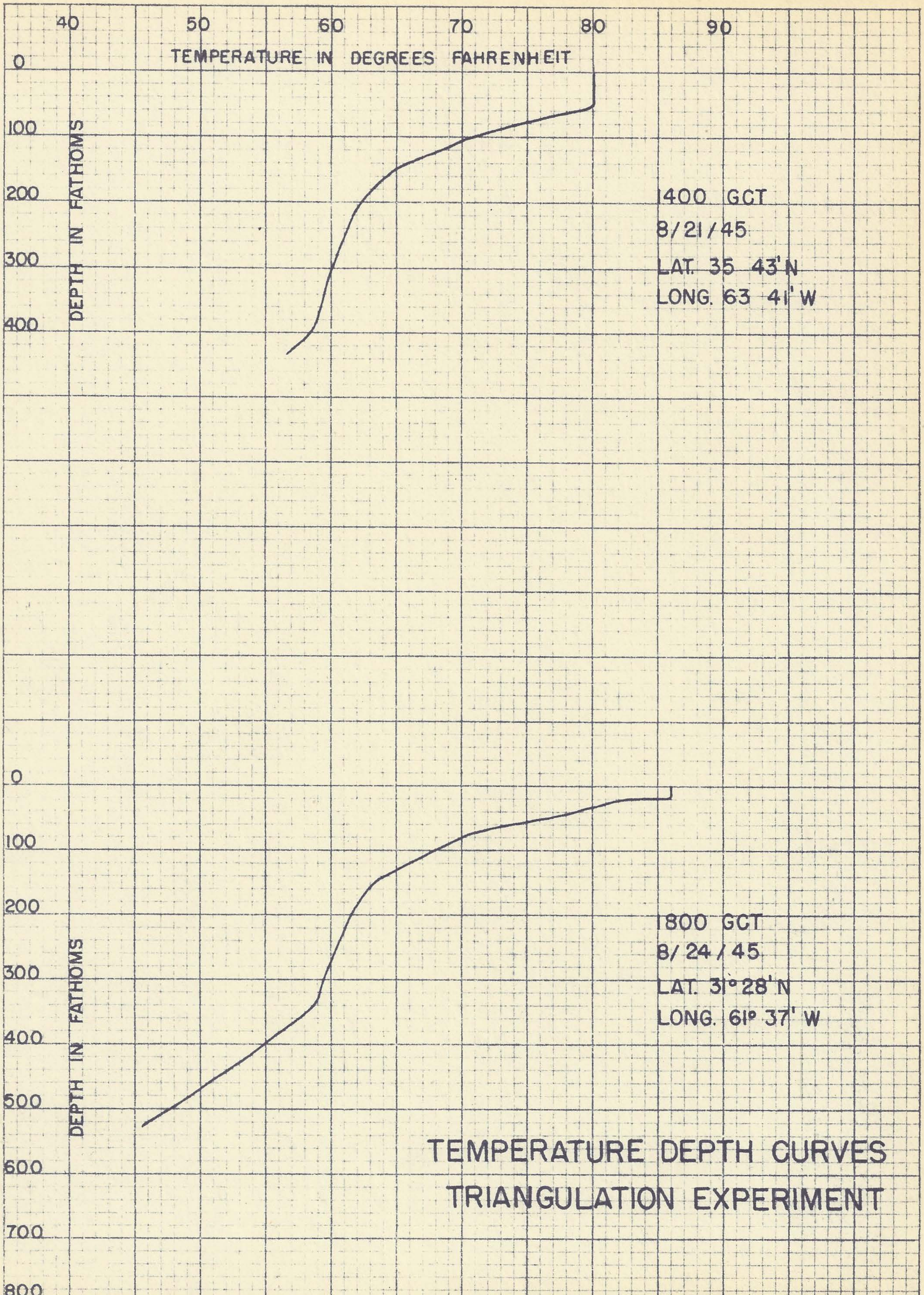


FIG.



TEMPERATURE DEPTH CURVES
TRIANGULATION EXPERIMENT

FIG.

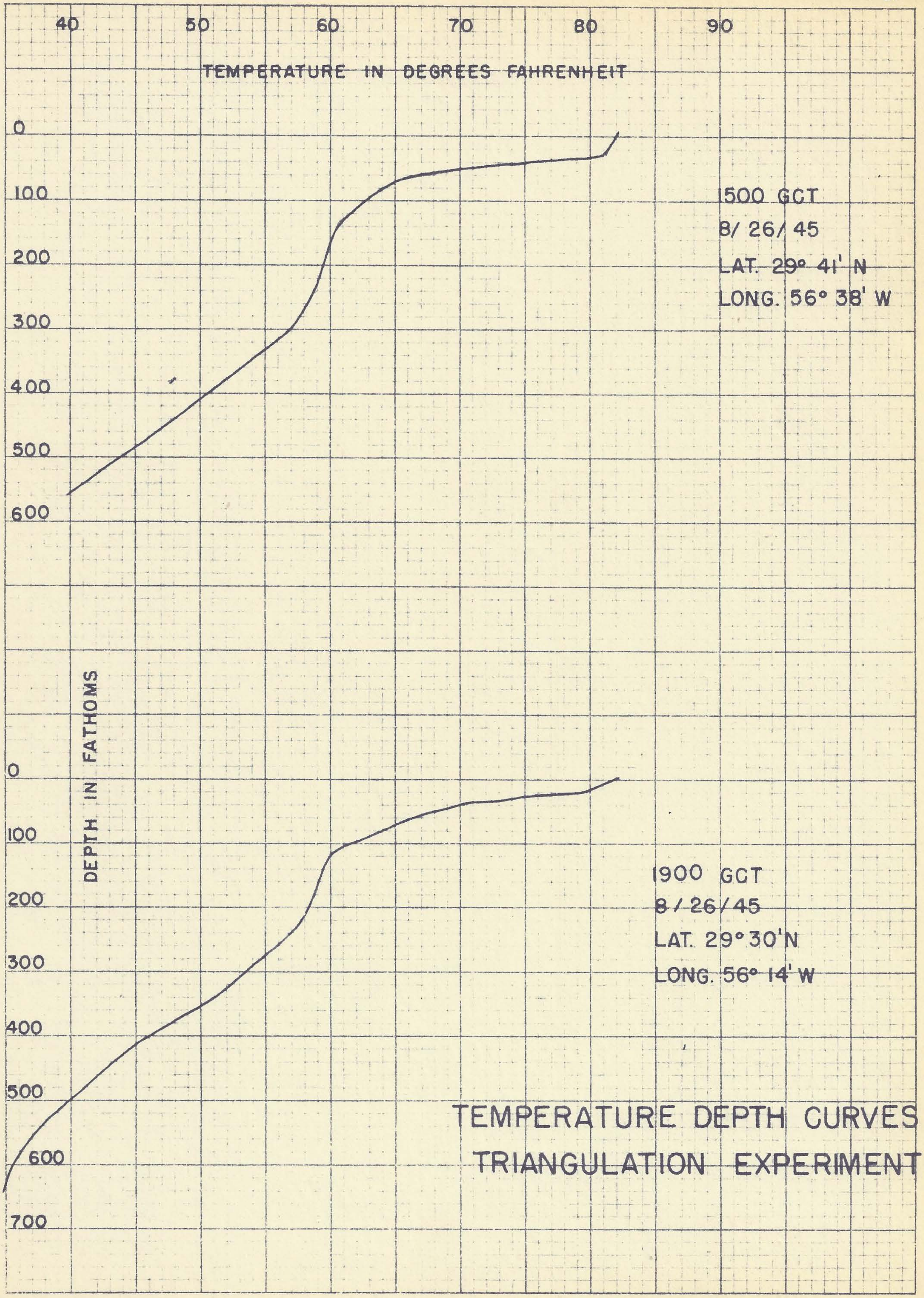
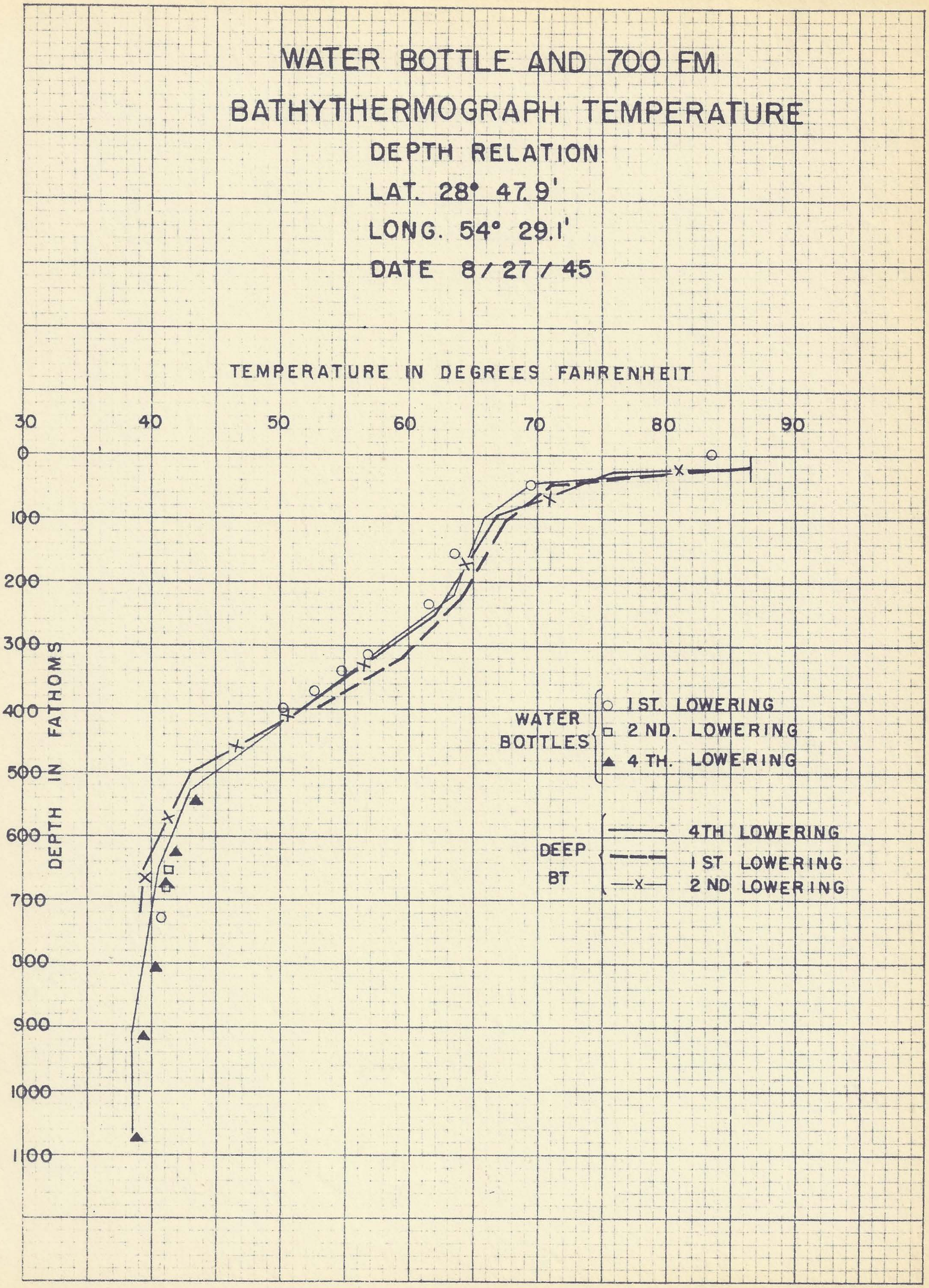

TEMPERATURE IN DEGREES FAHRENHEIT
40
50
60
70
80
90
DEPTH IN FATHOMS
0
100
200
300
400
500
600
700
1500 GCT
8/ 26/ 45
LAT. 29° 41' N
LONG. 56° 38' W
1900 GCT
8 / 26 / 45
LAT. 29° 30' N
LONG. 56° 14' W
TEMPERATURE DEPTH CURVES
TRIANGULATION EXPERIMENT

FIG.



WATER BOTTLE AND 700 FM.
BATHYTHERMOGRAPH TEMPERATURE
DEPTH RELATION
LAT. 28° 47.9'
LONG. 54° 29.1'
DATE 8/27/45
TEMPERATURE IN DEGREES FAHRENHEIT
30
40
50
60
70
80
90
0
100
200
300
400
500
600
700
800
900
1000
1100
DEPTH IN FATHOMS
WATER BOTTLES
1 ST. LOWERING
2 ND. LOWERING
4 TH. LOWERING
DEEP BT
4TH LOWERING
1 ST LOWERING
2 ND LOWERING